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Seismic response of multiblock structures with unilateral constraints

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ABSTRACT: this work, presents a method for the dynamic analysis of structures consisting of superimposed blocks. The hypotheses made are those of rigid blocks and monolateral Winkler-type deformable joints; an elastic joint made of separate infinitesimal springs placed side-by-side and non resistant to tensile stress is assumed to be between two adjacent blocks. By means of the total lagrangian approach the equation of motion, in the actual configuration, is obtained so that the coupling between the orizontal and the vertical equilibrium equations and the p-δ effect of the vertical forces are correctly taken into account. In order to best optimize the numerical procedures, an implicit integration method derived from the central difference method modified by a first order approximation method has been developed. Further, a time domain analysis for an existing column with a known horizontal acceleration assumed at the base, is reported.

1 INTRODUCTION

In the present work, a method for the dynamic analysis of structures consisting of superimposed blocks is presented. The hypotheses made are those of rigid blocks and monolateral Winkler-type deformable joints, so that the present discrete element model can be regarded as a particular rigid bodies-spring model by Kawai, (1978), generally abbreviated as RBSM.

In numerical analysis of the non linear behaviour of such block structures it is necessary to take into account the elastic deformation of each block and the separation-contact between adjacent blocks. Two basic approaches can be found in the literature: the Housner one deals with rigid blocks only, and entirely neglects elastic deformation; the second, derived from continuum mechanics, combines the finite element method with contact techniques such as the gap element method (Stadter and Weiss (1979)), the penalty method (Chaudhary and Bathe (1986)), or the lagrangian multiplier method (Hallquist et ... (1985)). In general, the latter approach, especially when the incremental formulation is used, is numerically unfavorable because of the excessive number of contact interfaces. On the other hand, the Housner approach is computationally quite simpler but binding assumptions have to be made in order to get a solution, and no information, about the stress and strain fields, can be obtained.

The method that simulates superimposed structures herein presented, assumes the contact interface between the blocks to be a Winkler-type monolateral bed. In other words an elastic joint made of separated infinitesimal springs placed side by side and non resistant to tensile stresses is assumed to be between two blocks. Obviously the spring constants represent the elastic property of the material of the two adjacent blocks. In the present discretization the springs play two different roles. They reproduce block elasticity and, at the same time, deal with contact-separation conditions. This approach is very similar to the RBS model first described by Kawai (1978) and, to some extent, recalls the method proposed by Blasi & Spinelli (1985), for superimposed blocks. In the following sections, using the total lagrangian approach, the incremental formulation of the equation of motion is obtained. Furthermore, a particular integration method is presented and some numerical examples showing the dynamic response of a column to seismic excitations is reported.

2 THE METHOD

As previously showed, contact between the blocks is ensured by joints made of infinitesimal springs, placed side by side incapable of resisting traction and each indipendent from one another. Further, the joints are considered to be rigid to shear deformation, (this ipothesis can be easily removed, see Ercolano Santorelli (1991)), so that an assembly of n superimposed blocks has 2n degrees of freedom. As unknowns, the horizontal displacements of the sections at the top of every single block and the vertical displacements of every baricenter were chosen. For a better understanding it is convenient to first write the equilibrium equations in the initial undeformed configuration..

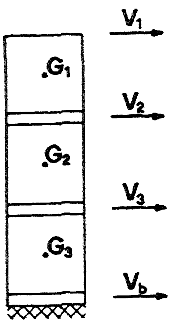

fig.1

The generic horizontal component of the absolute displacement of the baricenter v_{gi}, can be written as follows:

$$v_{gi} = v_b + \frac{h_i}{H_i} v_i + \frac{\bar{h}_i}{H_i} v_{i+1} \qquad (1)$$

where v_b is the horizontal displacement of the above mentioned base. The generic rotation θ_i is

$$\theta_i = (v_i - v_{i+1}) / H_i \qquad (2)$$

using matrix notation we obtain:

$$\mathbf{v}_g = \mathbf{v}_b + \mathbf{B}\,\mathbf{v} \qquad \theta = -\mathbf{H}^{-1}\,\mathbf{C}\,\mathbf{v} \qquad (3)$$

where $\mathbf{v}_b$ is the vector collecting the base displacement, $\mathbf{H}$ is the diagonal matrix of the block heights and $\mathbf{B}$ and $\mathbf{C}$ are matrices whose non-zero elements assume the following form:

$$\begin{array}{ll} B(i,i) = h_i / H_i & B(i,i+1) = \bar{h}_i / H_i \\ C(i,i) = 1 & C(i,i+1) = -1 \end{array} \qquad (4)$$

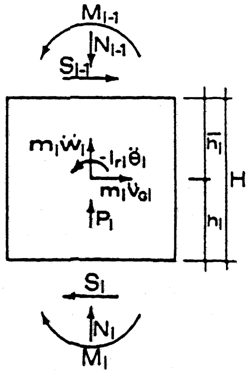

fig. 2

Let us now consider the generic rotation equilibrium equation around the baricentre of the i-th joint referred to all the overlying blocks:

$$\sum_{j=1}^{i} m_j \ddot{v}_{gj}\left(h_j + \sum_{k=j+1}^{i} H_k\right) - \sum_{j=1}^{i} I_{rj}\ddot{\theta}_j - M_i = 0 \qquad (5)$$

The system of the n type (5) equations can be written:

$$\mathbf{D\,M_D}\,\ddot{\mathbf{v}}_g - \mathbf{T\,I}\ddot{\boldsymbol{\theta}} - \mathbf{m} = 0 \qquad (6)$$

where $\mathbf{M_D}$ and $\mathbf{I_r}$ are the diagonal matrices of respectively, the masses and the rotational inertias; $\mathbf{m}$ is the vector of the bending moments.while $\mathbf{D}$ and $\mathbf{T}$ are low triangular matrices whose generic elements are unitary for $\mathbf{T}$ and assume the physical significance of distance between the i-th joint and the j-th block baricentre for $\mathbf{D}$. Substituting (3) into (6) we get:

$$\left(\mathbf{DM_DB} + \mathbf{T\,I_rH^{-1}C}\right)\ddot{\mathbf{v}} = \mathbf{m} - \mathbf{DM_D}\ddot{\mathbf{v}}_b \qquad (7)$$

Let us now examine the vertical equilibrium equation of all the blocks lying over the ith joint:

$$\sum_{k=1}^{i} -m_k\ddot{w}_k + p_K + N_i = 0 \qquad (8)$$

in matrix form:

$$\mathbf{M_T}\,\ddot{\mathbf{w}} = \mathbf{p}_{st} + \mathbf{n} \qquad (9)$$

where:

$$M_{T\,ij} = m_j \;:\; j \le i \;;\; p_{st\,i} = \sum_{k=1}^{i} p_K \;;\; n_i = N_i \qquad (10)$$

Observing the system of the equation (7) and (9) it is evident that $\mathbf{m}$ and $\mathbf{n}$ must be functions of both the relative displacements $\Delta\theta_i$ and Δw_i. For linear springs the following hold

$$N_i = \int_{A_{ri}} k_w[\Delta\theta_i\, y + \Delta w_i]\,dA \quad M_i = \int_{A_{ri}} k_w[\Delta\theta_i\, y + \Delta w_i]\,y\,dA \qquad (11)$$

where A_{ri} is the reacting área and k_{wi} is the spring constant. Equation (7) and (9) with positions:

$$\mathbf{M_o} = \mathbf{DM_D\,B} + \mathbf{T\,I_rH^{-1}C} \qquad \begin{array}{l} \mathbf{f}_v = \mathbf{m} - \mathbf{DM_D}\ddot{\mathbf{v}}_b \\ \mathbf{f}_w = \mathbf{p}_{st} + \mathbf{n} \end{array} \qquad (12)$$

can be conveniently written as follows:

$$\mathbf{M}\,\ddot{\mathbf{u}} = \mathbf{f} \qquad (13)$$

where

$$M = \begin{bmatrix} M_0 & 0 \\ 0 & M_T \end{bmatrix} \qquad u^t = (v, w) \qquad f^t = (f_v, f_w) \tag{14}$$

If (13) is written in the configuration characterized by the displacements **u** (6) becomes

$$[D + D_w]\, M_D\, \ddot{v}_g - T\, I\ddot{\vartheta} + E_v[p - M_D \ddot{w}] - m = 0 \tag{15}$$

where the non zero elements of $\mathbf{D_W}$ and $\mathbf{E_V}$ are the following:

$$D_{w\,ij} = \frac{1}{2}\Delta w_i + \sum_{k=j}^{i-1} \Delta w_K \,;\; E_{v\,ij} = v_{gi} + v_{i+1} \;:\; j \le i \tag{16}$$

(15), bearing (3) in mind, can be written as follows:

$$\begin{gathered}\left[(D + D_w)M_D\, B + T\, I_r H^{-1}\, C\right] \ddot{v} - E_v\, M_D \ddot{w} = \\ (D + D_w) M_D \ddot{v}_b + E_v\, p + m\end{gathered} \tag{17}$$

The vertical equilibrium equations become

$$S_v\, M_T \ddot{v}_g - M_T \ddot{w} + p_{st} + n = 0 \tag{18}$$

where the elements of the diagonal matrix $\mathbf{S_v}$ are defined as follows:

$$S_{v\,ii} = (\theta_i + \theta_{i+1}) / 2 \tag{19}$$

It is worthwile to remember that the shear vector **s** can be obtained by

$$s = -\, M_T \ddot{v}_g \tag{20}$$

Substituting (3) in (18) we get:

$$M_T \ddot{w} - S_v\, M_T B \ddot{v} = n + p_{st} + S_v\, M_T \ddot{v}_b \tag{21}$$

(17) and (21) can be written in the system form:

$$\left[M + \Delta M_u\right] \ddot{u} = f + \Delta f_u \tag{22}$$

where:

$$\Delta M_u = \begin{bmatrix} D_w M_D B & -E_v M_D \\ -S_v M_T B & 0 \end{bmatrix} \quad \Delta f_u = \begin{vmatrix} -D_w M_D \ddot{v}_b - E_v\, p \\ S_v\, M_T\, \ddot{v}_b \end{vmatrix} \tag{23}$$

Obviously:

$$\lim_{u \to 0} \Delta M_u = 0 \qquad \lim_{u \to 0} \Delta f_u = 0 \tag{24}$$

(22) can be integrated using any standard integration method. In this case, although we have a variation in both the mass and stiffness matrices, the most part of this variation lies in the second one because of the highly non linear constitutive relations. This does not suggest using an explicit integration method which needs matrix factorization at each step, but rather the central difference method, modified in order to avoid mass matrix factorization. Using time discretization

$$\left[M + \Delta M_u\right](u_{t-\Delta t} - 2u_t + u_{t+\Delta t}) = (f + \Delta f_u)\, \Delta t^2 \tag{25}$$

$$\left[M + \Delta M_u\right] u_{t+\Delta t} = \left[M + \Delta M_u\right](2u_t - u_{t-\Delta t}) + (f + \Delta f_u)\, \Delta t^2 \tag{26}$$

we finally get an equation of this kind:

$$\left[M + \Delta M_u\right] u_{t+\Delta t} = b_t \tag{27}$$

Premultiplyng (27) by $\mathbf{M}^{-1}$ we obtain:

$$\left[I + M^{-1} \Delta M_u\right] u_{t+\Delta t} = M^{-1} b_t \tag{28}$$

(28) is a typical structural reanalysis problem. It can be effectively solved using first order approximations or binomial series approximations, (see Phansalkar (1974) and Kirsch and Taye (1988)). First order approximation, in the Gauss-Seidel formulation is employed in the following.

Decomposing the matrix $\mathbf{M^{-1}\Delta M_u}$ into other three matrices; the lower triangular, the upper triangular and the diagonal

$$\begin{gathered} M^{-1} \Delta M_u = A = L + D + U \\ \text{with:} \quad L_{ij} = A_{ij} \;:\; j < i \,,\; D_{ii} = A_{ii} \,,\; U_{ij} = A_{ij} \;:\; j > i \end{gathered} \tag{29}$$

we obtain from (28),the following recursion:

$$[I + L + D]\, u^{i+1}_{t+\Delta t} = -\, U\, u^{i}_{t+\Delta t} + M^{-1} b_t \tag{30}$$

It can be easily shown how a necessary and sufficient condition for (30) to be convergent is

$$\lim_{k \to \infty} \left[(I + L + D)^{-1} U\right]^k = 0 \tag{31}$$

(31) holds if the spectral radius of $(I+L+D)^{-1}U$ is less than unity. In our case, because of the first order displacements ipothesys, this condition is, by far, ensured.

3 NUMERICAL EXAMPLE

Some results, were obtained analysing a three block column The column is 1407 cm. high and has a square section of 156 x 156 cm.. These dimensions were taken from the " Colonna di Foca " an existing column of the Imperial Forum in Rome. The Winkler coefficients were assumed to be equal to 533 kg/cm^3. and the total weight was estimated in 81000 kg.. A step by step analysis (the time step assumed was: $\Delta t = 10^{-4}$ sec.), was carried out in order to investigate the structural response. Two harmonic base accelerations with amplitude of 0.05g and 0.1g respectively were considered. Each analysis was carried out assuming a different frequency in the range 0.25-10 hz. and lasted 20 seconds. The maximum displacements at the top of each block are shown in figure 3.

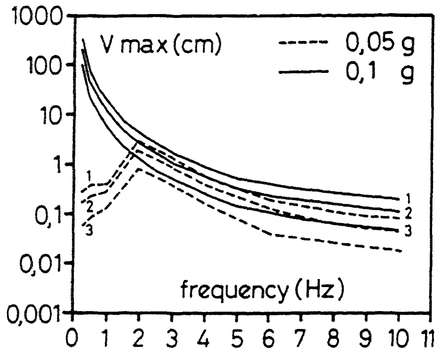

fig. 3

It is worthwile to notice that the maximum displacements , with g= 0.05 are reached at 2 hz.. while in the other case for frequencies below the 2 hz. the displacements are esponentially increasing.

4 CONCLUSIONS

The approach presented in the paper shows some interesting aspects. As said before, it is computationally less expensive than the others derived from FEM there is no discontinuity in the displacement derivatives and it is still possible to obtain some information on the stress field at the contact area, that the Housner approach.completely neglets.

Furthermore the monolateral bed can, to some extent, simulate the deformation of the two bodies in contact. Lastly, the numerical procedure, as shown in the example, can be easily applied to existing structures.

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