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Investigation of coupled lateral-torsional response in multistorey buildings

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ABSTRACT: The dynamic torsional behavior of an existing building that responded severely during service level earthquakes is presented in this paper. The building is a thirteen story "regular" space frame structure. The recorded responses of the building during different earthquakes were characterized by long duration, narrow banded motions with strong amplitude modulation; by large translational and torsional motions; by large amplification of the input ground motions; and by slow decay of the building's dynamic responses. Records are studied to obtain the building's dynamic properties and response envelopes. The causes for the severe response are identified from these studies. Three dimensional linear and nonlinear models of the building are developed to match the recorded response of the structure. Parametric studies are perform on the analytical models to study the effects of material and geometric nonlinearities, accidental eccentricities, bi-directional input ground motions and energy dissipation capacity in the response of the building. Results indicate that the severity of the torsional response in a eccentric multi-story structure is strongly influenced by the level of inelastic behavior, level of eccentricity, ground motion characteristics and the structure's energy dissipation capacity.

1 INTRODUCTION

The lateral-torsional coupled behavior of structures has been the subject of a large number of studies. Investigations of this behavior have usually been undertaken using highly simplified linear or nonlinear computer models. With the extensive installation of strong motion instruments in structures around the world, it has become possible to monitor the actual three dimensional behavior of buildings during earthquake events and to study lateral-torsional coupling in these structures under different conditions.

A structure that exhibited strong lateral-torsional coupling in its recorded response, among other response characteristics of interest, is an apparently regular thirteen story office building, located in San Jose, California, Fig. 1. This building is instrumented with twenty-two unidirectional strong motion accelerographs, positioned in five different levels (ground, 2, 7, 12 and roof) at the NW, SW and SE corners of the square frame plan (Lines B and 12, Fig. 1). The structural system consists of steel moment resistant space frames. A strong moment resisting frame, Lines 2-12-A-B, is located around a lighter moment resisting frame. A much lighter frame is located outside the strong frame, Lines 12-13 and A-B.

Several earthquake records have been obtained in this structure. The three most intense responses recorded to date are those obtained during the Morgan Hill earthquake of April 24, 1984 ($M_l = 6.2$), the Mt. Lewis earthquake of March 31, 1986 ($M_l = 5.8$) and the October 17, 1989 Loma Prieta earthquake ($M_l = 7.1$). Peak horizontal ground accelerations recorded for these events at the base of the building were 4, 4 and 11 % g, respectively. The building substantially amplified these base motions so that the maximum structural accelerations during the earthquakes were 17, 32 and 36 % g, respectively. Motions of the structure during all the earthquakes caused widespread damage to contents and disruption of services. The response records exhibit a strongly modulated pattern and locally indicate that the structure experienced substantial torsion, Figs. 2, 3 and 4. Another feature of the responses shown in these figures is that the structure continued to vibrate vigorously for more than 80 seconds. The input motion was much shorter in duration and maximum structural responses occurred generally long after the end of the strong motion portion of the base excitation.

2 RESPONSE OF SIMPLE ECCENTRIC FRAMES

Considerable insight on the effects of small eccentricities in nearly regular multi-story space frames can be obtained considering simple three degrees of freedom one story frame systems. The dynamic characteristics of a one story frame can be obtained by formulating the eigenvalue problem at the structure's center of mass (CM) in terms of the total global stiffness at the center of stiffness (CS) and the distance between these centers as follows:

$$\left[\begin{bmatrix} K_x & -K_x e_y & 0 \\ -K_x e_y & (K_\theta + K_x e_y^2 + K_y e_x^2) & K_y e_x \\ 0 & K_y e_x & K_y \end{bmatrix} - m w_n^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right] \begin{bmatrix} v_{xn} \\ v_{\theta n} \\ v_{yn} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

where: K_x is the story translational stiffness in the X direction at the center of stiffness, K_y is the story translational stiffness in the Y direction at the center of stiffness, K_θ is the story torsional stiffness at the center of stiffness, r is the story radius of gyration of the plan, e_x is the distance from the center of mass to the center of stiffness in the global X direction, e_y is the distance from the center of mass to the center of stiffness in the global Y direction, w_n is the natural frequency form mode "n" and $n = 1, 2, 3$, v_{xn}, $v_{\theta n}$, and v_{yn} are the translational and rotational components of mode n.

The general eigenvalue problem presented here cannot be solved in closed form because of its cubic characteristic equation. Nevertheless, this equation can be solved for the special case of identical translational stiffness in orthogonal directions. Then $K_x = K_y = K$. The solution of this special problem, with a single translational stiffness K, will provide insight into the more general problem.

The eigenvalue solution for this system can be expressed as:

$$\Omega_{1 \text{ or } 3} = \frac{1}{2}\left\{\left(1 + \left(\frac{e_o}{r}\right)^2 + \left(\frac{e}{r}\right)^2\right) \mp \left[\left(1 + \left(\frac{e_o}{r}\right)^2 + \left(\frac{e}{r}\right)^2\right)^2 - 4\left(\frac{e_o}{r}\right)^2\right]^{\frac{1}{2}}\right\}$$

$$\Omega_2 = 1 \tag{2}$$

where: $e^2 = e_x^2 + e_y^2$ is a measure of global eccentricity, $\Omega_n = (w_n/w)^2$ is ratio of coupled to uncoupled frequencies, $w = K/m$, and $e_o^2 = K_\theta/K$ is the ratio of torsional to translational stiffness at the center of stiffness. (Note that $(e_o/r)^2$ is equal to the ratio of translational to torsional periods of an uncouple system.)

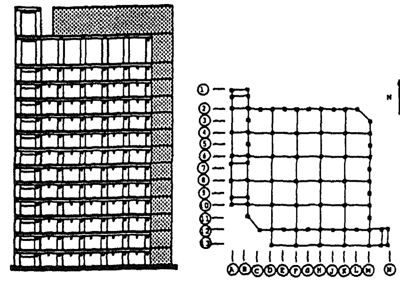

Figure 1: Building plan and framing.

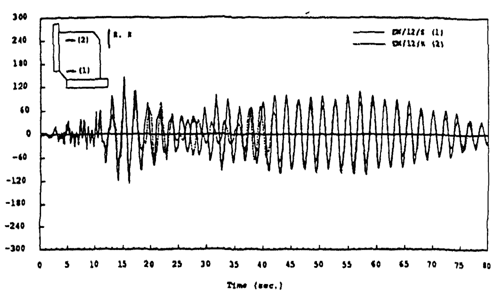

Figure 2: Twelfth floor acceleration records. EW direction. Mt. Lewis earthquake.

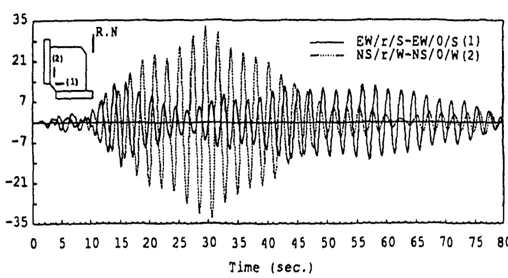

Figure 3: Roof SW corner relative displacements. Mt. Lewis earthquake.

After some numerical manipulation it can be shown that:

$$(1 - \Omega_1)(1 - \Omega_3) = -\left(\frac{e}{r}\right)^2 \tag{3}$$

These equations indicate that for similar uncoupled torsional and translational periods (values of e_o/r close to one), and small static eccentricities, the three coupled natural periods of the system could be extremely close.

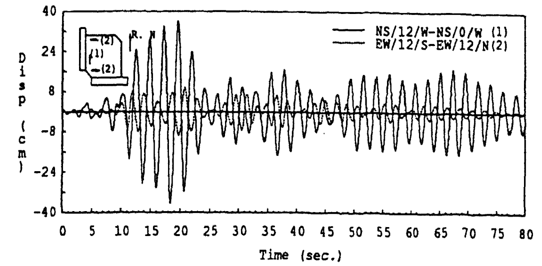

Figure 4: Twelfth floor NS relative displacements (SW corner) and torsion (EW records). Loma Prieta earthquake.

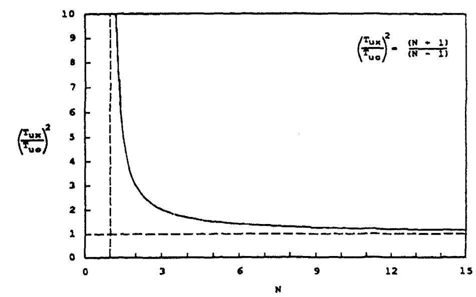

Figure 5: Ratio of uncoupled translational and torsional periods for a regular one story structure.

The mode shapes that correspond to this eigenvalue solution have the following form if $e_x \neq 0$ and $e_y \neq 0$:

$$\Phi = \begin{bmatrix} \frac{e_y/r}{1-\Omega_1} & \frac{e_x}{e} & \frac{e_y/r}{1-\Omega_3} \\ 1 & 0 & 1 \\ \frac{-e_x/r}{1-\Omega_1} & \frac{e_y}{e} & \frac{-e_x/r}{1-\Omega_3} \end{bmatrix} \tag{4}$$

Then the mode that corresponds to Ω_2 (=1) is a pure translational mode; the predominant direction of the mode is skewed relative to the reference axes, in accordance with the global eccentricity.

Close torsional and translational uncoupled periods are typically observed in systems that have uniform distribution of stiffness in plan (Newmark (1969)). For regular space frame structures these periods can then be quite close. It can be shown that for one story frames or multi-story frames with only three degrees of freedom per story (chain system) the ratio of translational to torsional uncoupled periods can be obtained by Equation 5 if the system has the following characteristics: a) multiple columns evenly distributed, b) uniform distribution of mass, c) coincident center of mass and stiffness, d) all columns with the same stiffness in a given direction, e) a shear behavior with three degrees of freedom per story (two horizontal translations and one in-plane rotation at

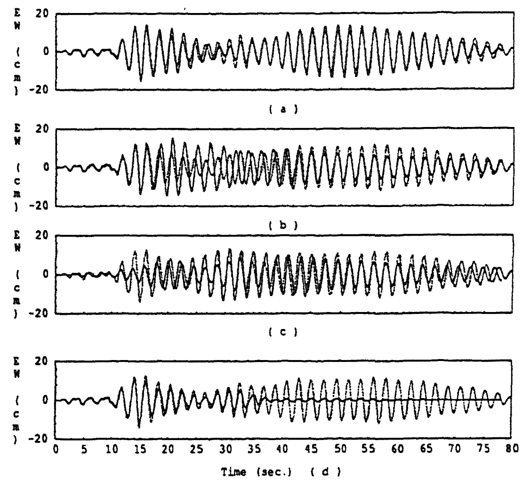

Figure 6: Twelfth floor motion Mt. Lewis event. Input ground motion 0-40 seconds. EW relative displacement, SW corner. a) First three modes, low damping model (1 %). b) First mode, low damping model. c) Second mode, low damping model. d) First three modes, moderate damping model (5 %). Model —— Record ······.

center of mass), d) all vertical elements with negligible torsional stiffness, and e) radio of gyration defined in terms of the external column position and distributed mass.

$$\left(\frac{T_{ux}}{T_{u\theta}}\right)^2 = \frac{[(\frac{d_x}{d_y})^2(\frac{k_y}{k_x})(N_x{}^2 - 1) + (N_y{}^2 - 1)]}{(N_x - 1)^2(\frac{d_x}{d_y})^2 + (N_y - 1)^2} \tag{5}$$

where: $N_{x\ or\ y}$ is the number of column lines in the x or y direction, $d_{x\ or\ y}$ is the spacing between two consecutive columns in the X or Y direction, $k_{x\ or\ y}$ is the stiffness of a individual column in the X or Y direction, T_{ux} is the uncoupled translational period in the X direction, and $T_{u\theta}$ is the uncoupled torsional period.

For the special case of a square building with equal structural systems in both directions, this formula can be simplified as follows:

$N_x = N_y = N$, $d_x = d_y$ and $k_x = k_y$,

so that

$$\left(\frac{T_{ux}}{T_{u\theta}}\right)^2 = \frac{(N+1)}{(N-1)} \tag{6}$$

It can be seen in Fig. 5 that this ratio quickly approaches one as the number of columns increases.

For regular frames with an even distribution stiffness in plan and small eccentricities the following assumption can be made $(e/r)^2 \ll 1$ and $(e_o/r)^2 \approx 1$. So Equations 2 and 4 can be approximated by:

$$\Omega_1 \approx 1 - \frac{e}{r}; \quad \Omega_2 = 1 \text{ and } \Omega_3 \approx 1 + \frac{e}{r} \tag{7}$$

and

$$\Phi = \begin{bmatrix} \frac{e_y}{e\sqrt{2}} & \frac{e_x}{e} & -\frac{e_y}{e\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{e_x}{e\sqrt{2}} & \frac{e_y}{e} & \frac{e_x}{e\sqrt{2}} \end{bmatrix} = [\Phi_1 \;\; \Phi_2 \;\; \Phi_3] \tag{8}$$

where: the mode shapes are normalized so that $\Phi_i^t \Phi_i = 1$, (see also Kelly (1990)).

Finally, the coupled natural frequencies of the system can be found from Equation 7 making use again of the assumption of small eccentricities:

$$w_1 \approx w\left(1 - \frac{1}{2}\frac{e}{r}\right); \; w_2 = w; \; w_3 \approx w\left(1 + \frac{1}{2}\frac{e}{r}\right) \tag{9}$$

The closeness of the predominant periods and the three dimensional characteristics of the modes shape will produce responses that can increase substantially the severity of the linear response of the coupled system (see Boroschek (1991)). Also the time history responses of this systems are strongly modulated (beating behavior).

3 BUILDING RECORDED RESPONSE

The recorded response of the San Jose building have been studied extensively, for example, Boroschek and Mahin (1989; 1990; 1990; 1991). Basic dynamic properties and response envelopes were obtained directly from the records. The dynamic properties of the building are presented in Table 1. The maximum response envelopes obtained during the three earthquakes studies are presented in Table 2. Figures 2 through 4 show some time histories obtained in the building.

It can be concluded, from the analysis of the response records, that the building presents a rather flexible structural system with relatively low damping. The predominant period was found to be near 2.2 seconds and modal damping is believed to be below 3% of critical. Because of the similar frame structural characteristics in both directions and the even distribution of stiffness in plan, the predominant periods of the system are quite close. The closeness of the periods together with small eccentricities present in the structure produced the strongly coupled lateral-torsional behavior observed in the records. Because of the spatial characteristics of the frame, the coupling affects both directions and the rotation for most of the modes studied. The eccentricity that produce the torsionally coupled response can be associated with the irregular distribution of the mass and framing irregularities caused by a greater number of structural and nonstructural elements on the west and south sides of the building.

Table 1: Natural periods and damping.

Predominant Direction	Mode	Period (sec)	Damping[a] (%)
EW	First	2.15-2.20	2-3
NS	Second	2.05-2.10	2-4
Torsion	Third	1.70	-
EW	Fourth	0.65-0.75	-
NS	Fifth	0.60-0.70	-

(*a*) Values based on the appearance of response wave forms.

Table 2: Response maxima.

Response Values	MH 1984	ML 1986	LP 1989
H Base Peak Rec. Acc. (g)	0.04	0.04	0.10
V Base Peak Rec. Acc. (g)	0.02	0.02	0.11
Max Str. Acc. (g)	0.17	0.32	0.34
Max Str. Amp.[a]	4.87	7.05	3.84
Max Str. Drift (cm)[b]	18.64	33.19	38.17
Max Torsional Disp (cm)[c]	7.28	12.22	12.32
Max Drift Index[b]	0.41	0.72	0.85
Input Duration EW (sec.)	55	27	28
Input Duration NS (sec.)	56	32	35
Base Shear EW (V/W)	0.08	0.06	0.18
Base Shear NS (V/W)	0.09	0.15	0.17

(*a*) Defined as the ratio of the peak acceleration at a location to the corresponding acceleration at the ground. (*b*) At recording position. (*c*) Maximum difference between recordings at same building side.

The structure responded strongly to these relatively minor earthquakes. It is believed that the intensity of the structural response was caused by the building's relatively low damping, the three-dimensional modes of the building constructively reinforcing one another during portions of the motion, the input duration, the possible resonance effect on the building caused by the close match of the dynamic characteristics of the site and the structure and the relatively large flexibility of the structure.

4 ANALYTICAL MODELING AND PARAMETRIC STUDIES

Three-dimensional linear and non-linear numerical models of the complete structure were developed to simulate the recorded responses and to perform parametric studies. Static and dynamic analyses were performed. The dynamic analyses consider unidirectional as well as bi-directional input motions with and without torsional input excitations. Several parameters were monitored during the analyses: maximum displacements, interstory drifts, base shear, maximum ductility demands, maximum cumulative ductility and element forces. The final model had 2418 elements.

4.1 *Linear models*

The model was developed using information from building plans and site inspections. A good match was obtained when the models included the center-to-center member dimensions (i.e., no rigid panels were included to model the flexibility of the beam-column joints), mass magnitude and distribution as estimated from structural plans, nominal element properties, and a modal damping ratio typically associated with steel structures responding in the linear range (1-3% of critical). By further adjusting the actual mass distribution and incorporating the deck contribution to the beam stiffness, computed global results were virtually identical to recorded values.

From analyses of different loading conditions it was concluded that in order to reproduce the building's response, both horizontal components of the ground records should be included. Bi-directional effects accounted for nearly 22% of the response in orthogonal directions. Torsional input motion had a small effect on the overall response of the structure.

The damping ratio did have an important effect on the response of the models. Because of the rapid fluctuations of spectral accelerations with periods present in lightly damped systems, the response of the low damping models used in the study were very sensitive to modeling uncertainties that influence the period estimates.

Also the models showed a strong sensitivity to the position of floor center of mass. Increasing the eccentricity by 5% of the building's largest plan dimension reduced displacements and shears in both directions by a maximum of 36% and increased floor rotations by nearly 144% and base torque by 160%. The ratio of maximum base torque to maximum base shear was increased by 180% when this additional eccentricity was included.

The modal coupling was quite sensitive to design model parameters. Small changes of stiffness in one direction reduce the coupling, in some modal components, by nearly 75%. This demonstrates the difficulty in reproducing the coupled behavior of the structure.

The linear models confirmed that the severity of the response was caused in part by the modal interaction of the three dimensional structural modes. Figure 6, for example, shows the response of a model with relatively low damping to the first 40 seconds of the Mt. Lewis earthquake. In part (a) of the figure the response of the analytical model with 1% viscous damping is compared with the recorded motion. Parts (b) and (c) of the figure show the first and second mode contribution to the displacement in the EW direction. Here it can be seen that the first and second modes individually have lightly attenuated responses after about 30 seconds of motion. However, the two modes go in and out of phase, resulting in constructive and destructive interference that produces a large dip in the combined response at second 30 and an increase in response up to second 60.

An analysis was also performed considering 5% viscous damping. Here (Fig. 6d), the response of the individual modes attenuate so quickly that virtually no beating under free vibration can occur and little significant motion occurs after 35 seconds.

4.2 *Nonlinear models*

The building studied did not suffer significant inelastic behavior, so the nonlinear characteristics of the models were not fit to any of the observed responses. Nevertheless the nonlinear model was used to study the effect of nonlinearities, additional eccentricities, damping and ground motion characteristics in the global response of the system. The elements developed by Riahi et al (1978) were used in the analysis. These elements are three dimensional beam-columns with a multidimensional interaction yield surface (P, M_y, M_x, M_z).

Initially a nearly triangular static lateral loading was applied to the structure. The load deformation curve showed that, due to the pattern of yielding, torsional rotations can grow more rapidly than the displacements at the center of mass in the direction of loading, Figs. 7 and 8. Nevertheless, after severe yielding of the system has occurred the displacement grows much faster than torsional rotations (energy dissipating mechanism is mainly translational). In other words, apparent coupling between translations and rotations in a multi-story structure is highly dependable on the story of loading, inelastic distribution and level of inelastic behavior.

For the dynamic studies five earthquake records were considered: the recorded base building records during the Morgan Hill (1984), Mt. Lewis (1986) and Loma Prieta (1989) events (because of their period characteristics these are considered as a soft site records), as well as the Mexico SCT (1985) and El Centro (1940) events. Both horizontal components of these earthquakes were used in all the analyses. The records were scaled to different values of effective peak acceleration to obtain the response of the

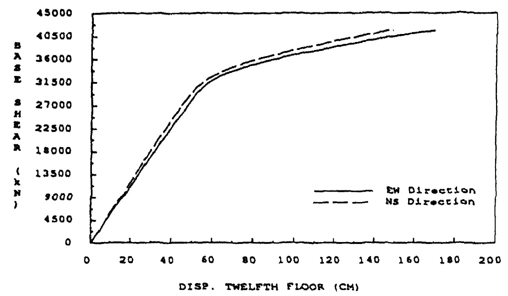

Figure 7: Base shear-twelfth floor displacement. EW and NS directions.

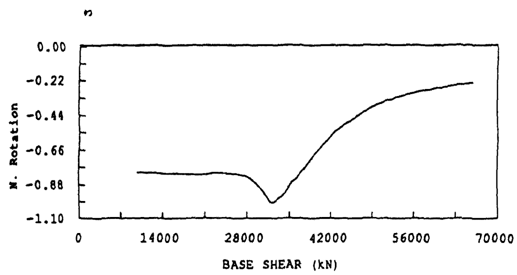

Figure 8: Normalized twelfth floor torsional rotations (θ/Δ_{EW}) at Center of Mass. Static-to-collapse analysis for the EW direction.

structure at different levels of inelastic behavior.

Analyses using different ground motions indicate that the properties of the input motion have a strong effect on the response of the models. These differences are more pronounced for elastic than for inelastic responses.

It was found that the ratio of in-plane torsional rotations to lateral displacements ("apparent coupling") could increase or decrease depending on the level of inelastic behavior and the characteristics of the input ground motion. Nevertheless, some scatter was found from the results. This indicates that more analyses are needed to identify a trend on the response and its relation to the observations regarding decoupling of torsional and lateral motions made on the basis of the static load to collapse studies and the effect of input motion predominant direction observed from the recorded torsional response of the building and simple linear models studied by Boroschek (1991).

Results from the analyses that considered added mass eccentricities indicate that, contrary to what was found for models subjected to unidirectional inputs, displacements at center of mass (or at a fixed point on the story plan) could decrease or increase depending on the building's characteristics and the properties of the input ground motion. An increase in eccentricity, from 0 to 10% of the maximum building dimension, had the effect of increasing the maximum rotational ductility demand on the strong frame girders (maximum increment was 22%) and reducing the cumulative ductility demand on these same elements (maximum reduction was 58%). On the other hand, for the elastic models, the variation of moment ratios for the strong frame girders increased (up to 40%) or decreased (25%) for the different ground motions studied. Similar trends were observed for the inter-story drifts.

Also, the addition of this eccentricity had the effect of redistributing inelastic demands between orthogonal directions. In some cases the maximum ductility demand for a given earthquake changed from one direction to the orthogonal direction, when the additional eccentricity was included.

5 CONCLUDING REMARKS

The results of this investigation agree with findings from the analyses of simple structures developed in other investigations. In general, the existence of torsional behavior in nearly regular space frames has the effect of increasing the stress or ductility demands in elements located far away from the center of rotation and changes the maximum translational displacements. These effects are more severe for elastic structures than inelastic structures and are highly dependent on the characteristics of the input ground motion.

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