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A proposed simple model for the study of seismic inelastic torsional coupling

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ABSTRACT: A simple general purpose model has been proposed for the study of seismic inelastic torsional coupling. The building code indirect influence in lessening torsional redundancy, as well as the necessity to satisfactorily capture the non-linear inelastic behaviour of torsionally coupled structures, have dictated the minimal characteristics required of this model. A simple model to conservatively estimate structural response is an acceptable alternative considering the stringent and impractical geometric and parametric requirements needed if identical structural response is to be ensured in the non-linear inelastic range.

1. INTRODUCTION

The considerable difference in the seismic dynamic response between symmetric structures and structures with stiffness and/or mass eccentricities in plan has long been recognized. Whenever the centres of rigidity and/or strength of a structure do not coincide with its centres of mass, translation in horizontal directions will be accompanied by torsional movement in plan, whether or not there is rotation in the ground motion.

It is well established that equations of motion of torsionally coupled structures are generally amenable to a format for which all structures sharing the same values for a few key parameters encapsulating their structural characteristics, irrespectively of their geometry, will share the same linear elastic response at a given reference point. Unfortunately, the same cannot be said of non-linear inelastic structures; a complex interdependence of the number, location, and hysteretic characteristics of structural elements directly impact on the behaviour of these structures.

A wide variety of structural models and configurations have been recently used to investigate the inelastic response of torsionally coupled structures. Some researchers replaced the physical structure by a single-element model having an interaction surface mapping the shear-torsion space (Kan and Chopra 1979). Others studied the behaviour of a unisymmetric structure having four identical columns of circular cross-section under bi-directional earthquake excitations (Tso and Sadek 1984). More frequently, the behaviour of monosymmetric structures having two or three lateral load resisting structural elements (LLRSE) sharing identical yield displacements and bilinear hysteretic model under unidirectional earthquake excitations was studied (Syamal and Pekau 1985, Bozorgnia and Tso 1986, Bruneau and Mahin 1990, among many). Resulting observations on the effect of various parameters on the inelastic response of torsionally coupled structures have generally not been in agreement. This can be partly attributed to the diverse analytical assumptions and approaches that were adopted in each study.

Consequently, a simple general purpose model for the study of seismic inelastic torsional coupling is needed, and some aspects of this problems are reviewed herein.

2. EQUATIONS OF MOTION

The general equations of motion around the centre of mass for single-story torsionally coupled structures are well known and have been derived by others (Kan and Chopra 1976). For monosymmetric structures (i.e. structures having one axis of symmetry) and neglecting torsional seismic excitation, the equations along the y-axis (axis of symmetry) are decoupled, and the resulting coupled translational-torsional equations of motion are simplified to:

$$\begin{bmatrix} m & 0 \\ 0 & mr^2 \end{bmatrix}\begin{bmatrix} \ddot{v}_x \\ \ddot{v}_\theta \end{bmatrix} + \begin{bmatrix} K_x & -K_x e \\ -K_x e & K_\theta \end{bmatrix}\begin{bmatrix} v_x \\ v_\theta \end{bmatrix} = \begin{bmatrix} -m\ddot{v}_{gx} \\ 0 \end{bmatrix} \qquad (1)$$

and, equivalently,

$$\begin{bmatrix} \ddot{v}_x \\ r\ddot{v}_\theta \end{bmatrix} + \omega_x^2 \begin{bmatrix} 1 & -e/r \\ -e/r & \Omega^2 \end{bmatrix}\begin{bmatrix} v_x \\ rv_\theta \end{bmatrix} = \begin{bmatrix} -\ddot{v}_g \\ 0 \end{bmatrix} \qquad (2)$$

with

$$\Omega = \omega_\theta / \omega_x = T_x / T_\theta \qquad (3)$$

$$\omega_x^2 = K_x / m \qquad (4)$$

$$\omega_\theta^2 = K_\theta / mr^2 \qquad (5)$$

where $\mathbf{K}_x$ and $\mathbf{K}_\theta$ are the system's translational (along X) and rotational (around θ) stiffness for the resulting two degrees-of freedom system, and **e** is the static eccentricity of this system. The mass of the floor is **m**, its radius of gyration **r**, $\mathbf{v}_x$ and $\ddot{\mathbf{v}}_x$ are the translation displacement and acceleration of the centre of mass in direction **x**, $\mathbf{v}_\theta$ and $\ddot{\mathbf{v}}_\theta$ are the rotational displacement and acceleration of the floor around a vertical axis, and $\ddot{\mathbf{v}}_{gx}$ is the ground acceleration in direction in direction **x**. The torsional stiffness of individual lateral load resisting elements is neglected.

Obviously, from Equation (2), in the linear elastic domain, all structures sharing the same (**e/r**), ω_x and Ω will have the same response v_x and rv_θ at their centre of mass. As will be demonstrated later, this does not hold in the non-linear inelastic domain.

3. BUILDING CODES' INDIRECT INFLUENCE

It is noteworthy that many current building codes indirectly promote the reduction of plan redundancy. This is illustrated in the short example following.

In an apparently symmetric building, the accidental eccentricity provision mandated by the equivalent static seismic lateral force design method of most building codes provides a minimum design eccentricity which is thought to account for uncertainties in mechanical properties, mass distribution, and ground motion. This accidental eccentricity is usually set by different codes to a small percentage of the maximum plan dimension.

For a monosymmetric structures, should only two lateral load resisting elements be present in the principal direction (Figure 1a), an accidental eccentricity of 5% of the maximum plan dimensions will increase the design forces in each element by 10%. Consequently, the design translational and torsional strength will also increase by 10%. If, instead, four equally spaced elements with equal stiffness are now considered (Figure 1b), the same accidental eccentricity requirements will increase the design forces by 18% for the edge elements and by 6% for the inside elements. The resulting design translational strength is thereby increased 12% and the net torsional strength in increased 17%, and, therefore, the more redundant structure is only achieved at a premium in material and labour. Consequently, strict adherence to building codes' seismic provisions could make the two element system a more economical design alternative, which is apparently discordant with earthquake engineering's traditional wisdom that redundancy improves the ultimate seismic resistance of structures.

This least cost observation suggests that a two LLRSE monosymmetric model is realistic. Hence, one such model is proposed for the study of seismic inelastic torsional coupling. For the chosen model illustrated in Figure 2, all floor diaphragms are assumed to be infinitely rigid in their own plane, elements in the orthogonal direction are ignored for the sake of simplicity and lateral load resisting elements are assumed to be equidistant from the centre of mass. Nonetheless, this simple general purpose model of minimal complexity must be able to capture the essence of the particular behavioral features of seismic inelastic torsional coupling in order to be acceptable. This remains to be verified.

4. CASE COMPARISON OF STATIC, DYNAMIC ELASTIC AND INELASTIC RESPONSES

In an ideal design process, where the engineer has unrestrained freedom on the structural layout and dimensioning, the LLRSEs can be proportioned such that the centre of resistance will coincide with the centre of mass (unless the centre of mass is not contained between the resisting elements, as would be the case for a building with a single eccentric core). In the special case where only two structural elements are provided for the lateral resistance system in the **x** direction, the resulting structure is statically determinate, and, in consequence, the lateral shear force must be distributed to the elements solely by the laws of equilibrium. In this case, a static lateral force applied at the centre of mass will be distributed to the lateral load resisting elements by geometric relations, and independently of the LLRSEs stiffness. Unfortunately, this may not always be possible, either due to imposed architectural constraints or to the detrimental structural consequences of added non-structural components ignored during conception, like for example, the addition of a non-structural masonry infill in a steel frame structure.

Assessments of the consequence of a larger than anticipated stiffness can be misleading if performed by traditional linear elastic analysis methods. Evaluation of the changed condition by static analysis, as shown in Figure 3, reveals that the displacements, while dramatically different than for the previous symmetric state, are now of equal or lesser magnitude, indicating that the design remains safe. The fallacy of this perception can be exposed by the simple example following where linear elastic and non-linear inelastic analyses were conducted for the same structure having two LLRSEs.

For these analyses, the N-S component of the 1940 El Centro earthquake record is scaled such that the symmetric two-element system reaches a ductility of exactly four from an inelastic step-by-step dynamic analysis. Then, the stiffness and strength of one of the elements is increased by 50% as on Figure 3. Elements are modeled as bi-linear hysteretic with 0.5% strain hardening. The results for both the elastic and inelastic analyses are presented in Figure 4. These results demonstrate the large amplification of edge displacement produced by the inelastic torsional coupling of the structure.

The large corresponding ductility demand on the weaker element obtained can not be predicted from either the static analysis or the elastic dynamic step-by-step analysis. This increase in ductility demand can be explained by examining the instantaneous state of the equation of motion.

5. INELASTIC STATUS OF EQUATIONS OF MOTION

During response to an earthquake excitation, the initiation of yielding in one of the LLRSEs will affect the instantaneous properties of the physical system, such that the equations of motion for the monosymmetric structure can be re-written as:

$$\begin{bmatrix} m & o \\ o & mr^2 \end{bmatrix} \begin{bmatrix} \ddot{v}_x \\ \ddot{v}_\theta \end{bmatrix} + \begin{bmatrix} K_x' & K_x' e' \\ K_x' e' & K_\theta' \end{bmatrix} \begin{bmatrix} v_x \\ v_\theta \end{bmatrix} = \begin{bmatrix} -m\ddot{v}_{gx} \\ 0 \end{bmatrix} \quad (6)$$

where all variables remain as previously defined with the exception that the primes (') are used to represent instantaneous properties. It is noteworthy that the stiffness matrix is equivalent to a tangent stiffness in which not only the translational and rotational stiffnesses (**K_x'** and **K_θ'**) are modified by the initiation of yielding, but also the value of the static eccentricity of the system (**e'**).

Re-arranging the equations, as per the previous section, leads to instantaneous values of **Ω'**, ω_x**'** and (e'/r). Obviously, the aforementioned conditions necessary to obtain equivalent elastic response $\mathbf{v_x}$ and $\mathbf{rv_\theta}$ at the centre of mass are not sufficient to ensure the same match in the inelastic domain. For two structures to share identical inelastic response, their tangent stiffness properties must match throughout a given earthquake excitation. This additional very restrictive condition makes the equivalence of complex structures with simpler models, or structures with different types of LLRSE models, virtually impossible, hence the incentive to select a simple general purpose model which will lead to conservative assessments of the seismic inelastic response of torsionally coupled structures. Toward that goal, selecting a two LLRSE system will ensure very large reductions in **K_x'** and **K_θ'**, coupled with large increases in **e'**, when yielding occurs in one of the LLRSE.

Thus, a minimal structural system with two LLRSE can apparently be useful in acquiring a fundamental understanding of how various parameters are affecting the inelastic torsional response. Conservatively, LLRSE oriented perpendicularly to the unidirectional earthquake excitation can be neglected; it is the authors' experience that those are not always located favourably to improve the torsional resistance.

6. GEOMETRIC EQUIVALENCE OF NON-LINEAR INELASTIC STRUCTURES

In spite of the very restrictive aforementioned conditions necessary to produce equivalent torsionally coupled structures through their inelastic response, some simple geometric constraints can still be established to define equivalent structures of similar plan layout but of different scale. For example, equivalent structures each with two LLRSE of equal strength are shown in Figure 5.

Assuming that LLRSEs have a hysteretic bi-linear model with no strain hardening (elastic-perfectly plastic), for those two structure of different geometry to share the same ω_X, ω_X', ω_θ, ω_θ, (e/r) and (e'/r), at all times, it implies that:

$$\omega_{xA}^{\;2} = \frac{K_{1A} + K_{2A}}{M_A} = \omega_{xB}^{\;2} = \frac{K_{1B} + K_{2B}}{M_B} \quad (7)$$

$$\omega_{xA}^{\;2'} = K_{1A}/M_A = \omega_{xB}^{\;2'} = K_{1B}/M_B \quad (8)$$

Thus

$$K_{1A}/M_A = K_{1B}/M_B \tag{9}$$

and

$$K_{2A}/M_A = K_{2B}/M_B \tag{10}$$

Also $\omega_{\theta A} = \omega_{\theta B}$ implies

$$\left[\frac{(K_{1B}+K_{2B})}{M_B}\right]\left[\frac{d_B}{r_B}\right]^2 = \left[\frac{(K_{1A}+K_{2A})}{M_A}\right]\left[\frac{d_A}{r_A}\right]^2 \tag{11}$$

and therefore

$$d_A/r_A = d_B/r_B = D = constant \tag{12}$$

where the subscripts A and B correspond to geometry A and B on Figure 5, and **d** is the distance from a LLRSE to the centre of mass (equidistant elements in this example). Thus, two LLRSE systems with the same ω_X, Ω, **(e/r)**, and same ratio **(d/r) = D** can be of different geometry and still have the same element response, i.e. identical element time histories and ductility demands can be obtained from a wider structure with a larger radius of gyration as long as the geometric ratio **(d/r)** is preserved. This ratio fixes the proportional geometric configuration of a structure as scaled by its radius of gyration. It is more restrictive than the (e/r) ratio.

Similar relationships could be developed for an infinity of plan layouts and multi-LLRSE systems without benefiting much the design process. While it is important to realize the additional constraints necessary for equivalencing inelastic structures, it is instead more productive to demonstrate that findings on the behaviour of relatively simple structures can be safely generalized to more complex structures.

7. CONSIDERATION OF ROTATIONAL INERTIA

The influence of the rotational inertia on the inelastic torsional response is best described when considering initially symmetric structures, i.e. structures where the normalized eccentricity **(e/r)** is zero. In this case, it is the Ω factor that reflects the significance of **r**, the radius of gyration of the floor plan, here taken around the centre of mass. This dimensional parameter, a physical representation of the mass-distribution around the centre of mass, is related to the selected floor plan configuration; although this property in practice is mostly inalterable by the engineer, the effects of variation in radius of gyration on the response of the structures at hand are of interest.

For a given floor translational mass **m**, a reduction in **r** will reduce the mass moment of inertia, **mr²**, and will simultaneously produce an increase in Ω, as

$$\Omega = \omega_\theta/\omega_x = (K_\theta m)/(mr^2 K_x) = K_\theta/(K_x r^2) \tag{13}$$

An initially symmetric structure will respond in a purely translational manner until yielding of one of the LLRSE, at which time the mass moment of inertia of the floor plan will provide an effective inertia (or resistance) against the introduction of torsional movement during that interval when the instantaneous physical properties of the structure provide a temporary mismatch between the centre of stiffness and centre of mass. If the mass moment of inertia is very small (large Ω), it is easy to produce a rotational movement as there is little resistance to the induction of angular motion. In the opposite fashion, if the mass moment of inertia is large (small Ω), considerable inertial resistance to angular motion exists and very little of it may develop. This phenomenon is graphically illustrated elsewhere (Bruneau 1992).

8. OTHER FACTORS

Since small changes in the characteristics of even simple structures are sufficient to ensure that identical non-linear response of lateral-load-resisting structural elements (LLRSEs) is not possible, then even the minimal structural system proposed and deemed sufficient to satisfactorily capture the non-linear inelastic characteristics of torsionally coupled structures, can be configured in a variety of different ways for which dissimilar response is unavoidable.

Admittedly, the type of hysteretic element model will have a considerable effect on the seismic inelastic response of torsionally coupled structures. Nonetheless, since current research still focuses on improving the basic understanding of the behaviour of torsionally coupled structures in the inelastic domain, the consideration of very complex hysteretic models remains premature. Some studies have briefly examined the influence of more complex models in relation to comprehensive parametric studies on simpler model, but at this time, bilinear hysteretic models have been the basis of most research on torsionally coupled structures.

The concept of strength eccentricity, (Sadek and Tso 1988) equivalent to the corollary concept of plastic centroid, provides a representation of the relationship between LLRSEs' strength within a same structure. It quantitatively expresses the observation that the relative yielding levels between different LLRSEs will directly affect the global inelastic behaviour. For a bilinear hysteretic model with two LLRSE and a given set of Ω, (e/r) and $\mathbf{T_X}$, the respective yield displacements between the two LLRSEs will completely define this inter-element model relation, and simultaneously locate the plastic centroid. By analogy with reinforced concrete theory, the plastic centroid is defined as the point where a static lateral load must be applied in order to produce a purely translational displacement when all elasto-perfectly plastic elements are yielded. The plastic centroid distance from the centre of mass can be used as another indicator of the severity of a structure's inelastic torsional behaviour. A plastic centroid distance of zero would produce simultaneous yielding of both LLRSEs under a monotonically increasing static loading, although under dynamic excitation it is not necessarily the case.

Interestingly, limited studies (Bruneau and Mahin 1991) have indicated that changes in plastic centroid distance seriously affect the ductility demand of the LLRSE whose yielding strength is varied, but have relatively little effect on the weak LLRSE whose yield strength is kept constant. Therefore, if the maximum response of the weak LLRSE is of concern, the proposed simple model with LLRSEs sharing equal yield displacements is generally adequate. If the strong LLRSE's response is also of interest, its high sensitivity to the plastic centroid distance makes the inter-element model relationship a more important issue.

Other preliminary findings indicate that:

- For bilinear hysteretic element model, modifications of the strain hardening value have unpredictable effects on the LLRSEs time history signatures; non-linear inelastic static analysis is found to be deficient in predicting the effect of strain hardening on the global behaviour. The high sensitivity of response to hysteretic model characteristics requires further consideration in future studies on inelastic torsional coupling.
- Conclusions obtained from the results of studies performed on stiffness eccentric structures could conservatively be extended to mass eccentric structures, assuming the comparison remains within the aforementioned limitations.
- Finally, the proposed model being of single story, may be unsuitable for the study of complex multistory structures without some modifications, especially in the case where ductility demand tends to concentrate on a few weaker stories; Analogous limitations also exist for ideal-symmetric structures. Additional research is needed.

9. CONCLUSION

A simple general purpose model has been proposed for the study of seismic inelastic torsional coupling. The building code indirect influence in lessening torsional redundancy, as well as the necessity to satisfactorily capture the non-linear inelastic behaviour of torsionally coupled structures, have dictated the minimal characteristics required of this model. A simple model to conservatively estimate structural response is an acceptable alternative considering the stringent and impractical geometric and parametric requirements required if identical structural response is to be ensured in the non-linear inelastic range.

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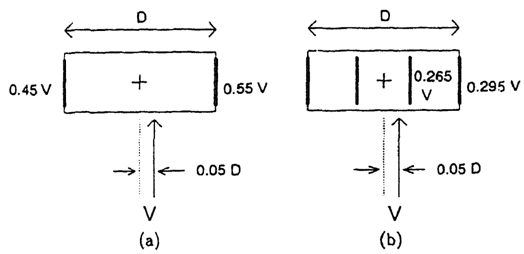

Figure 1. Monosymmetric structures with (a) two LLRSEs and (b) four LLRSEs, for the study of building codes' indirect influence on torsional redundancy - Plan views.

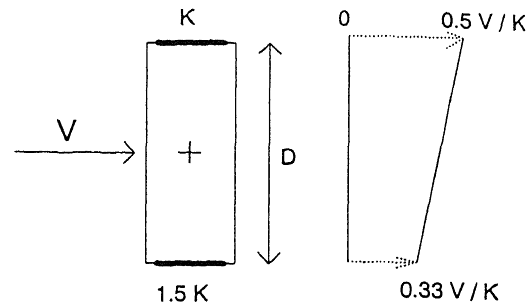

Figure 3. Effect of changed stiffness conditions using linear elastic stiffness analysis.

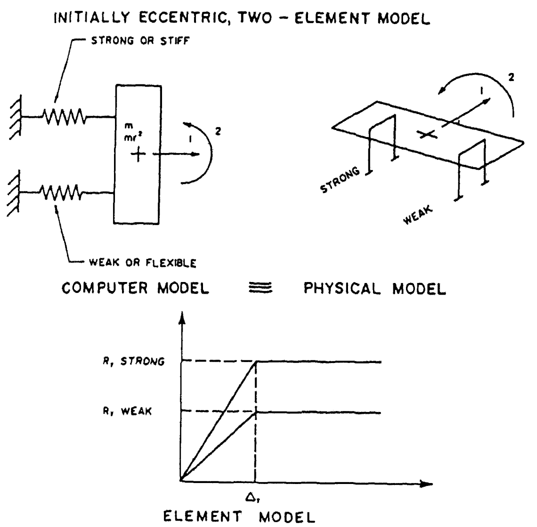

Figure 2. Proposed simple general purpose model for the study of inelastic torsional coupling.

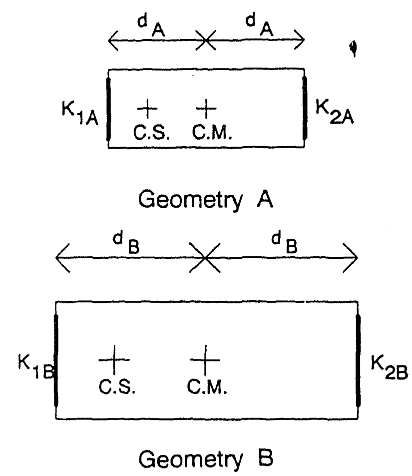

Figure 5. Geometrically equivalent non-linear inelastic structures

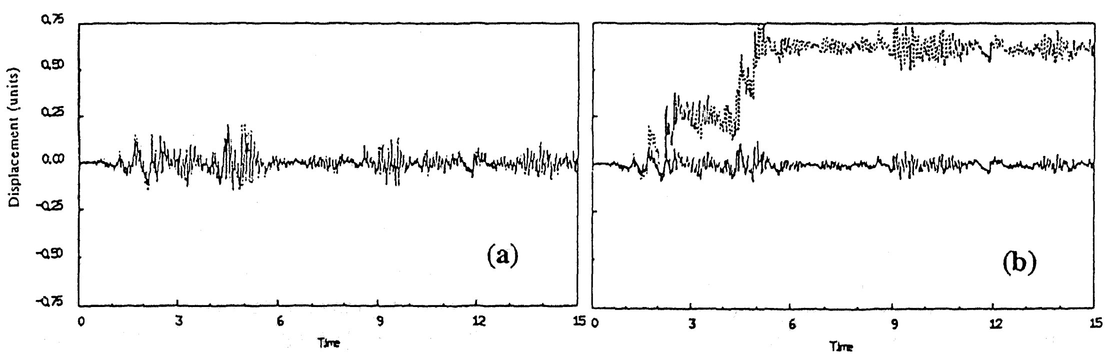

Figure 4. (a) Elastic and (b) Inelastic time history analyses of a two LLRSE structure with T_X=0.1 seconds, μ_{SDOF}=4, Ω=1.6, and element stiffnesses of k (dotted line) and 1.5 k (solid line); Yield displacement is 0.12 units.