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Effect of torsional coupling on the stability of structures

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ABSTRACT: The influence of torsion on instability during seismic response is investigated using a single story mono-symmetric structure. General expressions for the parameters of a single degree of freedom characterization of the structure are developed and used to identify the conditions where safety against instability may not be adequately assessed on the basis of a planar idealization. It is found that the planar model leads to unconservative predictions of instability when the failure mode involves rotation about an element. Although not exclusively, this failure mode is encountered when one or more elements of the structure have overstrength values significantly larger than the average for all the members. Some preliminary results on the importance of the three dimensional failure mode for multistory structures are also presented.

1 INTRODUCTION

The destabilizing effect of gravity on structures subjected to severe ground motion can lead to catastrophic collapse. Methodology to asses the safety factor against failure from instability, for structures that can be idealized as planar, has been recently presented by Bernal (1992). For structures with significant plan eccentricities, however, there is currently little information on how to make this assessment in the context of practical seismic design. This paper presents results of an ongoing study whose fundamental objectives are to identify the limits within which safety against instability may be evaluated on the basis of a planar idealization and to develop methodology on how to proceed when the two dimensional evaluation proves to be inappropriate.

To identify the fundamental parameters which affect the interaction between torsional response and instability, a single degree of freedom (SDOF) model, applicable to one story mono-symmetric systems, is developed. Observations on the expected influence of torsion on instability are made by inspecting the expressions that define the parameters of the equivalent system.

The importance of the shape of the controlling mechanism in the ability of two dimensional structures to undergo inelastic response without instability has been pointed out by Takizawa and Jennings (1980) and has been recently quantified by Bernal (1991). Based on the results presented by these authors, it is reasonable to expect that the three dimensional shape of the failure mode will be a key parameter when assessing instability in torsionally coupled multistory buildings. Unfortunately, however, the mono-symmetric one story structure does not capture the complexity of the three dimensional failure mode and can not be readily utilized to explore its importance. Notwithstanding, some preliminary results on the effect of the failure mode, derived from the analysis of a three story structure having one wall and two frames, are presented and discussed in the paper.

2 ONE STORY SYSTEM

A one story structure with eccentricity between the center of mass and the center of stiffness along the x-x axis is shown in Figure 1. The eccentricity about the y-y axis is taken as zero and, for simplicity, the influence of elements parallel to the x-x axis, as well as the ground motion component in this direction are neglected; the number of lateral force resisting elements parallel to the y-y axis is assumed arbitrary. To simulate the condition typically encountered in actual buildings, the vertical load is assumed to be transmitted to the foundation not only by the lateral force resisting elements but rather uniformly throughout the plan. The system described has two degrees of freedom which may be conveniently selected as the translation of the center of mass relative to the ground, u, and the rotation of the deck, θ. Neglecting damping, the equations of motion can be written in incremental form as;

$$M\begin{vmatrix}1 & 0\\ 0 & r^2\end{vmatrix}\begin{vmatrix}\Delta\ddot{u}\\ \Delta\ddot{\theta}\end{vmatrix}+Kt\begin{vmatrix}1 & -e\\ -e & Kr/Kt\end{vmatrix}\begin{vmatrix}\Delta u\\ \Delta\theta\end{vmatrix}-Mg/H\begin{vmatrix}1 & 0\\ 0 & r^2\end{vmatrix}\begin{vmatrix}\Delta u\\ \Delta\theta\end{vmatrix} = -M\begin{vmatrix}\Delta\ddot{y}\\ 0\end{vmatrix} \quad (1)$$

where M is the mass of the deck, r is radius of gyration about the center of mass, Kt is the summation of the tangent stiffness of the various members, Kr is the tangent rotational stiffness about the center of mass, e is the distance between the instantaneous center of stiffness and the center of mass (positive when the center of mass is to the right of the center of stiffness), g is the acceleration of gravity, H is the height, $\ddot{y}$ is the ground motion acceleration and the symbol Δ is used to indicate increment.

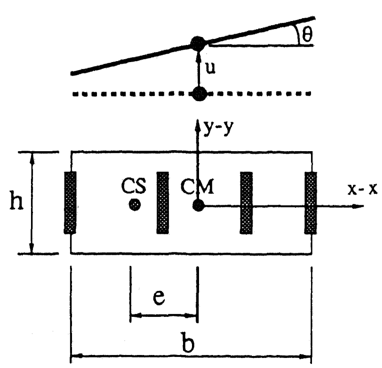


Fig.1 One story torsionally coupled system

Unless otherwise noted, the yield strength of the various elements (whose hysteresis are taken as elasto-plastic) are assigned based on the elastic distribution of member forces which results from the application of a static lateral load at the center of mass; this approach is consistent with the provisions contained in the Uniform Building Code (1990) and the NEHRP Recommendations (1988), wherein dynamic amplification of the static eccentricity is not considered when the design is based on the equivalent lateral force procedure. Accidental eccentricity is neglected, however, because the analyses do not contemplate uncertainties in member properties, nor include the rotational ground motion component.

2.1 *Single degree of freedom model*

It is well known that the linear response of the one story structure of Figure 1 is governed by three independent parameters which are often selected as: the uncoupled translational period T, the ratio of uncoupled translational to rotational period Ω and the ratio of the static eccentricity e_0 to the radius of gyration, r. Attempts to characterize the inelastic response of the system in terms of these three parameters, however, have essentially been unsuccessful (Tso and Sadek 1985, Kan and Chopra 1981). The introduction of an additional parameter computed as the distance between the center of mass and the location of the resultant of the full plastic capacity of the structure (strength eccentricity) (Gomez et al. 1985, Sadek and Tso 1989, Goel and Chopra 1990, Bruneau and Mahin 1987) has lead to improvements in some cases but the inelastic response of systems with identical elastic parameters and strength eccentricities can still be quite different (Chandler and Duan 1991). On the basis of the preceding discussion, it is evident that a characterization of the torsionally coupled one story structure should strive to account for the actual arrangement of resisting elements. One way to account for the actual distribution and yet condense the system to a small number of characterizing parameters is to transform it into an equivalent SDOF model. This approach is also attractive because the available information on dynamic instability of SDOF systems can be directly utilized to explore the behavior of the torsionally coupled structure.

Reduction of the structure to a SDOF system can be viewed in this case as the introduction of a predefined relationship between the displacement of the center of mass and the angle of rotation. In general, the prescribed relation can itself be specified as a function of the amplitude of the displacement at the center of mass. The basic assumption in the reduction to a SDOF can be mathematically expressed as;

$$\begin{vmatrix}\Delta u\\ \Delta\theta\end{vmatrix} = \Delta u\begin{vmatrix}1\\ f_{(u)}\end{vmatrix} \quad (2)$$

where $f_{(u)}$ is a predefined function. A convenient approach for selecting $f_{(u)}$ is to assume that it is given by the ratio of the incremental rotation to the incremental displacement at the center of mass, obtained from a static analysis where a lateral load is applied at a predefined eccentricity and increased from zero until the attainment of maximum strength. Needless to say, the selection of the location of the point of application of the load plays a role in the computed effective resistance function. For any position, however, the effective resistance is a multilinear curve having N-1 segments prior to the attainment of maximum resistance, where N is the number of lateral load resisting elements. The period of the equivalent SDOF system (based on the initial slope of the resistance curve) can be shown to be given by;

$$T_e = T \lambda \tag{3}$$

where,

$$\lambda = \left(\frac{\phi^2 + 1}{\phi^2 - 2\phi e_0/r + \Omega^2}\right)^{0.5} \tag{4}$$

and

$$\phi = \frac{\Omega^2 + \beta\ (e_0/r)^2}{e_0/r\ (\beta + 1)} \tag{5}$$

In equation (5) β is a factor which when multiplied by the initial static eccentricity gives the position of the applied load measured from the center of mass. Another parameter that is fundamental for predicting instability in a SDOF system is the slope of the effective restoring force for deformations beyond the attainment of maximum capacity; this slope, when normalized by the initial elastic slope is known as the stability coefficient, Θ. For the system considered here it can be shown that the stability coefficient is given by;

$$\Theta_e = \Theta_0\ \lambda^2 \tag{6}$$

where Θ_0 is the stability coefficient of the associated symmetric structure, which can be computed from;

$$\Theta_0 = \frac{T^2\ g}{4\ \pi^2\ H} \tag{7}$$

Given that T_e and Θ_e depend on the selected value of β, it is of interest to examine how this selection affects the overall predictions of instability. This can be done by using statistical expressions which have been derived to predict the minimum strength to prevent instability as a function of period, effective stability coefficient and key ground motion parameters. For elasto-plastic systems, the minimum strength for stable response per unit mass, S_{ac}, is given by Bernal (1992) as;

$$S_{ac} = \frac{5\ \Theta^{.75}}{T^{1.42}}\ PGV\ \sqrt{t_{.9}} \tag{8a}$$

but need not be larger than

$$S_{ac} = \frac{36\ \Theta^{.75}}{T^{1.86}}\ PGD\ t_{.9}^{.2} \tag{8b}$$

where PGV and PGD are the peak ground motion velocity and displacement respectively and $t_{.9}$ is the effective ground motion duration (time needed to deliver the central 90% of the total energy in the record) (Trifunac and Brady 1975). With the period and the duration in seconds the units for S_{ac} are those of the PGD divided by sec^2 or of the PGV divided by seconds; the expressions given in equation (8) provide mean level predictions.

For a given system, the required strength S_{ac} can be computed from equation (8) once a value of β is selected. Figure 2 shows the ratio computed by dividing S_{ac} by the value corresponding to the associated symmetric structure, plotted versus β for some selected values of e_0/r, Ω, T and Θ_0. The results shown are typical of those found for a wide range of parameters and indicate, not only that the predicted minimum required strength is insensitive to β, but also that this strength can often be taken as that corresponding to the associated symmetric structure. It is important to clarify that the similarity between the S_{ac} values of the symmetric and torsionally coupled SDOF representations do not necessarily imply a small influence of torsion on the safety against instability. This is so because the safety margin depends on the ratio of capacity to demand and the available "capacity" of the eccentric system for a given β value is not the same as the strength of the symmetric structure.

The ultimate capacity of the equivalent SDOF model, expressed as a base shear coefficient, can be shown to be given by;

$$C_e = \sum_{i=1}^{N-1} \Delta F_i \left(\frac{\rho_i + \beta \cdot e_0/r}{\rho_i}\right) \tag{9}$$

where

$$\rho_i = \frac{f_{(u)}}{r} \tag{10}$$

and ΔF_i is the increment in force needed to induce the ith yielding event. The summation

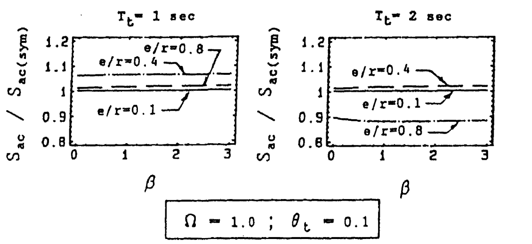


Fig.2 Ratio of S_{ac} to S_{ac} of the associated symmetric structure, as a function of β, for some selected systems.

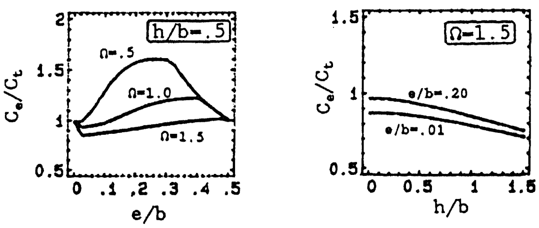


Fig.3 Ratio of C_e to C_t for different values of Ω, e/b and α, for η=1.

involves N-1 terms because the structure of Figure 1 becomes a mechanism when there is one bar left to yield. It is worth noting that the value of ρ_i is the same as that given by equation (5) if instantaneous values of Ω and the eccentricity are used.

An assessment of the equivalent strength computed from equation (9) can not be done in general because the results are dependent on the particular distribution of the resisting elements. Some observations of practical interest can be made, however, if the strength distribution is prescribed and if equation (9) is simplified by assuming that the term in parenthesis remains equal to the initial value for all the terms in the summation. Numerical results have indicated that the foregoing approximation typically does not introduce undue error. Consider the particular case where the strength distribution is proportional to member forces which result from the application of a load at the center of mass, except for the member at the edge of the deck nearest to the center of stiffness, which we assume to be stronger than called for by the design. Provided the edge element is sufficiently strong the structure will fail by pivoting about this element and one can obtain the following expression;

$$C_e/C_t = \frac{(\lambda + \beta)^2}{\eta(\lambda^2 + \beta)(1 + 2\cdot\beta\cdot(e_0/b))} \qquad (11)$$

where

$$\lambda = \frac{\Omega}{(e_0/r)} \qquad (12)$$

and C_t is the summation of the yield capacities of all the resisting elements divided by the weight of the deck; η is the global overstrength introduced by the excess capacity of the edge element about where the pivoting ocurrs. It should be noted that η is simply equal to the actual C_t divided by the value of C_t corresponding to an overstrength of one in the edge element. Values of the strength ratio computed from equation (11), for representative values of Ω, e_0/b, and the aspect ratio of the deck h/b, are plotted in Figure 3 for the case η=1. These curves are computed using values of β that minimize the C_e/C_t ratio. As can be seen from Figure 3, the reduction of the strength capacity of the equivalent SDOF system compared to total capacity which is available when the system is treated as planar is generally negligible for practical purposes. Nevertheless, the ratio can be much smaller than one if the pivoting element has an overstrength ratio which is much larger than the average. The previous observation is illustrated in Figure 4 which plots the ground motion scale factor needed to induce collapse versus η, for a system with three bar subjected to the EW component of El Centro (1940); for comparison, the collapse intensity obtained when the structure is treated as planar is also shown. It should be noted that the results for the eccentric system in Figure 4 are not obtained from a SDOF solution but by direct integration of the coupled equations. As can be seen from the figure the collapse intensity for the eccentric system becomes independent of the total strength once the failure is controlled by pivoting about the strong element. In contrast, when the system is analyzed as planar, the collapse intensity is linearly related to the total strength provided. The behavior depicted in Figure 4 in all likelihood carries over to the case of multistory structures, and appears to be one of the fundamental reasons why it may not be possible to estimate instability thresholds on the basis of results from planar models.

Another interesting case is that when the failure mode involves pivoting about an internal element that is close to the geometric center of the deck. Unfortunately, in this case the equivalent strength becomes dependent on the number and distribution of the resisting elements obviating the possibility for deriving a general expression. Although preliminary results appear to indicate that this type of failure can result in significant reductions in the collapse intensity, the strength-stiffness relationship of the resisting elements where this failure has been observed have generally been quite unrealistic.

3. PRELIMINARY OBSERVATIONS OF THE EFFECT OF TORSION ON INSTABILITY OF MULTISTORY BUILDINGS

The structure shown in Figure 5a is utilized here to derive some preliminary observations on the effect of torsion on instability of multistory structures. As shown, the structure is composed of two identical frames and one shear wall and is three stories high; the typical assumption of rigid diaphragm action for the floor slabs is utilized. Resisting elements perpendicular to the frames and wall considered and the corresponding component of the ground motion are neglected in this preliminary investigation. The analyses are carried out using DRAIN-TABS (Guendelman Israel and Powell 1977) using 5% of critical damping on each of the first two modes of the structure.

The collapse intensity for the EW component of El Centro is first computed introducing a constrain against plan rotation. Using iterations it is found that the critical ground motion scale factor is 2.10 and the mechanism along the height is found to be global as one would expect due to the presence of the shear wall. Following this analysis the constrain against plan rotation was removed and the iterative process repeated to compute the critical intensity for the actual eccentric structure. In this case the critical scale factor was found to be 1.36 and the failure mode was essentially a pivoting about the wall with first story mechanisms in the two frames. An illustration of the failure mode is presented in part (b) of Figure 5. It is worth noting that in this example the strength of the frames and the wall have been apportioned in accordance with the results of a standard analysis. Given the pivoting failure mode observed, increases in wall capacity, which would appear to be beneficial in a standard planar analysis would actually lead to little or no increases in the collapse intensity.

4. CONCLUSIONS

The results of the investigation appear to indicate that the intensity of ground motion needed to induce collapse in one story structures is not likely to be significantly affected by torsional eccentricity, provided the overstrength of the various elements do not deviate too much from the average for the system. In the cases where the previous condition is not satisfied, the structure typically fails by pivoting about the strong element at an intensity that can be much lower that than which would be inferred if the system were analyzed as planar. Based on results obtained for planar structures and on a pilot study carried out using a three dimensional three story building it appears that the shape of the failure mode is likely to play a fundamental role in the instability of actual multistory structures.

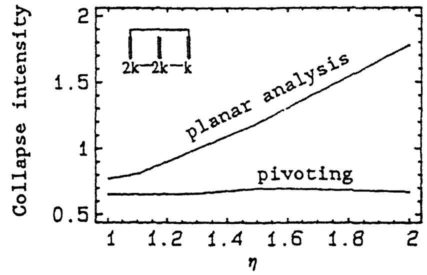
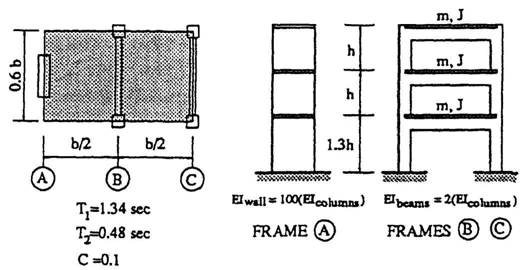
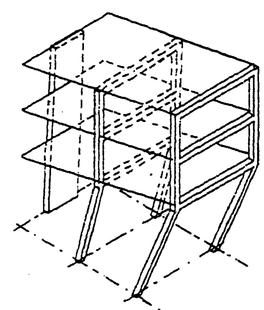


Fig.4 Collapse intensity versus overstrength factor for a three member eccentric structure (T=1sec, C_t=0.1, h/b=0.5 and 5% damping).



(a)



(b)

Fig.5 (a) Three story structure used to examine instability (b) Three dimensional mechanism failure mode.

5. REFERENCES

Bernal, D. 1991. Dynamic instability in buildings. *Proceedings of the Sixth Canadian Conference on Earthquake Engineering*, Toronto: 85-92

Bernal, D. 1992. Instability of buildings subjected to earthquakes. *Journal of Structural Engineering*, American Society of Civil Engineers. (Schedule for publication: August 1992)

Bruneau, M. and Mahin, S.A. 1987. Inelastic seismic response of structures with mass or stiffness eccentricities in plan. *Earthquake Engineering Research Center, Report No. UCB/EERC-87/12*, University of California, Berkeley, California

Chandler, A.M. and Duan, X.N. 1991. Evaluation of factors influencing the inelastic seismic performance of torsionally asymmetric buildings. *Earthquake Engineering and Structural Dynamics*, Vol.20: 87-95

Goel, R.K. and Chopra, A.K. 1990. Some aspects of inelastic earthquake response of one-story asymmetric-plan systems. *Proceedings of Fourth U.S. National Conference on Earthquake Engineering*, Vol.2, Palm Springs, California: 259-268

Gomez, R., Ayala, G. and Jaramillo, J.D. 1987. Influencia de la excentricidad en resistencias de los elementos estructurales sobre la respuesta sismica de edificios. *Proceedings of the Seventh Mexican National Conference on Earthquake Engineering*, Queretaro, Mexico:C15-C29

Guendelman-Israel, R. and Powell, G.H. 1977. DRAIN-TABS: A computer program for inelastic earthquake response of three dimensional buildings. *Earthquake Engineering Research Center, Report No. UCB/EERC-77/8*, University of California, Berkeley, California

Sadek, A.W. and Tso, W.K. 1989. Strength eccentricity concept for inelastic analysis of asymmetrical structures. *Eng. Struct.*, Vol.11: 189-194

Takizawa, H. and Jennings, P. 1980. Collapse of a model for ductile reinforced concrete frames under extreme earthquake motions. *Earthquake Engineering and Structural Dynamics*, Vol.8: 117-144

Trifunac, M.D. and Brady, A.G. 1975. A study of the duration of strong ground motion. *Bulletin of the Seismological Society of America*, Vol.65: 581-626

Tso, W.K. and Sadek, A.W. 1985. Inelastic seismic response of simple eccentric structures. *Earthquake Engineering and Structural Dynamics*, Vol.13: 255-269