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Seismic response of horizontal setback buildings

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ABSTRACT: The seismic response of multistorey horizontal setback buildings with L- and V-shaped plans has been studied including in-plane flexibility of the floor diaphragms. The building has been modelled as a continuum system whose equations of motion are solved for the appropriate boundary conditions. The model yields some interesting insights into the dynamics of such buildings. Even though the floor flexibility does not significantly affect the natural frequencies and the overall base shear in the building, its effect on the shear distribution in individual frames may be quite significant. It is seen that if the overall longitudinal and transverse stiffness of the individual wings are equal, both rigid floor as well as flexible floor modes of vibration exist; however, those with floor deformation are not excited by spatially uniform ground motion.

INTRODUCTION

Buildings with horizontal setback have suffered significant damage in the past earthquakes. Examples include the two-storey V-shaped West Anchorage High School in Alaska (earthquake of March 27, 1964) (Meehan 1967; Jain 1983), the four-storey L-shaped San Marcos building in Santa Barbara, California (earthquake of June 29, 1925) (Dewell and Willis 1925; Arnold and Reitherman 1982); and the four-storey L-shaped Seminario, in Puerto Montt, Chile (earthquake of May 22, 1960) (Steinbrugge and Flores 1963; Clough 1965). Such buildings pose two main problems: (i) the floors may exhibit significant in-plane flexibility and therefore the usual assumption of rigid floor diaphragm may not always be valid, and (ii) at the re-entrant corner where two or more wings meet, stress concentration caused by floor flexibility leads to severe damage during a strong ground shaking. Also, for such buildings the usual two-axis seismic analysis may not be sufficient (e.g., Arnold and Reitherman 1982).

Dynamic analysis of such buildings, including the effect of floor flexibility, can be carried out by the finite element method. However, due to a large number of degrees of freedom, this results in expensive analysis, thus limiting its use in design. Moreover, it is useful to have a 'feel' of the dynamics of such buildings through a more generalised solution in order to properly interpret and verify the finite element analysis results.

The building configuration studied in this paper consists of a multistorey frame building of L- or V-shaped plan. The in-plane floor flexibility has been included in the analysis. Jain (1988) summarises the earlier contributions on analysis of different types of building with flexible floor diaphragms. However, there are hardly any studies available in the literature on buildings with horizontal setbacks including in-plane flexibility of floors.

ANALYTICAL MODEL

The structure is treated as linear and elastic. Torsional stiffness of floors and frames is neglected. The structure is considered to be rigidly held at the ground, and hence, the effects of soil-foundation interaction and foundation flexibility are ignored. The damping has not been included in frequency and mode shape calculation, but can be directly incorporated in modal equations.

The analytical model consists of treating long, narrow buildings having a number of identical frames (or walls) and a number of identical floors spaced uniformly as an "equivalent vertically-oriented anisotropic plate" for ground motion in transverse direction. This plate is such that a thin vertical strip cut out of it has only shear flexibility (or bending flexibility) and thus behaves like a shear (or bending) beam representing the transverse frames (or the walls). A thin horizontal strip in this plate behaves like a bending beam representing the floors. The mass and stiffness of the building are distributed uniformly throughout its length and height. For ground motion in the longitudinal direction, such a structure can be adequately modelled as a shear beam. The model of anisotropic plate predicts the seismic response of long, narrow rectangular buildings (Jain and Mandal 1992) which matches with that obtained by discrete lumped mass type models (Maybee, et al 1966; Jain 1984).

The structures with L- or V-shaped plans studied herein have long and narrow wings identical in all respects. Vibration of such a building involves longitudinal as well as transverse motion of both the wings. The longitudinal response of a wing has been modelled by a shear beam while in the transverse direction the wing has been modelled as a vertically-oriented bending-shear anisotropic plate. Such a model, though approximate for buildings that are not uniform in configuration, has the advantage that essential features of the building's main structural components are retained in the analysis, while avoiding a very detailed formulation of the problem. Thus, it is useful in understanding the general behavior of such structures in a simple manner.

Figures 1, 2 & 3 show the isometric view, the schematic plan of the building, and the coordinate systems, respectively. Let L be the center line length of each wing, and H be the height of the building. The angle between the wings is represented by 2θ. Let $v_1(x,z,t)$ and $u_1(z,t)$ be the deflections at point (x,z) of wing OA at time t in the transverse and longitudinal directions, respectively. $v_2(x,z,t)$ and $u_2(z,t)$ are the corresponding displacements for wing OB. As the structure is symmetric about one axis, it possesses symmetric and antisymmetric modes. Thus only one wing needs to be analyzed by imposing symmetry or antisymmetry conditions at the junction of the two wings. Considering the wing OA, the equations of motion for free vibration are:

$$D_1 \frac{\partial^4 v_1(x,z,t)}{\partial x^4} - K_1 \frac{\partial^2 v_1(x,z,t)}{\partial z^2} = - m_1 \frac{\partial^2 v_1(x,z,t)}{\partial t^2} \quad (1)$$

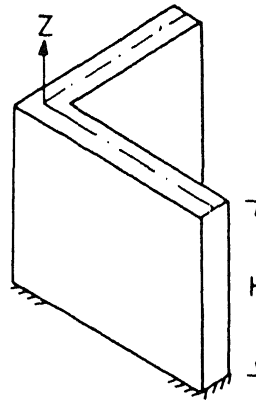

Figure 1. Isometric view of a V-shaped building.

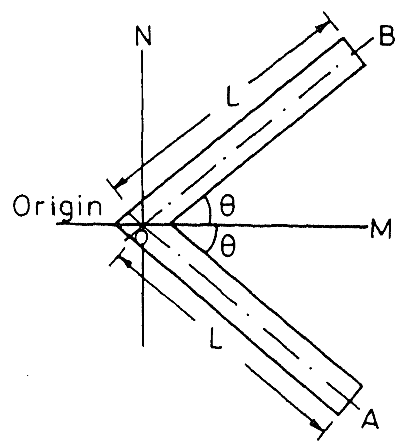

Figure 2. Schematic plan.

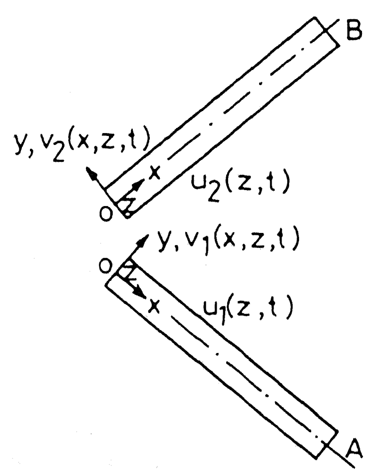

Figure 3. Coordinate systems and directions of deflection.

$$K_o \frac{\partial^2 u_1(z,t)}{\partial z^2} - m_o \frac{\partial^2 u_1(z,t)}{\partial t^2} = D_1 \frac{\partial^3 v_1(x=0,z,t)}{\partial x^3} \tan\theta \quad (2)$$

(for symmetric moes)

$$K_o \frac{\partial^2 u_1(z,t)}{\partial z^2} - m_o \frac{\partial^2 u_1(z,t)}{\partial t^2} = - D_1 \frac{\partial^3 v_1(x=0,z,t)}{\partial x^3} \cot\theta \quad (3)$$

(for antisymmetric moes)

Here $K_o = k' A_o G$ = shear stiffness of the wing in the longitudinal direction; A_o = cross-sectional area of the equivalent plate in x-y plane; and m_o = mass per unit height of the wing. The last term in equations 2 and 3 appears from transverse shear in wing OB at the junction, which is related to the transverse shear in wing OA at the junction through the symmetry and antisymmetry conditions.

The boundary conditions for wing OA are:

1. $v_1(x, z=0, t) = 0$
2. $\partial v_1(x, z=H, t)/\partial z = 0$
3. $\partial v_1(x=0, z, t)/\partial x = 0$
4. $\partial^2 v_1(x=L, z, t)/\partial x^2 = 0$
5. $\partial^3 v_1(x=L, z, t)/\partial x^3 = 0$
6. $v_1(x=0, z, t) = u_1(z,t) \tan\theta$ (for symmetric modes)
   $v_1(x=0, z, t) = - u_1(z,t) \cot\theta$ (for antisymmetric modes)
7. $u_1(z=0, t) = 0$
8. $\partial u_1(z=H, t)/\partial z = 0$

Boundary condition 3 assumes that overall twist of the building in the antisymmetric modes is negligible.

Equations (1), (2) and (3) have been solved by the method of separation of variables for the above boundary conditions (Jain and Mandal, 1992). This results in transcendental frequency equations, mode shape expressions, and modal participation factors. With these, the structure can be analyzed for earthquake response either by the time-history analysis or by the response spectrum techniques.

The resulting mode shapes and participation factors reveal some interesting insight into the response of such buildings. It is seen that when total shear stiffness in the transverse and longitudinal directions of each wing are unequal, only flexible floor modes exist. When total shear stiffness in both the directions of each wing are equal, rigid floor modes as well as flexible floor modes exist. However, the flexible floor modes of the latter case are not excited by spatially uniform ground motion, and hence the floor flexibility need not be considered in the latter case. This observation is important from the design point of view. It indicates that the problem of stress concentration, and the resulting damage at the junction of two wings, can be avoided if the structural configuration is planned in such a way that the overall lateral stiffness of the wings in the transverse and longitudinal directions is equal.

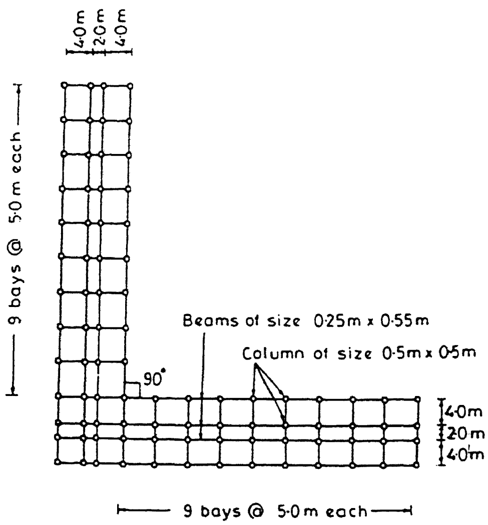

Figure 4. Plan of the example building.

NUMERICAL EXAMPLE

As an illustration of the method a six-story horizontal setback building with L-shaped plan (Figure 4) has been analyzed. Building properties taken in this example are: center line length of wings (L) = 50.0 m; height of the building (H) = 21.6 m; angle between the wings (2θ) = 90^o; mass per unit area in the plane of each wing (m_1) = 4,000 kg/m^2; modulus of elasticity of concrete (E) = 2.55×10^{10} N/m^2; moment of inertia of horizontal bending beam (I) = 4.46 m^4/m height; shear stiffness for shear beam in the transverse direction (K_1) = 52.0×10^6 N/m length; and shear stiffness for shear beam in the longitudinal direction (K_o) = 2.0×10^9 N.

Natural frequencies and periods for first four modes of the example structure obtained from the flexible and rigid floor models are given in Table (1). The natural periods for flexible floor model are higher than those for the rigid floor model, though the difference is insignificant.

First four symmetric and antisymmetric modes are plotted in Figure 5. The first two modes involve first vertical shear beam mode while the next two modes involve second vertical shear beam mode. Difference among the first two and the latter two modes is due to the different horizontal bending beam modes.

It is observed that in the first and third modes, floor displacement at the junction is more than that at the free end. As the total shear stiffness in the longitudinal direction is less than that in the transverse direction, each wing is deflecting more in the longitudinal direction, which increases the transverse deflection of the other wing near the junction by pulling and pushing effect. The second and fourth modes involve the second horizontal bending beam mode; and in these modes floor displacement at the free-end is higher than that at the junction.

Base shear has been obtained by taking the product of shear stiffness and slope at base. The ground motion is assumed to be characterized by a constant spectral acceleration of 0.2*g*. Various total base shears and different frame base shears are given in Table (2). It is seen that base shears are

Table 1. Natural frequencies and periods for flexible floor (rigid floor) model.

Mode No.	Frequency (rad/sec)	Period (sec)
I	7.78 7.80	0.808 0.806
II	12.10	0.519
III	23.03 23.39	0.273 0.269
IV	26.17	0.240

same for the symmetric and antisymmetric modes for ground motion respectively along and perpendicular to the line of symmetry. Ground motion in any of these two directions excites either the symmetric or the antisymmetric modes only. An examination of total base shears (Table 2) reveals that the contribution of first mode to the overall base shear is much higher than that of the higher modes. Next higher contribution is of the third mode. The total base shear for the flexible floor model is comparable to that from the rigid floor model. However, junction frame shares about 23% more load than that shared by the free-end frame. It is likely that for different building properties but similar configuration, this difference may become even more significant. Therefore, design based on the rigid floor model may be unsafe for certain frames even though the natural frequencies and the total design shear are reasonably accurate and hence quite acceptable.

Table 2. Base shears for flexible floor (rigid floor) model.

Mode No. ⟶	I	II	III	IV	SRSS VALUES
Total transverse base shear for each wing (N)	5.27×10^6 5.49×10^6	1.00×10^5	4.24×10^5 6.10×10^5	1.49×10^5	5.29×10^6 5.53×10^6
Total longitudinal base shear for each wing (N)	4.40×10^6 4.22×10^6	6.46×10^4	5.28×10^5 4.69×10^5	3.36×10^4	4.43×10^6 4.25×10^6
Total base shear for the whole building (N)	1.37×10^7 1.37×10^7	5.13×10^4	1.35×10^6 1.53×10^6	1.60×10^4	1.37×10^7 1.38×10^7
Junction frame base shear (N)	5.20×10^5	7.48×10^3	6.21×10^4	4.13×10^3	5.23×10^5
Middle frame base shear (N)	4.83×10^5	6.83×10^3	4.06×10^4	1.12×10^4	4.85×10^5
Free-end frame base shear (N)	4.23×10^5	3.30×10^4	7.96×10^3	3.87×10^4	4.26×10^5

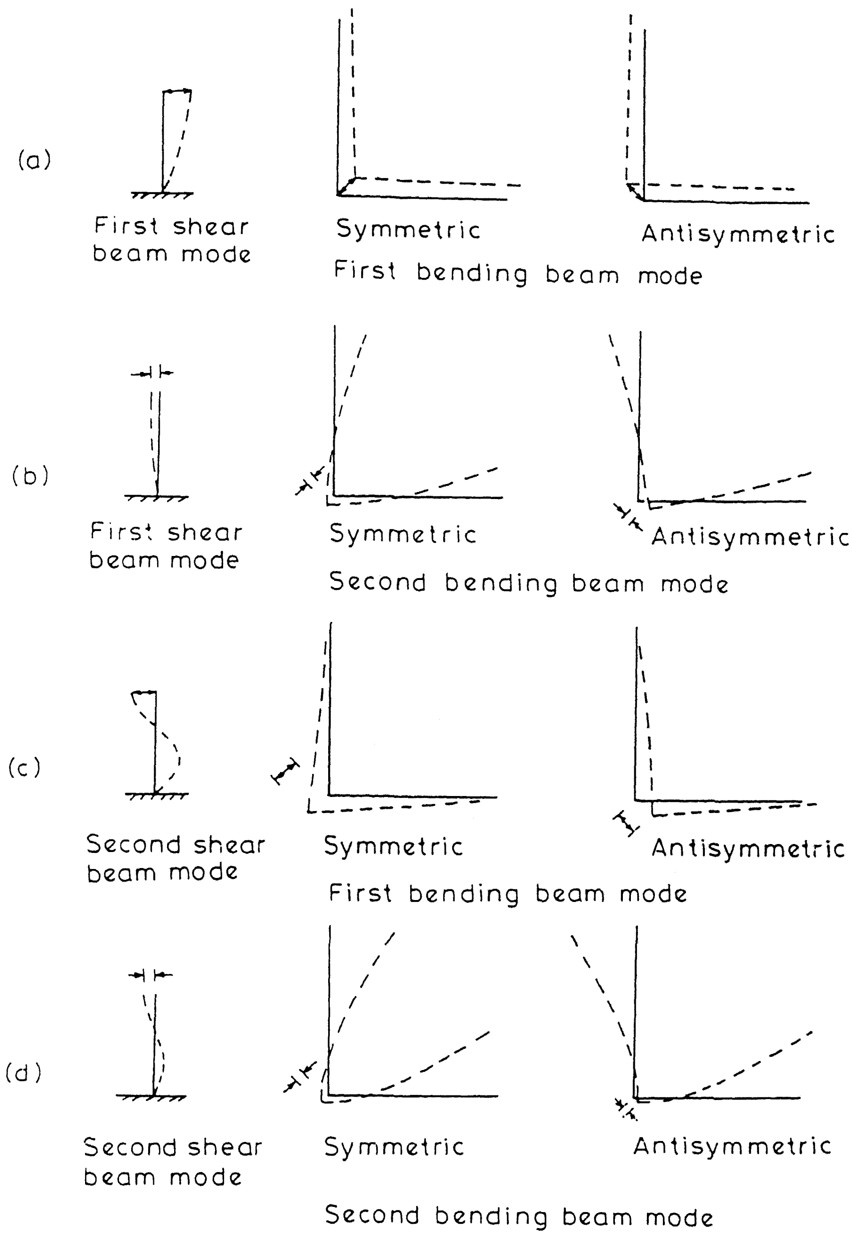

Figure 5. Mode shapes (a) first mode, (b) second mode, (c) third mode, and (d) fourth mode.

CONCLUSIONS

For a V-shaped building with unequal transverse and longitudinal stiffness of a wing, all natural modes of vibration involve floor deformation. However, if the two stiffnesses are equal, rigid floor as well as flexible floor modes exist but the latter are not excited by spatially uniform ground acceleration. Thus the problem of stress concentration at the re-entrant corners of such buildings can be avoided by planning the structural configuration such that the overall transverse and longitudinal stiffnesses of the individual wings are equal.

The floor flexibility does not significantly affect the fundamental period and the overall base shear in such buildings. However, it does affect the shear distribution among transverse frames, thus leading to unsafe design for some frames. When total transverse stiffness is more than the longitudinal stiffness, first mode involves more deflection at the junction than that at the free-end, and vice-versa. Thus, in the first case more shear is resisted by the transverse frames near the junction than those at the free-end, while in the latter case, the frames at free-end will be more severely loaded.

The effects of floor flexibility tend to be more significant with increase in difference between transverse and longitudinal stiffness, increase or decrease of angle between the wings for V-shaped structures from 90^{o}, increase of length-to-width and decrease of height-to-length ratio.

A parametric study of such buildings (Jain and Mandal 1992) reveals that for the L-shaped buildings (i.e., $2\theta = 90^{o}$) floor flexibility effects are of the same order for symmetrical and antisymmetrical modes. However, as θ increases (decreases), floor flexibility effects decrease (increase) for the symmetrical modes and increase (decrease) for the antisymmetrical modes. Hence, 2θ equal to 90^{o} is the optimal angle for V-shaped buildings. Also, these effects increase significantly with an increase in aspect ratio (length-to-width ratio) of the wings, and with decrease in the building height. In this example building, total transverse shear stiffness (K_1L) is greater than the longitudinal shear stiffness (K_o). However, for the situation where $K_o > K_1L$, the curvature of floor deflection in the first and the third modes becomes reverse of the earlier case. This results in the transverse frames at the free end experiencing more shear than the junction frames.

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