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P-delta effects in multi-storey structural design

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ABSTRACT: A method of designing for P-delta effects in ductile multi-storey buildings is presented. With this approach the strength required to prevent an increase in ductility demand when P-delta effects are included in an analysis, is determined from the performance of a single degree of freedom oscillator. Non-linear numerical integration time history analyses of a series of ductile frame structures indicate that the approach forms the basis of a practical method of design for P-delta effects.

1. INTRODUCTION.

In the seismic design of multi-storey structures allowance should be made for "P-delta" effects. These are the additional overturning moments applied to the structure resulting from the seismic weights, "P", supported by the structure, acting through the lateral deflections, Δ, which directly result from the horizontal seismic inertia forces. They are second order effects which increase the displacements, the member actions and lengthen the effective fundamental period of the structure.

Previous research by Montgomery (1981), Bernal (1987), and Chung (1991) has shown that P-delta effects are much more significant in ductile than in elastically responding structures. For design purposes, in most cases seismic induced P-delta actions can be neglected in structures which remain elastic throughout the earthquake. Although some common linear structural computer programs such as ETABS (1986), P-frame (1987), and Autosteel (1992) incorporate P-delta effects into their analysis procedures, it has been shown by Bernal (1987) and Chung (1991) that the increase in member action that they predict underestimates the influence of P-delta effects on the response of a structure behaving nonlinearly during the passage of a large earthquake. Codes of practice have provided little guidance on how to cope with seismic induced P-delta actions in structures.

In this paper a design method which allows for P-delta effects in multi-storey structures, which have been designed to prevent column sway mechanisms forming, is outlined. It is assumed that the software for linear elastic static and optionally, modal response spectrum analyses is available. The basic P-delta information for the procedure is provided from single degree of freedom analyses in much the same way design response spectra are used in multi-storey structural design.

P-delta effects in a structure may be controlled by increasing its lateral stiffness, increasing its strength or by a combination of these. Relying on increasing the lateral stiffness alone could require the structural form to be changed and as such this can lead to a significant increase in cost. In this paper, the design approach to control the influences of P-delta effects has been chosen on the basis of increasing structural strength.

2. BASIC MECHANICS OF A SINGLE DEGREE OF FREEDOM OSCILLATOR.

The response spectrum used in the seismic design of multi-storey structures is based on the analysis of single degree of freedom models without P-delta effects. The design procedure, outlined later in this paper, builds upon this concept and incorporates the influence of P-delta effects into the behaviour of the basic single degree of freedom (sdof) oscillator model.

The model is shown in Fig. 1. It consists of a mass, m, with a weight force, P, supported by a rigid column of height, h, with a flexural spring at its base. A dashpot is attached to the mass to allow viscous damping to be included. A lateral force, V, applied to the mass causes a deflection, δ. It is assumed that the base spring is bilinear so that in the situation with

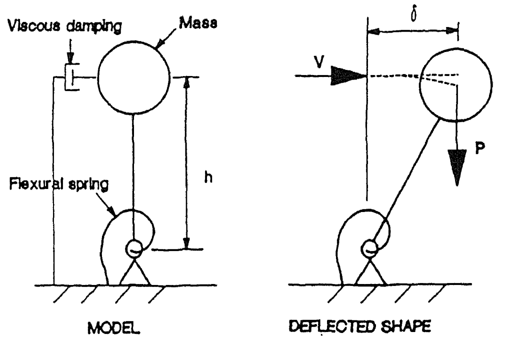

Figure 1. The basic single degree of freedom model.

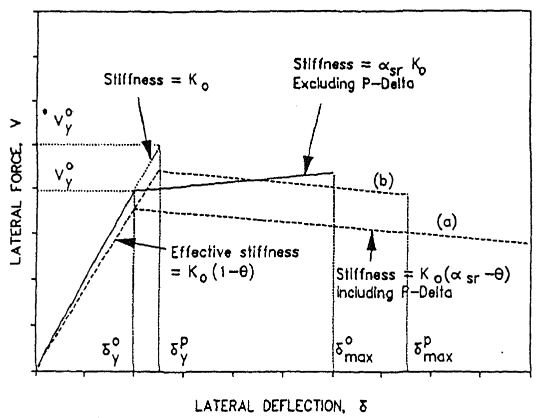

Figure 2. Monotonic force-deflection curves for sdof model.

no vertical load acting, the monotonic lateral force - deflection curve is given by the solid line in Fig. 2.

The initial stiffness of this curve is K_o, which reduces to $\alpha_{sr}K_o$ after yield occurs at a lateral force V^o_y. The nonlinear behaviour of the oscillator under an earthquake ground motion is governed by its natural period of vibration T, the fraction of critical viscous damping ξ, the strain hardening ratio α_{sr} and the value of the non-dimensionalised yield strength V^o_y/mg of the spring. The ductility demand is given by

$$\mu = \frac{\delta^o_{max}}{\delta^o_y} , \qquad (1)$$

where δ^o_y is the deflection at yield and δ^o_{max} is the maximum deflection.

The inclusion of the vertical weight P into the oscillator model reduces the lateral stiffness by P/h and the effective strength for resisting lateral forces to $V^o_y - P\delta^o_y/h$.

The lateral force versus deflection response of the oscillator with P-delta induced actions is shown in Fig. 2 by the dashed line (a). The dashed curve (b) is that of the oscillator with P acting and the strength of the spring enhanced to a value $^*V^o_y$ so that the ductility μ, which is equal to $\delta^P_{max}/\delta^P_y$, is maintained equal to that found when P-delta effects were neglected; this is $\delta^o_{max}/\delta^o_y$. It is useful to note that for the same ductility demand the oscillator with the vertical load has a larger maximum displacement than the initial oscillator.

The nondimensional ratio of the P-delta over-turning moment to the resisting strength, which has been defined as the stability factor, θ, (Bernal (1987), ATC 3-06 (1984)), is given by

$$\theta = \frac{P\delta}{Vh} . \qquad (2)$$

The reduction in effective stiffness of the oscillator caused by the influence of the P-delta load means that the period of oscillation is increased from T to T′. As the period is inversely proportional to the square root of the effective initial stiffness, the stability coefficient given by Eq. 2 may alternatively be calculated as

$$\theta = 1 - \left[\frac{T}{T'}\right]^2 . \qquad (3)$$

The increased strength required to ensure that the ductility demand of the oscillator with the vertical load is the same as that without this load can be measured in two ways. Bernal (1987) defines a strength amplification factor as

$$\alpha = \frac{^*V^o_y}{V^o_y} . \qquad (4)$$

A typical strength amplification spectrum is shown in Fig. 3 from Chung (1991).

Alternatively Chung (1991) defines a "P-delta" amplification factor, "β" by the expression

$$^*V^o_y = V^o_y + \beta \frac{P\delta^o_{max}}{h} . \qquad (5)$$

This second expression provides a definition which is useful in the design of multistorey buildings. V_y^o represents the required basic strength and δ^o_{max} the anticipated maximum displacement of the structure in a design level earthquake. These are both calculated excluding the influence of P-delta effects. "β" represents a factor required to magnify the P-delta effects resulting from the initial calculations. The $P\delta^o_{max}/h$ term may be considered to be an additional strength which when scaled by β and added to V^o_y provides the required enhanced strength, $^*V^o_y$. The basic value, $P\delta^o_{max}/h$ is defined as the "$\beta = 1$" strength.

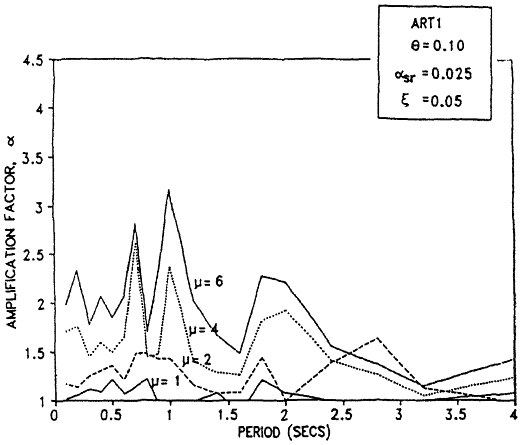

Figure 3. Amplification spectrum for ART1 ground motion, an artificial ground motion.

From the definitions provided, α and β are related by the expression

$$\beta = \frac{\alpha - 1}{\mu\,\theta} \tag{6}$$

Using the previous definitions, the prediction of the maximum displacement of the strength enhanced oscillator with P-delta actions in terms of the actions without P-delta acting can be developed as

$$\delta^{P}_{max} = \delta^{o}_{max} + \mu\,\beta\,\theta\,\delta^{o}_{max} \tag{7}$$

3. STEPS FOR MULTI-STOREY DESIGN

(i) Perform an elastic analysis of the structure neglecting P-delta effects using either the modal response spectrum or equivalent static method. The structural actions in the potential plastic hinge zones and the lateral deflections at each level (Δ_i) are the structural equivalents of V^{o}_{y} and δ^{o}_{y} respectively.

(ii) Determine the first mode period, T_1. If the computer programs available perform only static analyses, the Rayleigh formula may be used.

(iii) Calculate T_1', the first mode period with P-delta effects included. A number of readily available computer programs are able to achieve this. Alternatively, a modified Rayleigh formula given in the appendix can be used.

(iv) Calculate the stability factor, θ, for the structure using Equation 3.

(v) Obtain a design value for β using T_1, μ, and θ. Specific values may be calculated using a sdof oscillator from a specified earthquake record as shown in Fig. 3. Ideally the β values, like design spectral values, should be developed from a statistical basis and have associated with them a probability of exceedence for the design life of the structure.

(vi) Calculate the "$\beta = 1$" structural strengths. Equation 5 indicates that the enhanced strength required, $^{*}V^{o}_{y}$, is equal to the basic design strength, V^{o}_{y}, plus β multiplied by what are nominated as the "$\beta = 1$" structural strengths. These are the structural actions required to support the P-delta forces acting on the structure displaced to a reference shape. This corresponds to δ^{o}_{max} in the sdof oscillator. To aid calculation of the P-delta actions two basic functions of a structure, which are to provide support for gravity loads and resistance to lateral displacement, may be visualised as being provided by two different sets of structural components. This approach is illustrated in Fig. 4, where the vertical load carrying function is provided by an initially vertical member made up of rigid struts that are pinned at each level. When this is deflected into some profile, horizontal forces which can be calculated from equilibrium considerations, are required at each level to support it. These are provided by the lateral force resisting system and they give rise to the "β=1" structural actions.

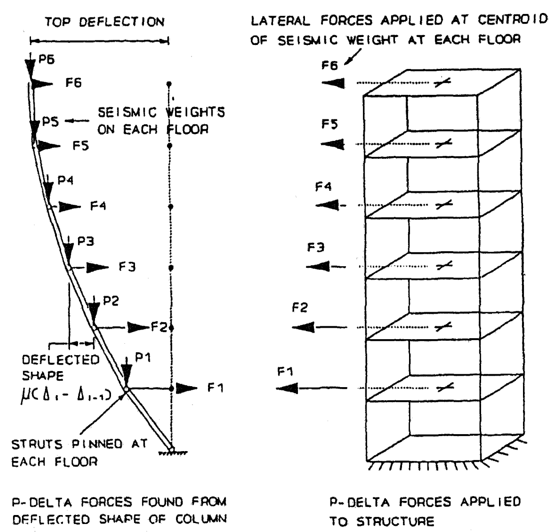

Figure 4. Two components of structural system.

The deflected profile used to calculate the lateral forces, may be taken as μ times the elastic deflection of centre of mass at each level, (Δ_i), as calculated in step (i). These can be calculated by either an equivalent static analysis or be taken as the envelope of deflections found in a modal response spectrum analysis.

(vii) Scale the "$\beta=1$" strengths by β and add to basic strengths calculated in step (i). Proceed with the rest of the design.

ASSESSMENT OF DESIGN PROCEDURE FOR P-DELTA EFFECTS.

A number of analyses of walls and frames have been made to assess the proposed procedure, Davidson (1990). The results of analyses on a series of reinforced concrete frames designed to provide seismic resistance for a number of multi-storey buildings are presented in this section. They were proportioned to provide lateral support to a number of structures, all of which had the same floor plan. Two sets of structures were designed, one for a "normal" and one for a "soft" soil site as prescribed by DZ4203 (1990). Each set consisted of structures of 3, 6, 12, 18, and 24 stories designed to a nominal ductility of 6. The idealized plan for these is shown in Fig. 5. Seismic actions in the X direction only were considered and torsional actions were neglected.

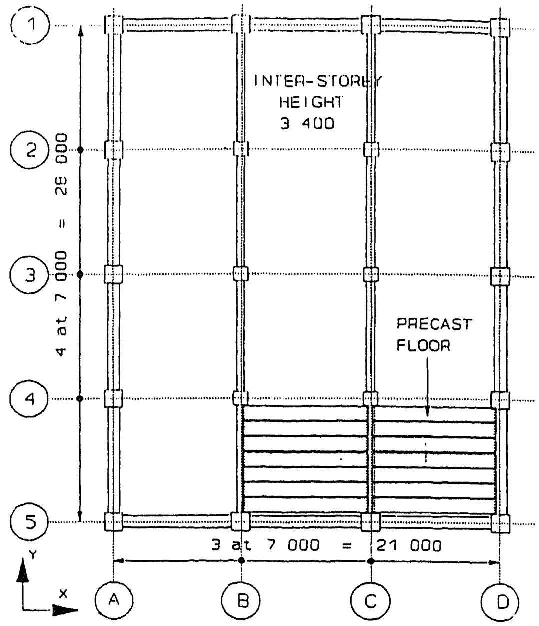

Figure 5 : Typical Floor Plan of Structures

The internal framing was assumed to support the gravity loads but provide no lateral resistance. The arrangement of the precast flooring was such that the beams of the perimeter frames on lines 1 and 5 were not subjected to any significant gravity loading. This arrangement was used to avoid any complications arising from redistribution of gravity load actions, Fenwick (1987). With these assumptions the behaviour of the frames could be modelled using the 2D analysis program DRAIN-2DX (1988).

The frames were designed so that column sway modes could not develop and plastic hinging was restricted to the column faces of the beams and to the bases of the columns. For the basic design, in which P-delta actions were neglected, the flexural strengths of the beams were taken as the average values at each floor level as calculated from a modal response spectrum analysis. In this analysis, the gross section properties of the columns were used and to allow for cracking in the beams the effective moment of inertia was taken as half the gross value. The yield strengths at the base of the columns were taken as 1.26 times the combined modal bending moment at those positions. This corresponds to the minimum dependable strength recommended in NZS 3101 (1982). A complete description of the frames and the structural properties is given in Chung (1991).

The calculation of the enhanced strength required for the frames to resist P-delta effects depends upon T_1, θ, and μ. T_1 and θ were determined as described in the previous section. The nominal design ductility, μ, of each frame, which was based on the lateral deflections of the top level, was six. As the nonlinear seismic response of the strength enhanced and P-delta influenced frame was to be compared with that of a basic frame, in which P-delta actions were excluded, the ductility chosen on which to base the comparison was that value obtained from the non-linear time history analysis of the basic structure. This value, $\mu_{calc.}$, was approximately equal to six, but varied depending upon how closely the design spectrum matched that of the chosen earthquake.

The predicted β value required for strength enhancement "β_{pred}" was obtained from an analysis of a single degree of freedom oscillator with the properties; T_1, θ, and $\mu_{calc.}$ to the earthquake ground motion used to analyse the frames. The predicted required strength of the frame was then taken as its basic strength, plus β_{pred} times the "$\beta=1$" set of strengths calculated as described in step (vi) of the previous section.

To assess whether this predicted additional strength was sufficient to enable the frames to support the additional P-delta forces, without increasing the ductility demand, the nonlinear response of each frame with P-delta forces acting was found for the chosen ground motion. The strength used for the frame was the basic strength plus "β_{req}" times the "$\beta=1$" strengths. The amount, "β_{req}", was adjusted in successive analyses so that the ductility demand of the frame with P-delta forces was the same as that of the basic frame, $\mu_{calc.}$. The equivalent ductility requirement of the initial and strength enhanced structures was judged on the basis of

Table 1 Summary Results for frames designed for "Normal" site spectrum.

Item	No. of Levels									
	3		6		12		18		24	
Period (secs)	1.02		1.43		2.55		3.01		3.33	
Av.seismic design beam shear *(kN)	36.8		55.3		58.7		73.4		89.3	
θ	0.019		0.041		0.060		0.57		0.51	
Artificial earthquake / El Centro.	Art	ElC	Art	ElC	Art	ElC	Art	ElC	Art	ElC
μ_{calc}	4.60	5.04	5.56	3.77	6.67	4.45	6.67	3.93	6.71	3.61
β_{pred}	1.81	0.67	0.67	0.23	0.73	1.04	0.59	0.61	0.47	0.53
β_{req}	2.21	0.79	0.28	0.20	0.71	0.71	0.29	0.78	0.43	0.73
Predicted increase in beam shear *(kN)	14.7	5.43	10.1	3.73	21.6	30.7	20.4	21.0	17.0	19.2
Required increase in beam shear *(kN)	17.9	6.41	4.5	3.24	21.0	21.0	10.0	27.0	15.6	26.5
Required % increase in beam shear	49	17	8	6	36	36	14	37	7	30
Max. drift without P-Δ+	.011	.013	.011	.008	.015	.010	.014	.008	.012	.006
Max. drift with P-Δ+	.019	.014	.012	.007	.020	.016	.019	.011	.017	.009
Max. drift with P-Δ, strength enh.+	.016	.014	.012	.007	.018	.012	.015	.009	.013	.007

Table 2 Summary Results for frames designed for "Soft" site spectrum.

Item	No. of Levels									
	3		6		12		18		24	
Period (secs)	1.02		1.43		2.55		3.01		3.33	
Av.seismic design beam shear *(kN)	33.7		52.4		56.7		69.1		85.1	
θ	0.019		0.041		0.060		0.050		0.051	
Artificial earthquake / El Centro.	Art	ElC	Art	ElC	Art	ElC	Art	ElC	Art	ElC
μ_{calc}	9.90	5.34	9.46	3.86	5.74	4.70	4.95	4.22	4.81	3.83
β_{pred}	1.34	0.60	1.53	0.31	0.96	0.82	1.24	1.01	1.09	0.76
β_{req}	1.16	0.77	1.57	0.20	1.10	0.73	0.99	0.84	1.17	0.80
Predicted increase in beam shear *(kN)	9.9	4.4	20.6	4.2	23.4	20.0	32.7	26.6	31.4	21.9
Required increase in beam shear *(kN)	8.6	5.7	21.1	2.7	26.8	17.8	26.1	22.2	33.7	23.1
Required % increase in beam shear	25	17	40	5	47	31	38	32	40	27
Max. drift without P-Δ+	.022	.012	.017	.007	.012	.009	.010	.008	.009	.006
Max. drift with P-Δ+	.033	.013	.021	.007	.019	.015	.012	.012	.011	.009
Max. drift with P-Δ, strength enh.+	.026	.014	.020	.007	.013	.012	.010	.010	.010	.007

* Average beam shear in lower ⅓ of beams + Interstorey drift

two criteria. The first related to the top deflection and the second to the maximum interstorey drift. The ductility demands were taken as equal when

$$\delta^p_{max} = \delta^o_{max} + \mu_{calc} * \beta_{req} * \theta * \delta^o_{max}, \qquad (8)$$

where the symbol δ represents the top deflection or the maximum interstorey deflection. In all cases the equivalent ductility demand was made on the most severe of these criteria. Generally both gave similar results.

The results of these analyses are summarised in Tables 1 and 2. Table 1 summarises the results for frames designed for a "normal" site whereas Table 2 shows the results for a "soft" site. Both sets of frames have been analysed for two ground motions; an artificial motion which has a response spectrum that approximately matches the design spectrum, and El Centro 1940 NS. In both tables the results for the artificial ground motion are given first. The accuracy of the method outlined is measured by the closeness of the "β_{pred}" and "β_{req}" values. In most cases the required amount of strength enhancement is within 20% of the amount predicted. The differences are caused by a number of factors, one of which is that the "$\beta=1$" strengths are calculated from an elastic deflected shape profile which can be different from the profiles observed with non-linear response.

It is pertinent to note that the magnitude of the increase in strength required to support the P-delta forces ranges up to 50% of the basic design strengths of the beams in the lower third of the structure. These and other values in Tables 1 and 2 correspond to equal ductility demand. As the strengths have been increased the interstorey drifts are greater than for the basic frame. If an equal drift requirement was employed, a larger increase in strength would be required.

The results presented for the El Centro ground motion illustrate that the method is applicable for earthquakes other than those that have response spectra similar to the design spectra used. Although for any frame the "β" values for the El Centro earthquake differ from those for the artificial earthquake, the "predicted" and "required" values are still similar.

5. DISCUSSION AND CONCLUSION

The method outlined in this paper is able to use the results generated from a single degree of freedom analysis to predict the additional strength requirements of a multistorey structure to resist P-delta effects. The accuracy of the method has been illustrated by comparing "predicted" and "required" additional strengths for specific earthquake ground motions.

The strengths given to the structure that were analysed were minimum values to comply with design codes requirements. Most structures in practice have a reserve strength in excess of these values, which in practice can be used for resisting P-delta actions.

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APPENDIX: Approximate Method for the Calculation of the Fundamental Period with P-delta effects.

The fundament period allowing for P-delta effects, T'_1, is given by the equation -

$$T_1' \doteq 2\pi \sqrt{\frac{\sum w_i \Delta_i^2}{g\left(\sum F_i \Delta_i - \sum_{i=1}^{n} d_i^2 h_i \sum_{j=i}^{n} w_j\right)}} \quad \text{(A1)}$$

where d_i is the interstorey drift in storey i which is defined by $d_i=(\Delta_i-\Delta_{i-1})/h_i$, h_i is the interstorey height, w_i is the seismic weight at level i, and F_i is the equivalent static force giveing rist to the lateral deflections, Δ_i.

If Δ_i are obtained from a modal response spectrum analysis, F_i may be calculated as the equivalent inertia forces acting at each level, thus

$$F_i = \frac{w_i}{g}\frac{\Delta_i}{} \left(\frac{2\pi}{T_i}\right)^2 \quad \text{(A2)}$$