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Random response of plane SCWB frames under severe seismic excitation

Y. Lei & F. Ziegler
Department of Civil Engineering, Technical University of Vienna, Austria

ABSTRACT: According to the essential feature of earthquake resistant design, frames with strong columns and weak beams (SCWB) are preferable. The probabilistic theory of effective loading developed recently for elasto–plastic structures is extended to analyze the random response of such plane frames subjected to in–plane earthquake ground acceleration. For redundant framed structures, it is shown by simulation results that the plastic hinge approximation is simple and reliable. The total displacement responses are split into elastic parts and inelastic parts. The elastic parts are calculated from the associated linear background structures with initial stiffness but under updated effective excitations while the inelastic parts are reasonably well modeled by Markov processes with properties estimated from the responses of the background structures.

1 INTRODUCTION

Vanmarcke (1976) developed a semi-probabilistic approach that was somewhat improved by Irschik (1986) though the consideration of the updated effective earthquake loading of the linear background structure for shear-beam frames. Since shear-beam designs with plastic zones developed in the columns are hazardous, we extend the approach to the frames with strong columns and weak beams which are preferable, because the failure of a column is often catastrophic but that of a beam will not cause the collapse of the whole structures. Plastic zones in the model are concentrated in the beams. The force–displacement relation shows hardening. The latter is linear in the case of the plastic hinge model. See also the recent paper by Bhartia and Vanmarcke (1991) where elastic -plastic structures with tri-linear hardening model are considered.

2 STRUCTURAL MODELING

A multi-story plane frame, shown in Fig 1, under seismic excitation is modeled as follows :

a) The structure is excited by the horizontal component of the ground motion which is assumed to be a random process. Axial deformations of the columns are neglected.

b) The masses are lumped at the floor levels and their rotational inertia is neglected.

c) Following the strong column-weak beam (SCWB) design philosophy of moment resisting steel frames (Eliopoulos and Wen (1991)), columns are designed to be stronger than beams. Thus all columns remain elastic and all the plastic deformation is concentrated in the beams. A linear column - beam element at each story and a rotational inelastic spring at each floor level represent this behavior, Fig. 2.

* Chairman of the Austrian Association for Earthquake Engineering (OGE)

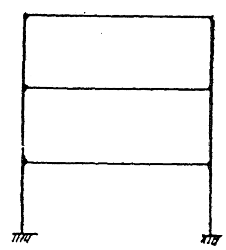

Fig. 1

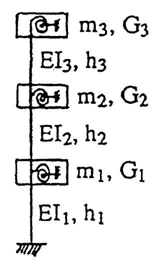

Fig. 2

d) The constitutive relation of the steel beam is assumed to be ideal elastic-plastic, i.e., $\sigma = E(\varepsilon - \varepsilon^*)$.

e) The lateral displacements are not very large and therefore the geometric nonlinear effects are neglected. Damage is not included while this effect has recently been studied by the authors (1991).

3 RANDOM RESPONSE OF SINGLE–STORY FRAME (SDOF – SYSTEM)

3.1 Equation of motion

Using the principle of virtual work, the equation of motion for the lateral displacement $\overline{w}(t)$ of a one-story frame, as shown in Fig.3, is derived as

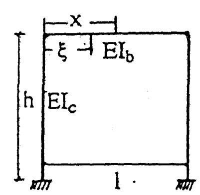

Fig. 3

$$\ddot{\overline{w}}(t) + \omega_0^2 \overline{w}(t) = -\ddot{w}_g(t) + 2\omega_0^2 \int_0^l M_h(\xi)\kappa^*(\xi)d\xi \quad , \qquad (1)$$

where $\omega_0^2 = \dfrac{1}{m\delta}$; $\kappa^* = \dfrac{1}{I_b}\int_A z\,\varepsilon^* dA$,

δ is the flexibility of the frame (the lateral displacement of the elastic frame due to a horizontal unit force at the floor level) and M_h is the corresponding moment distribution along the beam.

$\overline{w}(t)$ can be split into $w^*(t)$ and $w(t)$ as

$$w^* = 2\int_0^{\frac{l}{2}} M_h(\xi)\kappa^*(\xi)d\xi \quad , \qquad (2a)$$

$$\ddot{w}(t) + 2\zeta_0\omega_0\dot{w}(t) + \omega_0^2 w(t) = -\ddot{w}_g(t) - \ddot{w}^*(t) \quad . \qquad (2b)$$

In Eq.(2b), light viscous damping is added.

3.2 Deterministic solution and simulation

A fast and efficient deterministic algorithm is needed for reliable simulation. Contrary to the standard incremental stiffness formulation for structures under severe excitation, Irschik and Ziegler (1989) introduced the concept of updated internal loadings which act upon the associated linear background structures with the primary linear stiffness assigned.

3.2.1 Finite spread of plastic zone

In Eq. (2), the non-compatible curvature is treated as a fictitious additional loading to be updated in the course of time. A time-stepping solution is done in an incremental formulation. Assuming the initial values for $\Delta\kappa^*$ at time t_p, Δw_p^* is estimated form Eq. (2a) and using a linear time-ramp function within Δt, Δw_p can be evaluated from Eq.(2b).

Analogous to Eq. (2), the increment of vertical displacement of the beam at point ξ becomes

$$\Delta v_p(\xi) = 2\int_0^l M_v(\xi,x)\Delta\kappa^*(x)dx + \frac{\delta_v(\xi)}{\delta}\Delta w \quad , \qquad (3)$$

where $\delta_v(\xi)$ is the lateral displacement due to a vertical unit force applied at point ξ on the beam and $M_v(\xi,x)$ is the relevant bending moment at point x in the beam, (Fig.3). Thus the increment of curvature and strain are

$$\Delta\kappa_p = -\Delta v_p''(\xi) \quad ; \quad \Delta\varepsilon_p = z\Delta\kappa_p \quad . \qquad (4)$$

The nonlinear equations are solved by means of the modified Newton-Raphson algorithm for the ideal elastic plastic stress-strain relation, dividing the beam into cells, Hayek et al (1990).

The generation of the input is done by Shinozuka and Goerge (1991)

$$\ddot{w}_g(t) = \sum_k \sqrt{2S(\omega_k)\Delta\omega_k}\cos(\omega_k t - \theta_k) \quad , \qquad (5)$$

where θ_k is uniformly distributed between 0 and 2π.

3.2.2 Plastic hinge model

In the plastic hinge model, plasticity is concentrated at two plastic hinges at the two ends of the beam, close to the frame's corner.

$$w^* = 2M_h(0)\theta \quad , \qquad (6a)$$

$$\ddot{w}(t) + 2\zeta_0\omega_0\dot{w}(t) + \omega_0^2 w(t) = -\ddot{w}_g(t) - \ddot{w}^*(t) \quad . \qquad (6b)$$

The incompatible angle of rotation is again treated as an internal loading. Analogously, the incremental rotation angle is derived in a time-stepping procedure. Nonlinear equations are solved in an iterative manner considering the ideal elastic-plastic $M - \theta$ relation in the plastic hinge model.

3.3 Results and analysis

Structural parameters of the frame (Fig.3) are selected $EI_c / EI_b = 4$, $E = 210$ kN/mm^2 , $h = l = 10$ m, ζ=0.02, m=2089 kg, $\varepsilon_y = 0.002$, $M_y = 9.45*10^5$Nm

Ground acceleration is assumed to be stationary wide-banded white noise with (one-sided) power spectrum density S_o. A non-dimensional input parameter H is introduced, Hayek et al (1990)). $H = \omega_0^3 a^2/S_0$ and $a = M_y / M_h*\delta$ is the yield-limit of $\overline{w}$ in the plastic hinge model.

For the above two cases, simulation have been performed. It is noted that although the individual system response may differ for the above two cases, there is only a slight difference between the sample averages. Fig. 4 compares the standard-deviation of w^* for different H .

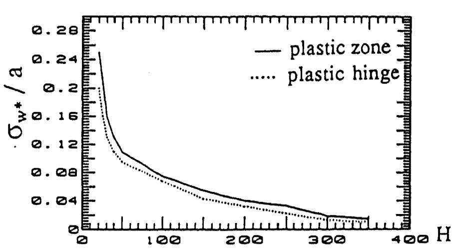

Fig. 4

However, simulation work in the plastic hinge model saves much computational time.

Therefore, in the analysis of the random response of framed structures with SCWB model, the plastic hinge approximation is simple and reliable.

It is also noted that there exist stationary random responses for SCWB framed structures under stationary excitations due to the elastic stiffness of the columns. On the contrary, drift responses increase with time for shear-beam frames in the plastic hinge model (Irschik (1986)).

3.4 Approximate stochastic analysis

Based on the simulation results above, the analysis is carried out in the plastic hinge model.

3.4.1 *Stationary response under stationary wide-banded white noise excitation*

3.4.1.1 Response of associated background structure

In Eq.(2b), w*(t) appears as an additional fictitious external loading of the associated linear background structure. The loading, however, is changed due to plastic events and an effective input concept is introduced. We characterize the effective excitation including the nonlinear influence, by changing the given external power spectrum density through a reduction factor $\psi_0 \in [\,0,1\,]$. This idea can also be interpreted from the concept of overall energy balance, Bhartia and Vanmarcke (1991).

The variance of w is then given by

$$\sigma_w^2 = \frac{\psi_0 \pi s_0}{4\zeta\omega_0^3} \quad . \tag{7}$$

For the elastic frame, the rotation angle at the corner and the lateral displacement are related by $\theta = \left(\frac{\tilde{\theta}}{\delta}\right) w$, where $\tilde{\theta}$ is the angle of rotation at the corner *due to a horizontal unit force at the floor level.*

$$\sigma_\theta^2 = \left(\frac{\tilde{\theta}}{\delta}\right)^2 \sigma_w^2 \quad . \tag{8}$$

In the plastic hinge model, the yield-barrier for θ is $\theta_y = M_y / M_h \tilde{\theta}$.

In order to get ψ_0, a power balance relation is considered.

$$\frac{m\pi s_0}{2}(1-\psi_0) = 2M_y\left[\mu^+\langle\Delta\theta^{*+}\rangle - \mu^-\langle\Delta\theta^{*-}\rangle\right] \quad , \tag{9}$$

where $\langle\Delta\theta^{*+}\rangle$, $\langle\Delta\theta^{*-}\rangle$ are mean increments of the nonlinear rotation angle in positive and negative yield excursion respectively, and μ^+ , μ^+ are the relevant mean rates of excursion .

With a certain θ^* value, it can be shown from the analysis of structural elastic deformation that the equilibrium position for θ is

$$\theta' = \theta^* / 2\upsilon \ , \ \upsilon = 2\,(1+6hEI_b / lEI_c) \tag{10}$$

For low yield-barrier, *yield excursions occur in* clumps and a clump of yield excursions may be viewed as a "single" event (Vanmarcke (1976)), then

$$\mu^\pm = \frac{\omega_0}{2\pi E\{N^\pm\}} \exp\left[-\frac{(\pm\theta_y - \theta')^2}{2\sigma_\theta^2}\right] \tag{11}$$

where $E\{N^+\}$, $E\{N^-\}$ are the mean clump sizes and Lutes et al (1980) gave an improved semi-empirical approximation for $E\{N^\pm\}$.

The mean yield increments in Eq.(9) are determined by considering further the energy relation during a yield excursion.

$$T_2 - T_1 = \Delta W_p - \Delta U \tag{12}$$

where $T_2 = 0$, $T_1 = -\frac{1}{2} m \dot{w}^2$, ΔU is the change of potential energy of the frame in its yielding phase. This corresponds to the change of elastic strain energy in the two columns since the strain energy of the beam does not change. It can be proved that during a yielding phase,

$$U(\theta) = \frac{4EI_c}{3h}\theta^2 + \frac{2}{3}M_y\theta + \frac{M_y^2 h}{6EI_c} \quad , \tag{13}$$

$\Delta W_p = -2M_y|\Delta\theta^*|$ is the e plastic work in the hinge

With a certain θ^* value, the prescribed background structure responds in the elastic region about the mean value θ'. Thus yield excursions in the positive *and negative directions must be considered* separately.

$$\frac{m\dot{w}^2}{2} = U(\theta^* + \theta_y + \Delta\theta^{*+}) - U(\theta^* + \theta_y) + 2M_y\Delta\theta^{*+}, \tag{14a}$$

$$\frac{m\dot{w}^2}{2} = U(\theta^* - \theta_y + \Delta\theta^{*-}) - U(\theta^* - \theta_y) - 2M_y\Delta\theta^{*-}. \tag{14b}$$

The increments during a yield excursion is assumed to be exponentially distributed (Bhartia and Vanmarcke (1991)). Taking expectations on both sides of Eq. (14) leads to the quadratic equations for $\langle\Delta\theta^{*+}\rangle$, $\langle\Delta\theta^{*-}\rangle$.

Finally, with certain θ^* value, the above nonlinear equations for ψ_0 can be solved iteratively to get conditional statistics for the background structure.

3.4.1.2 Nonlinear response

There is a change in the nonlinear part response during each yield excursion. From Eq.(14), this change depends only upon the most recent value of θ^* and therefore it is reasonable to model $\theta^*(t)$ by a Markov process (Grossmayer et al (1981), Bhartia and Vanmarcke (1991)).

The corresponding FPK equation for the transition probability density of $\theta^*(t)$ is

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial \theta^*}\left[\beta_1(\theta^*)\,p\right] + \frac{1}{2}\frac{\partial^2}{\partial \theta^{*2}}\left[\beta_2(\theta^*)\,p\right] , \qquad (15a)$$

$$\text{with } \beta_1(\theta^*) = \lim_{\tau\to 0}\frac{E\left[\Delta\theta^* \mid \theta^*(t)=\theta^*\right]}{\tau} = \mu^+\langle\Delta\theta^{*+}\rangle + \mu^-\langle\Delta\theta^{*-}\rangle , \qquad (15b)$$

$$\beta_2(\theta^*) = \lim_{\tau\to 0}\frac{E\left[\Delta\theta^{*2} \mid \theta^*(t)=\theta^*\right]}{\tau} = 2\left[\mu^+\langle\Delta\theta^{*+}\rangle^2 + \mu^-\langle\Delta\theta^{*-}\rangle^2\right] .$$

For stationary response, solving FPK equation yields

$$p(\theta^*) = \frac{c}{\beta_2}\exp\left[\int_0^{\theta^*}\frac{2\beta_1(\theta)}{\beta_2(\theta)}d\theta\right] , \qquad (16)$$

where c is the normalization constant.

Then the unconditional statistical value for the associated linear structure can be evaluated further on the basis of the total probability theorem.

3.4.1.3 Results and comparison

Comparison between the approximate results and the sample averages is made in Figs.(6–7).

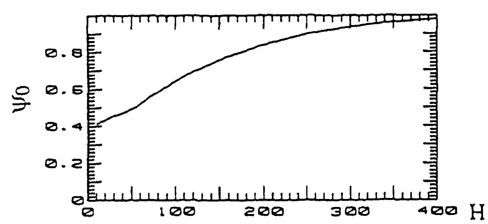

Fig. 5

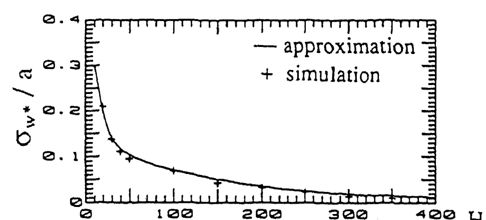

Fig. 6

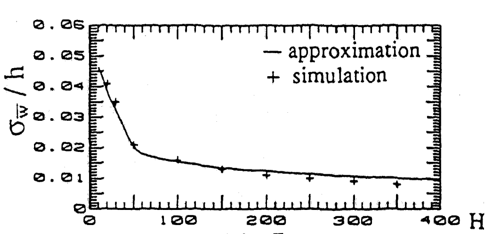

Fig. 7

3.4.2 Nonstationary response

The strong motion phase of the seismic input is most realistically modeled as a nonstationary random process. Such a process is commonly characterized by an evolutionary power spectrum constructed from a stationary random process with a one-sided spectral power density $S_0(\omega)$ and a slowly varying deterministic envelope $A(t,\omega)$, $S(t,w) = A^2(t,\omega)S_0(\omega)$.

Usually, the frequency dependence of A is not important and the time evolution of A which accounts for the build-up and die-off segments of typical earthquake records may be assumed in the form: $A(t) = c\,[\exp(-\alpha t) - \exp(-\beta t)]$, $\beta > \alpha$.

$S_0(\omega)$ is assumed to be wide-banded: band-limited white noise and the filtered white noise representation are frequently used, Irschik and Ziegler (1991).

3.4.2.1 Wide-banded white noise

We characterize the effective excitation by changing the given time envelope function and including frequency dependence : $\varphi(t,\omega) = A^2(t)\psi_0(t,\omega)$. Consequently, the evolutionary power spectrum becomes : $\varphi(t,\omega)S_0$.

The response variance has been approximated by Spanos (1980).

$$\sigma_w^2 = \frac{\pi s_0}{2\omega_0}e^{-2\zeta\omega_0 t}\int_0^t \exp(-2\zeta\omega_0\tau)\,\varphi(\tau,\omega)\,d\tau \qquad (17)$$

In order to evaluate ψ_0, the power balance equation is generalized, Irschik (1986), while μ^+, μ^- can be derived analogously to Eq.(11) with the semi-empirical expression for $E\{N^\pm\}$ adapted for the nonstationary case (Lutes et al (1980)).

$\langle\Delta\theta^{*+}\rangle$, $\langle\Delta\theta^{*-}\rangle$ are still evaluated from the energy relation during the yielding phase. They depend also on the most recent value of $\theta^*(t)$.

With certain $\theta^*(t)$ value, the conditional statical values of background structure are estimated in time-stepping procedure. (Irschik (1986)).

The FPK equation for the transition probability density of nonlinear $\theta^*(t)$ is given by Eq.(15) except the two coefficients are time dependent.

The parabolic partial differential equation for the transition probability density of $\theta^*(t)$ can be solved numerically, e.g., the implicit finite difference scheme of Crank-Nicolson type can be applied. Evaluation is also made in a time-stepping procedure compatible with the procedure for the background structure.

The nonstationary response is analyzed. Time envelope modulation function is specified by $A(t) = 10.13\,(\exp(-0.5t) - \exp(-t))$, $H = 20$.

Comparison between the approximate results and simulation values is shown in Figs.(9-10). Approximate results are fairly satisfactory.

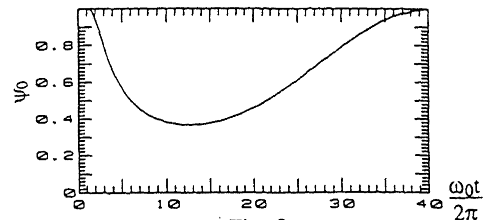

Fig. 8

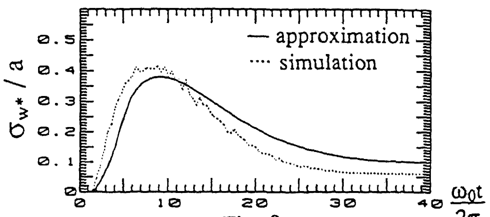

Fig. 9

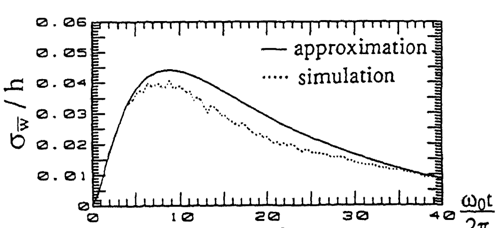

Fig. 10

3.4.2.2 Filtered white-noise representation

The Kanai-Tajimi representation is frequently used for $S_0(\omega)$, with $S_0(0) = S_0 \neq 0$.

By analyzing the available records of seismic ground motion, Boore (1983), however, obtained the asymptotic behavior : $S_0(\omega) \sim \omega^2$, as $\omega \to 0$.

Paparizos and Iwan (1988) proposed an improved model

$$S_0(\omega) = \frac{4\zeta_g(\omega/\omega_g)^2 S_0}{[1-(\omega/\omega_g)^2]^2 + 4\zeta_g(\omega/\omega_g)^2} \qquad (18)$$

In the power balance equation, the input power , by using a formula of Ohi and Tanaka (1984), becomes,

$$E\{p_i\} = \frac{m\pi S_0}{2} A^2(t) F(\omega_0) \ ,$$

$$F(\omega_0) = 4\omega_0\omega_g^2\zeta_g^2(\omega_0\zeta_g + \omega_g\zeta) \, / \, \{\zeta_g[(\omega_g^2 - \omega_0^2) + 4\omega_g\omega_0\zeta_g^2\zeta\,(\omega_g^2 + \omega_0^2) + 4\omega_g^2\omega_0^2(\zeta_g^2 + \zeta^2)]\} \qquad (19)$$

The nonstationary random response of the frame under the same time modulation is also analyzed similarly. Parameters of the Iwan spectrum are chosen as $\omega_g/\omega_0 = 1.5$; $\zeta_g = 0.25$, H= 20 .

As shown by Figs.(11-12), fairly good results are obtained from the approximate theory compared with simulation.

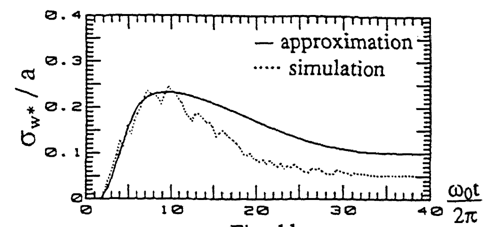

Fig. 11

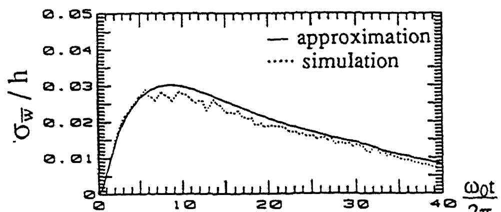

Fig. 12

4 RANDOM RESPONSE OF MULTI-STORY FRAMES (MDOF–SYSTEM)

we take a two-story frame out of the Fig.1 and Fig. 2 as an example and use the plastic hinge model.

4.1 Equations of motion

Using the principle of virtual work, the equations of motion for lateral displacements are derived as

$$\ddot{\bar w}_1 + \frac{m_2\delta_{12}}{m_1\delta_{11}}\ddot{\bar w}_2 + k_1\bar w_1 = -\left(1 + \frac{m_2\delta_{12}}{m_1\delta_{11}}\right)\ddot w_g + 2k_1(M_{12}\dot\theta_2 + M_{11}\dot\theta_1) \qquad (20a)$$

$$\ddot{\bar w}_2 + \frac{m_1\delta_{21}}{m_2\delta_{22}}\ddot{\bar w}_2 + k_2\bar w_2 = -\left(1 + \frac{m_1\delta_{21}}{m_2\delta_{22}}\right)\ddot w_g + 2k_2(M_{21}\dot\theta_1 + M_{22}\dot\theta_2) \qquad (20b)$$

where δ_{ij} are elements of the flexibility matrix, $k_i = 1/m_i\delta_{ii}$ and M_{ij} are elements of Green´s influence matrix for rotations of the column-beam connections.

The lateral displacements $\bar w_1$, $\bar w_2$ can be split $\bar w_1(t) = w_1(t) + w_1^*(t)$; $\bar w_2(t) = w_2(t) + w_2^*(t)$,

$$\begin{Bmatrix} w_1^* \\ w_2^* \end{Bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{Bmatrix} 2\theta_1^* \\ 2\theta_2^* \end{Bmatrix} \tag{21a}$$

$$\begin{bmatrix} 1 & \frac{m_2\delta_{12}}{m_1\delta_{11}} \\ \frac{m_1\delta_{21}}{m_2\delta_{22}} & 1 \end{bmatrix} \begin{Bmatrix} \ddot{w}_1 \\ \ddot{w}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix} = \tag{21b}$$

$$- \begin{bmatrix} 1 & \frac{m_2\delta_{12}}{m_1\delta_{11}} \\ \frac{m_1\delta_{21}}{m_2\delta_{22}} & 1 \end{bmatrix} \begin{Bmatrix} \ddot{w}_g + \ddot{w}_1^* \\ \ddot{w}_g + \ddot{w}_2^* \end{Bmatrix}$$

4.2 Deterministic solution and simulation

In Eq. (21), the nonlinear effect renders fictitious external loading acting on the background structure and thus modal expansion technique can still be applied, $\vec{w} = [\phi]\vec{q}$. Then, the principal coordinates q_j are represented by uncoupled oscillators.

$$\ddot{q}_j + 2\zeta_j\omega_j\dot{q}_j + \omega_j^2 q_j = f_j \ , \tag{22}$$

where a light viscous modal damping is added and

$$f_j = -(\mathscr{L}_j\ddot{w}_g + \sum_{i=1}^{2} \mathscr{L}_{ji}\ddot{w}_i^*) / m_j^{*2} \ . \tag{23}$$

Modal masses $m_j^* = \vec{\phi}_j^T [m]\vec{\phi}_j$ and $[m]$ is mass matrix in Eq.(23b). Participation factors are given by

$$\mathscr{L}_j = \vec{\phi}_j^T [m]\vec{I} \ , \quad \vec{I} = \sum_{i=1}^{2} \vec{e}_i \ , \qquad \mathscr{L}_{ji} = \vec{\phi}_j^T [m]\vec{e}_i \ , \tag{24}$$

respectively, where $\vec{e}_i$ denotes the i-th unit vector.

Using a time-stepping procedure with incremental formulation, $\overrightarrow{\Delta w^*}$ can be estimated with Eq.(21a) and $\Delta\vec{w}$ can be calculated from Eq.(21b) with linear solution strategies.

Analogous to Eq.(20), equations of motion for the rotation angles at the corners can also be derived and therefore, the increments of rotation angles can be obtained. The ideal elastic-plastic $M_i - \theta_i$ relation for each plastic hinge is used to calculate the response iteratively.

4.3 Approximate stochastic analysis

For simplicity, the stationary response of the frame under wide-banded white noise excitation is analyzed

In Eq.(24), effective modal loading input is introduced and the effective power spectral densities are $\psi_j S_0$. Then

$$\sigma_j^2 = \frac{\psi_j \pi S_0 \mathscr{L}_j^2}{4\zeta\omega_j^3 m_j^{*2}} \tag{25}$$

and the variance of the relative story elastic displacement: $\tilde{w}_1 = w_1$; $\tilde{w}_2 = w_2 - w_1$ are

$$\sigma_i^2 = \sum_{j=1}^{2} \alpha_{ji}\, \beta_{ji}^2\, \sigma_j^2 \quad ; \quad \beta_{ji} = \phi_{ji} - \phi_{j\,i-1} \ , \tag{26}$$

where $\alpha_{ji} = 1 + \sum_{l=1}^{2} (\mathscr{L}_l m_j^* \beta_{li} / \mathscr{L}_j m_l^* \beta_{ji})\, A_{jl}|_{l\neq j}$,

A_{jl} have been derived by Vanmarcke (1976).

For elastic frames, the rotation angles at the corners and the lateral displacements are related by

$$\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = [c] \begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix} = [c][\phi] \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = [c'] \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \ , \tag{27}$$

where $[c]$ is a matrix determined by structural parameters, therefore,

$$\sigma_i^2(\theta) = \sum_{j=1}^{2} \alpha'_{ji}\, c'^2_{ji}\, \sigma_j^2 \ , \tag{28}$$

where $\alpha'_{ji} = 1 + \sum_{l=1}^{2} (\mathscr{L}_l m_j^* c'_{li} / \mathscr{L}_j m_l^* c'_{ji})\, A_{jl}|_{l\neq j}$.

In order to get ψ_j, a modal power relation is considered

$$\frac{m_j^* \pi s_0 \mathscr{L}_j^2}{2m_j^{*2}} (1-\psi_j) = \sum_{i=1}^{2} 2M_i^y \left[\mu_i^+ \langle \Delta\theta_i^{*+} \rangle - \mu_i^- \langle \Delta\theta_i^{*-} \rangle \right] \Gamma_{ji} \tag{29}$$

where Γ_{ji} are the mode participation factors that have been derived by Irschik and Ziegler (1989).

With certain $\vec{\theta^*}$ values, it can be shown from the elastic deformation that the equilibrium positions for $\vec{\theta}$ are $\vec{\theta'} = [D]\vec{\theta^*}$, where $[D]$ is a matrix defined by the structural parameters. Therefor μ_i^+ and μ_i^- are

$$\mu_i^{\pm} = \frac{\upsilon_{i0}}{E\{N_i^{\pm}\}} \exp\left[-\frac{\left(\pm\theta_i^y - \theta_i'\right)^2}{2\sigma_i^2(\theta)} \right] \ , \tag{30}$$

where υ_{i0} have been derived by Vanmarcke (1976).

The mean yield increments are determined from the energy relations during a yielding phase. Assuming i–th story to yield alone,

$$T_{2i} - T_{1i} = \Delta W_{pi} - \Delta U_i \tag{31}$$

where $T_{2i} \approx 0$, $T_{1i} = \frac{1}{2} m_i \dot{\vec{w}}_i^2$, $\Delta U = U_{i2} - U_{i1}$, is calculated from the deformation of the frame caused by the inertia force and yielding moment at the i-th story level, and $\Delta W_{pi} = -2M_i^y * |\Delta\theta_i^*|$.

Yielding in the positive and negative have also to be considered separately and $\langle \Delta\theta_i^{*+} \rangle$, $\langle \Delta\theta_i^{*-} \rangle$ depend on their most recent $\vec{\theta^*}$ values.

Finally, with certain $\vec{\theta^*}$ values, the conditional statistics for $\vec{w}$ can be calculated iteratively.

$\vec{\theta^*}(t)$ is also reasonably modeled by a vector Markov process with the FPK equation

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial \dot{\theta}_i}[A_i p] + \frac{1}{2}\frac{\partial^2}{\partial \dot{\theta}_i \partial \dot{\theta}_j}[B_{ij} p] \quad (32a)$$

where
$$A_i = \lim_{\tau \to 0} \frac{E\left[\Delta\theta_i^* \mid \vec{\theta^*}(t) = \vec{\theta^*}\right]}{\tau} = \mu_i^+ \langle \Delta\theta_i^{*+} \rangle + \mu_i^- \langle \Delta\theta_i^{*-} \rangle \quad (32b)$$

$$B_{ij} = \lim_{\tau \to 0} \frac{E\left[\Delta\theta_i^* \theta_j^* \mid \vec{\theta^*}(t) = \vec{\theta^*}\right]}{\tau} = \begin{cases} 2\left[\mu_i^+ \langle \Delta\theta_i^{*+} \rangle^2 + \mu_i^- \langle \Delta\theta_i^{*-} \rangle^2\right] & i=j \\ 0 & i \neq j \end{cases}$$

5 CONCLUSIONS

The idea of updated internal loading is incorporated in an efficient algorithm for statistical averaging of the random response of SCWB frames. The plastic hinge model is shown to be simple and reliable.

The probabilistic approximate theory of effective loading is modified and extended to analyze both the stationary and nonstationary random response of framed structures with SCWB model under seismic excitation. The validity of the approximation has been verified by the statistical averaging. The advantage of the approximation is the possibility of calculating the inelastic responses based on the responses of the associated linear background structures to make it be more straightforward and suitable in engineering application. The inelastic statistics obtained are also of importance for further structural reliablity analyses.

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