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Higher order peaks in response of multistoried buildings

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ABSTRACT: Knowledge of the higher order peak amplitudes becomes an essential input to the seismic design of buildings when the maximum stresses may repetitively exceed the elastic design limit. It is useful to understand how these higher order peak amplitudes depend on various governing parameters and what are their amplitudes in terms of the largest peak amplitudes.

INTRODUCTION

In the design of earthquake-resistant buildings, it is not feasible to design for elastic response to moderate and strong earthquakes. Therefore, current design practices allow excursions of the response into the nonlinear range during very strong ground shaking, and so it becomes important to ensure the safety of the structure during these excursions. This may be achieved in part, with simplified design procedures, and involving only small excursions into the nonlinear range, with the knowledge about the amplitudes and the number of these excursions. Then, one needs to estimate not just the largest peaks in the structural response, but also the second, third, the fourth and so on, largest peaks. A comparison of the peak values with the chosen design amplitude level can furnish the information about the number and the extent of the excursions of nonlinear response (Amini and Trifunac, 1985; Lee and Trifunac, 1986).

The purpose of this paper is to study the behavior of these higher order response peaks, normalized by the corresponding largest peak amplitudes, and along with their dependence on various structural and earthquake parameters. For this, a recently developed statistical approach has been employed (Gupta and Trifunac 1987, 1988, 1989, 1990a,b, 1991). It can give the response amplitudes of the higher order peaks for a given probability of exceedance. For this study, fixed base buildings have been considered with different number of stories, and different mass and stiffness distribution along the height. Those have been subjected to synthetic accelerograms corresponding to artificial earthquakes occurring at different hypocentral distances. The expected amplitudes have been obtained for the first, second, third and fourth order peaks of various response functions.

PROBABILISTIC ESTIMATES OF RESPONSE PEAKS

Stochastic modeling of a seismic response function in a building, say $f(t)$, can be done by assuming that it forms a part of a zero mean stationary random process, and that its peaks are statistically independent and identically distributed. If $f(t)$ is represented by the sum of an infinite number of sine waves with random amplitudes and phases, it is possible to derive the probability density function for the peaks of $f(t)$, normalized by a_{rms}, the root-mean-square (r.m.s) value of $f(t)$, and in terms of ϵ which is a measure of the width of the mean square spectral density, $S(\omega)$, of $f(t)$ (Cartwright and Lonquet-Higgins, 1956). For $\epsilon = 0$ and 1, this distribution becomes Rayleigh and Gaussian respectively. Further, using the order statistics, the distribution of the peaks of $f(t)$ can be used to obtain the distribution and then the expected values of its largest or a higher order peak out of the total of N peaks (Gupta and Trifunac, 1988). The parameters a_{rms}, ϵ and N are obtained from the zeroth, second and fourth moments of the energy spectrum density of $f(t)$, $S(\omega)$, and from the time duration, T, of $f(t)$.

The determination of the spectral density function $S(\omega)$ is central to estimation of the peak amplitudes of $f(t)$. For a n-story building, modeled as a shear beam with lumped masses and discrete springs (Fig. 1), for the i^{th} floor displacement, this function can be written as (Gupta and Trifunac, 1989, 1990),

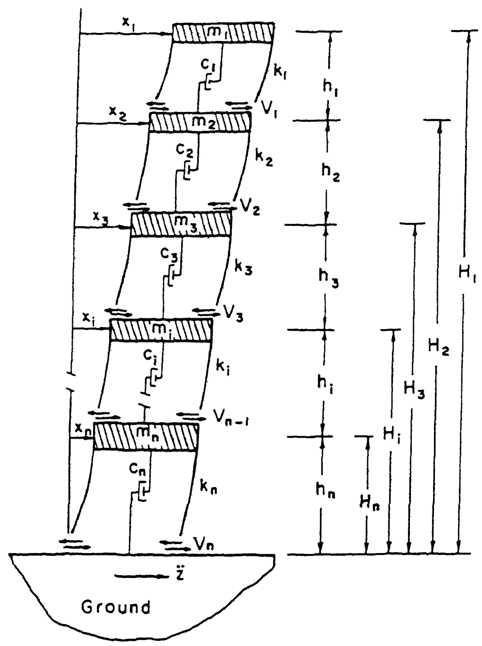

Fig. 1 Shear beam model of a multi story building.

$$S_i(\omega) = \overline{S}(\omega)\left[\sum_{j=1}^{n}(\phi_i^{(j)})^2\alpha_j^2|H_j(\omega)|^2\{1+\delta_{ij}\}\right] \quad (1)$$

where $\overline{S}(\omega)$ is the power spectrum density function of the ground acceleration $\ddot{z}(t)$. For "not-too-short" earthquake excitations, this can be expressed in terms of $|Z(\omega)|$, the Fourier transform of $\ddot{z}(t)$, as (Udwadia and Trifunac, 1974)

$$\overline{S}(\omega) = \frac{1}{\pi T}|Z(\omega)|^2. \quad (2)$$

Further, in Eq. (1), $\phi_i^{(j)}$ is the ith element of the jth mode, δ_{ij} is a correction term applied to account for the interaction between various modes of vibration, and $H_j(\omega)$ is the transfer function of the relative displacement response of the jth equivalent single-degree-of-freedom (SDOF) oscillator defined as

$$H_j(\omega) = \frac{1}{\omega_j^2 - \omega^2 + 2i\varsigma_j\omega_j\omega} \quad (3)$$

with ω_j and ς_j being the natural frequency and damping ratio in the jth mode.

In the computations of ε and N for the displacement response at the ith floor, say ε_i and N_i, from Eq. (1), the correction factors δ_{ij} may turn out to have negligible effect, as these parameters involve the ratios of various moments of the spectral density function. Further, the absence of stationarity in the response function may not enter these considerations explicitly, if we study the higher order peak amplitudes as fraction of the first order peak amplitude, and thus do not involve the computation of the r.m.s. value. Spectral density expressions for shear and overturning moment response can be obtained from Eq. (1) via minor modifications (Gupta and Trifunac 1989, 1990).

ANALYTICAL FORMULATION AND RESULTS

The ratio of a higher order peak amplitude to the first order peak amplitude in this approach is a function of the parameters ε and N only. Hence, it is useful to make approximations and to simplify the expressions of ε_i and N_i for the ith floor displacement response. Assuming that the Fourier spectrum $|Z(\omega)|$ is reasonably flat over the dominant frequencies and that the first few natural frequencies of the building fall within this band of frequencies (Newland 1984), we can approximate $|H_j(\omega)|^2$ by a delta function and obtain

$$\varepsilon_i \simeq \left[1 - \frac{\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{\{\phi_i^{(j)}\}^2\{\phi_i^{(k)}\}^2\alpha_j^2\alpha_k^2}{(\omega_k/\omega_j)(\varsigma_k/\varsigma_j)}}{\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{\{\phi_i^{(j)}\}^2\{\phi_i^{(k)}\}^2\alpha_j^2\alpha_k^2(\omega_k/\omega_j)}{(\varsigma_k/\varsigma_j)}}\right]^{1/2} \quad (4)$$

and

$$N_i \simeq \frac{T}{2\pi}\left[\frac{\sum_{j=1}^{n}\{\phi_i^{(j)}\}^2\alpha_j^2(\omega_j/\omega_1)/(\varsigma_j/\varsigma_1)}{\sum_{j=1}^{n}\{\phi_i^{(j)}\}^2\alpha_j^2/(\omega_j/\omega_1)(\varsigma_j/\varsigma_1)}\right]^{1/2}. \quad (5)$$

These expressions indicate that for a fixed n, the parameters ε_i and N_i at any floor depend on i) the total duration T of the input excitation, and on ii) the distribution of floor masses and story stiffness along the building height. The latter one influences the building mode shapes and modal participation factors, with the ratios $(\varsigma_k/\varsigma_j)$, $(\varsigma_j/\varsigma_i)$, (ω_k/ω_j) and (ω_j/ω_i) remaining nearly constant. ε_i and N_i will however vary with the story level, i, due to the presence of modal shape terms in Eqs. (4) and (5), and in that the higher modes are likely to contribute significantly. If we study the relative effects of the variations in ε and N on the ratio of a higher order peak value to the largest peak value, it will be observed that this ratio decreases with increase in ε or decrease in N. Further, for the range of ε_i normally encountered (i.e. less than 0.9), variation in ε_i values along the building height is not likely to substantially affect the normalized peak values. On the other hand, the values of N_i range somewhere between 20 and 50, and for this range, there may be significant variation of the ratios of peak values along the building height. Though the effects of N and ε are opposite in nature, the ratio $E[a_{(r)}]/E[a_{(1)}]$ (i.e. the expected value of the r^{th} order peak amplitude to the expected value of the largest peak amplitude) increases with the increase in N irrespective of the variation in ε. These trends have been illustrated by considering an example of a 16-story fixed-base building with linear variation of mass and stiffness values along the building height, uniform story height a damping ratio, and fundamental period of vibration being equal to (0.1 $\times$ 16 =) 1.6 sec. The base excitation is represented by a typical Fourier spectrum, as shown in Fig. 2. Further, the total excitation duration (Trifunac and Brady, 1975) is assumed to be 20 seconds.

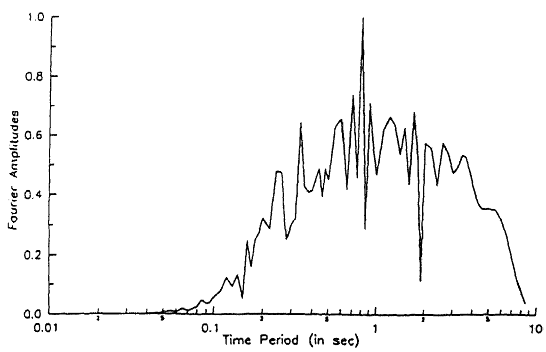

Fig. 2 Normalized Fourier spectrum of the translational (horizontal) component of example excitation.

Fig. 3 shows the variation of N_i and ε_i with the floor level for the displacement, shear force and overturning moment responses, and Fig. 4 shows the variation of the normalized expected amplitudes of the second, third and fourth order peaks for these response functions. On comparing these two figures, it may be noted that the variation along the building height of the normalized amplitudes indeed depends mainly on the total number of peaks, N. Thus, higher modes dominate near the building base in case of shear and moment responses. For this reason, buildings with smaller number of stories are likely to have less pronounced variation of normalized amplitudes along the building height. This is illustrated by Fig. 5 which shows the normalized amplitudes for a 4-story building with linear variation of masses and stiffness along the building height and with (0.1 $\times$ 4 =)0.4 sec fundamental period of vibration. Buildings with a smaller number of stories have greater tuning between the response functions at different floors, whereas in the case of a greater number of stories, various adjacent stories group together for coherent behavior, with each group having a little different behavior from the other groups. For a given response function, the difference between minimum and maximum normalized amplitudes over the building height is found to increase with the number of stories for the given order of a peak, and with the order of the peak for a given building. This makes it feasible to approximate the variation of normalized amplitudes by a constant value, only in case of short and medium tall buildings and just for the second and, probably, the third order peaks of the response functions. This includes most of the cases encountered in practice, and for these, it may be sufficient to calculate the normalized peak value at a particular floor and assume it to apply for other floors. For a given order, the normalized peak amplitudes may decrease so much (say, to 0.70) with the increase in number of stories (even in the case of the third order of peak), that designing the structure for these forces may cause the response to go too far into the nonlinear range. In those cases, a more rigorous nonlinear analysis may be necessary.

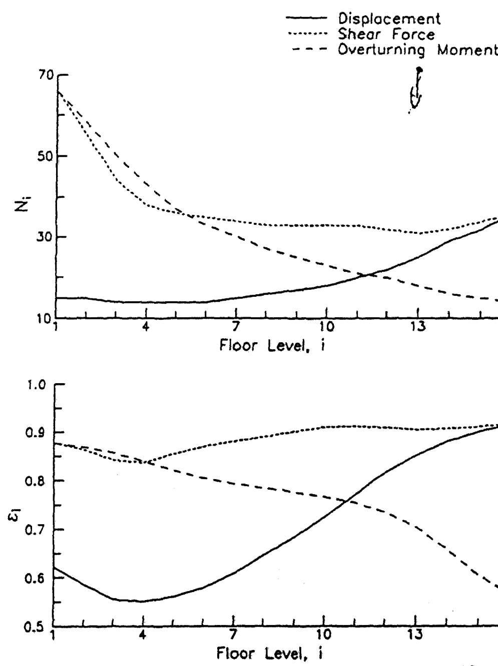

Fig. 3 Variation of ε_i and N_i with the floor level i.

The selected effects of floor mass and story stiffness distributions have been investigated by considering the example 16-story building also for uniform distribution with one step change at mid height of

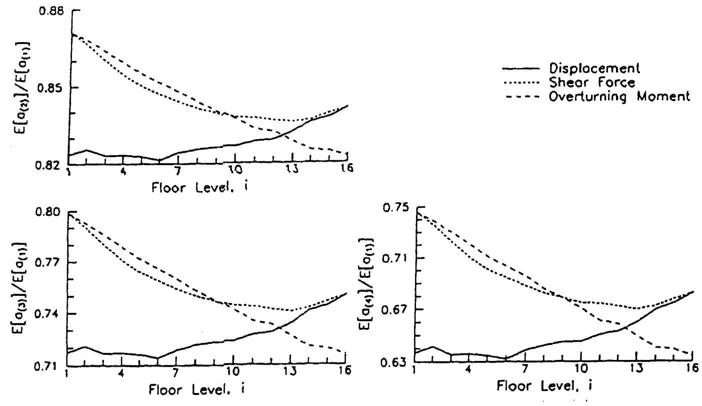

Fig. 4 Variation of normalized higher order (2, 3, 4) peak amplitudes with the floor level, for 16-story building.

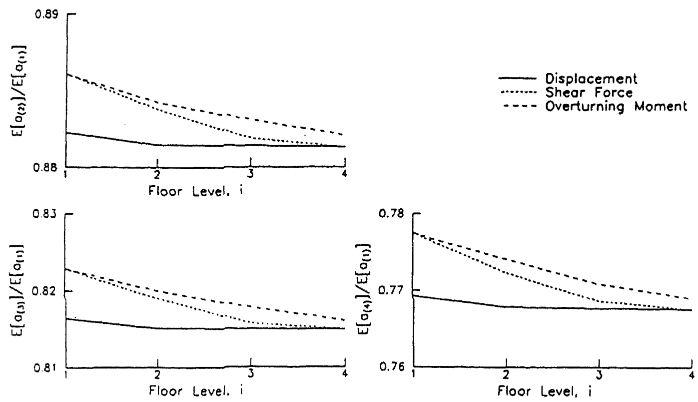

Fig. 5 Variation of normalized higher order (2, 3, 4) peak amplitudes with the floor level, for a 4-story building.

the building, and with the other parameters remaining unchanged Fig. 6 shows results of the second normalized peak for $\beta = 0.1$, 0.4, 0.7 and 1.0 where β represents the story mass and stiffness values of the top 8 stories, as fractions of those for the bottom 8 stories. It is seen that minor variations in the distribution of mass and stiffness values, from linear to those with step change, do not lead to significant changes in the peak amplitudes at any floor level.

The effects of the excitation duration T have been investigated by generating synthetic records as in Lee and Trifunac (1985, 1987), for a site at Westmoreland, Imperial Valley, California. Four cases have been considered by taking the hypocentral distances as 1, 5, 15 and 50 km. The earthquake magnitude and the confidence levels for no exceedance have been uniformly taken as 6.5 and 0.5 for these records. The resulting values of T are respectively 6.3, 9.38, 18.42 and 39.24 sec. The example 16-story building has been subjected to these excitations and the normalized second order peak amplitudes have been computed for the displacement, shear force and overturning moment responses. Fig. 7 shows that the second and higher order peak amplitudes increase with the duration of excitation, i.e. the second largest peak comes closer to the largest peak in magnitude as the disturbance lasts longer. Further, for shorter durations, these curves have greater undulations since at low values of N, the change in N is associated with a greater variation in the peak amplitudes. Thus, for larger hypocentral distances, dependence of the peak amplitudes on the story level is reduced, and the approximation by a constant may be justified.

CONCLUSIONS

Higher order peaks in the earthquake response of multistoried buildings have been investigated by studying their amplitudes as fractions of the corresponding highest peak, for a parametric variation of

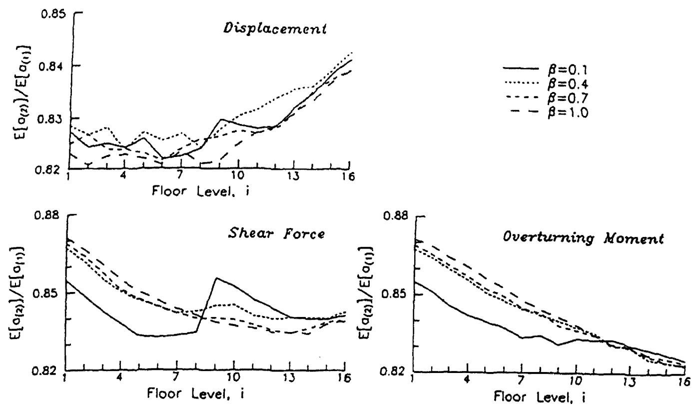

Fig. 6 Effects of floor mass and story stiffness distribution in case of step reduction by factor β.

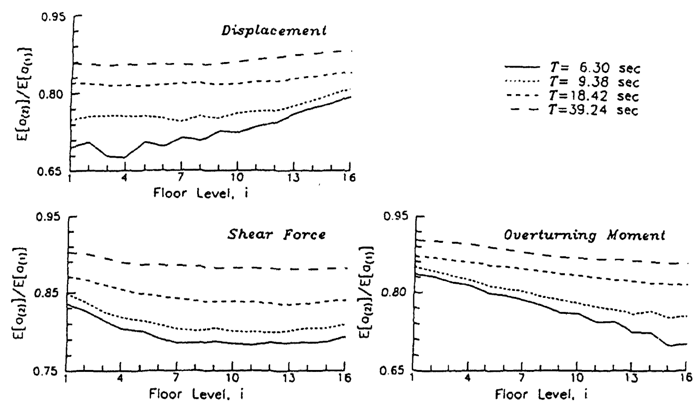

Fig. 7 Effects of excitation duration on the normalized peak amplitudes.

the building and excitation characteristics. It has been seen that the single most important parameter which significantly affects the (normalized) amplitudes of higher order peaks, is the duration of excitation. The longer is the duration, the greater are these amplitudes. The variation of the peak amplitudes along the building height for any response function is, however, influenced by the mode shapes, and by their relative participation for a building with fixed number of stories. This largely depends on the distribution of floor masses and story stiffness along the building height. It is possible to approximate such a variation by a constant value provided the building is low, the site is not located near the epicenter of the earthquake, and the order of peaks considered is not higher than three.

To the extent that this type of equivalent linear analysis can be used to describe the early stages of the response of nonlinear systems, the above examples suggest that: 1) For shorter and impulsive excitation closer to the earthquake source, the designer should aim to reduce the ratios $E[a_{(i)}]/E[a_{(1)}]$ for $i = 2, 3$ (and perhaps 4 and 5), such that for the distribution of expected excitations this ratio is nearly constant at all levels of a multi story structure. This will help to distribute the extent and the distribution of nonlinear deformation uniformly throughout the structure, and it may be accomplished in part by ingenious selection of the structural system, and the type and the geometrical distribution of the components resisting the lateral loads. 2) The characteristics of the optimal structural configuration depend not only on the level of shaking, but also on the impulsive (near) versus long (distant) earthquake shaking. To the extent that such characteristics can be identified in and around many seismic areas of the world, by using the uniform risk spectrum approach (Trifunac, 1988), for example, it is seen that the typical code design methods with only one "spectral shape" function, cannot provide uniform and balanced level of seismic resistance for all new construction.

REFERENCES

Amini, A. and M.D. Trifunac 1985. Statistical Extension of Response Spectrum Superposition, *Int. J. Soil Dynamics and Earthquake Eng.* 4(2): 54-63.

Cartwright, D.W. and M.S. Longuet-Higgins 1956. The statistical distribution of maxima of a random function. *Proc. Roy. Soc. London A* 237: 212-232.

Gupta, I.D. and M.D. Trifunac 1987. Order statistics of peaks in earthquake response of multi-degree-of-freedom systems. *Earthq. Eng. Eng. Vib.* 7(4): 15-50.

Gupta, I.D. and M.D. Trifunac 1988. Order statistics of peaks in earthquake response. *J. Eng. Mech. (ASCE)* 114(10): 1605-1627.

Gupta, V.K. and M.D. Trifunac 1989. Investigation of building response to translational and rotational earthquake excitations. *Report CE 89-02, University of Southern California, Los Angeles, California.*

Gupta, I.D. and M.D. Trifunac 1990a. Probabilistic spectrum superposition for response analysis Including the Effects of Soil-structure interaction. *J. Prob. Eng. Mech.* 5(1): 9-18.

Gupta, V.K. and M.D. Trifunac 1990b. Response of multistoried buildings to ground translation and rocking during earthquakes. *J. Prob. Eng. Mech.* 5(3): 138-145.

Gupta, V.K. and M.D. Trifunac 1991. Effects of ground rocking on dynamic response of multistoried buildings during earthquakes. *Struct. Eng. / Earthquake Eng. JSCE* 8(2): 43-50.

Lee, V.W. and M.D. Trifunac 1985. Torsional accelerograms. *Int. J. soil Dynamics and Earthquake Eng.* 4(3): 132-139.

Lee, V.W. and M.D. Trifunac 1986. A note on time of maximum response of single degree of freedom oscillator to earthquake excitation. *Int. J. Soil Dynamics and Earthquake Eng.* 5(2): 119-129.

Lee, V.W. and M.D. Trifunac 1987. Rocking strong earthquake acceleration. *Soil Dynam. Earthq. Eng.* 6(2): 75-89.

Newland, D.E. 1984. An introduction to random vibrations and spectral analysis. *Longman, New York*: 32-80.

Trifunac, M.D. 1988. Seismic microzonation mapping via uniform risk spectra. *9th World Conf. Earthquake Eng.* VII, 75-80, Tokyo-Kyoto, Japan.

Trifunac, M.D. and A.G. Brady 1975. A study on the duration of strong earthquake ground motion. *Bull. Seism. Soc. Amer.* 65: 581-626.

Udwadia, F.E. and M.D. Trifunac 1974. Characterization of response spectra through the statistics of oscillator response. *Bull. Seism. Soc. Amer.* 64: 205-219.