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A substructure technique for spatial analysis of multistorey buildings

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ABSTRACT:The paper proposes a new substructure type made up of a floor or floor partions,considered as behaving like a rigid washer in the median plane and horizontal and vertical structural elements.In a static analysis the substructure has $3n_A +3$ degrees of freedom(d.o.f),if n_A is the floor inplane nodes number;the number of these d.o.f can be reduced to only three for a dynamic(seismic)analysis.The writing and solving of conditioning equations for the whole structure within both types of analysis are thus simplified.

1.INTRODUCTION

The spatial analysis of multistory complex buildings acted upon by static and/or dynamic(seismic)forces with programs F.E.M.,case in which six d.o.f. are accorded to each node, leads to the solving of ample algebraic problems and difficulties in the interpretation of results.
If we accept the hypothesis of floors behaving like rigid washers in the median plane and the substructure technique,the dimensions of these algebraic problems can be reduced;the substructures consist in plane or spatial frames,planar or unplanar diaphgrams,single or in frames,etc. for which the stiffness matrices may be determined accurately or approximately [1,2,3]

In[1] it is necessary that the flooring should stretched over the whole surface of the storey and for it the automatic computing program ETABS(CASE)was elaborated;in[2.3,] we admitted floor portions and even individual bar nodes(fig.1)for which automatic computing prgrams are to be drawn up .

By accepting the same hypothesis of flooring behaviour the paper proposes a new type of substructure made up of the flooring or a partion of it, the beams and/or the lintels from its plane and the columns and/or the diaphragms that support it.The stiffness matrix of the substructure and of the whole structure is elaborated both for the static and dynamic structural analysis.

2.SUBSTRUCTURES DEGREES OF FREEDOM

The mass centre of the floor or the floor portion "A",is accorded three main d.o.f. in the general(structural) reference system,two translations in the horizontal plane,U_A,V_A,and a rotation about the vertical axis ϕ_A, which define the subvector D_{MA}(fig.1)

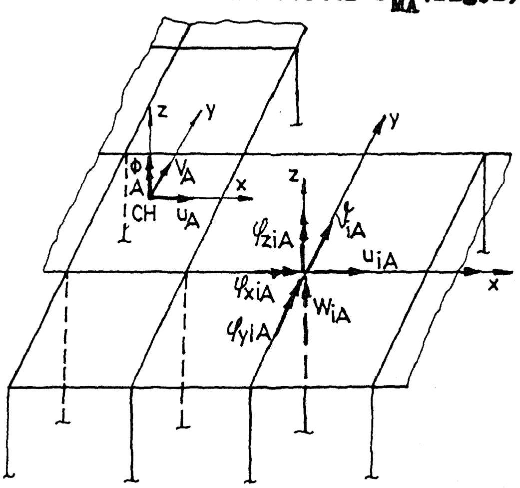

Fig.1 Substructure Degrees of Freedom

A node, i_A, on the floor, defined as the intersection of beam axes and/or columns, is accorded six d.o.f. in the local reference system made up of the axes of the bars concurring in the node and the main axes of their sections (fig.1); due to the admitted assumption, three d.o.f. of the node are dependent (secondary) on those of the mass centre by a transform matrix (the inplane floor translations U_{iA}, V_{iA} and the rotation φ_{ziA} about the vertical axis that define the subvector $d_{iA}{}^*$), while the other d.o.f. are independent (semi-principal) (with a translation after the vertical axis W_{iA} and the rotations about the axes in the floor plane φ_{xiA}, φ_{yiA}, defining the subvectpr d_{iA}).
If the number of nodes on the floor is n_A, then the number of d.o.f. used in the static analysis will be $3n_A+3$ (the independent and principal ones) because the secondary d.o.f. $3n_A$ in number, can be eliminated; at the same time in the dynamic analysis the $3n_A$ independent d.o.f. can be eliminated so that only the principal three d.o.f. of the floor should be left.

3. THE PHYSICAL RELATION OF AN INPLANE FLOOR BAR

The inplane floor bars (beams and/or lintels) undergo rigid body displacements, corresponding to the principal d.o.f. and are planarily deformed due to the independent (semiprincipal) d.o.f. of each boundary (fig.2.)

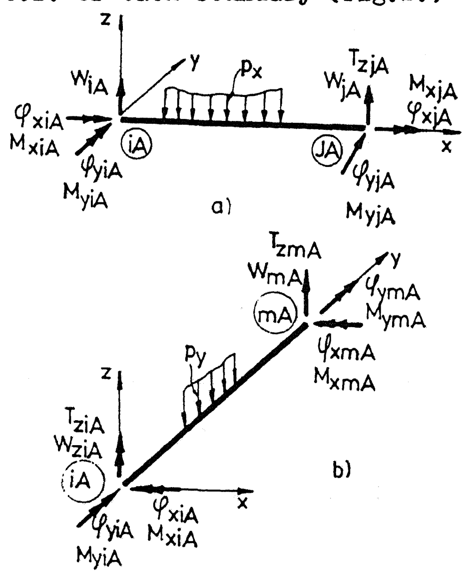

Fig.2. Characteristic Elements of Beam

As related to the local reference system for a given bar, $i_A j_A$, corresponding to the six d.o.f. from the two boundaries, the physical stress-strain relation is:

$$\begin{bmatrix} k_{iA,iA} & k_{iA,jA} \\ k_{jA,iA} & k_{jA,jA} \end{bmatrix} \begin{Bmatrix} d_{iA} \\ d_{jA} \end{Bmatrix} = \begin{Bmatrix} S_{iA} \\ S_{jA} \end{Bmatrix} + \begin{Bmatrix} R_{iA} \\ R_{jA} \end{Bmatrix} \quad (1)$$

in which the stiffness matrix is 6x6 and symmetrical, while S_{iA} and R_{iA} are the subvectors in the boundary i_A of the three stresses, respectively responses to the loads applied along the bar.

This physical relation remains unaltered if the local reference system of the bar has parallel axes to those of the general reference system (the case of orthogonal beam grids); if not, the physical relation will be affected by the stress-strain transform matrices between the two reference systems and/or the transform matrices imposed by the existence of some stiff bar partions so as to reach the theoretical nodes iA, jA.

For each vertical structural element iA -iB (column of diaphragm) corresponding to the six d.o.f. of each boundary (three dependent d.o.f. and three independent d.o.f.) and in relation to the local system made by the principal inertia axes of the cross section and the bare axis (fig. 3) the physical relationship is:

$$\begin{bmatrix} k_{iA,iA} & k_{iA,iA^*} & k_{iA,iB} & k_{iA,iB^*} \\ k_{iA^*,iA} & k_{iA^*,iA^*} & k_{iA^*,iB} & k_{iA^*,iB^*} \\ k_{iB,iA} & k_{iB,iA^*} & k_{iB,iB} & k_{iB,iB^*} \\ k_{iB^*,iA} & k_{iB^*,iA^*} & k_{iB^*,iB} & k_{iB^*,iB^*} \end{bmatrix} \begin{Bmatrix} d_{iA} \\ d_{iA^*} \\ d_{iB} \\ d_{iB^*} \end{Bmatrix} = \begin{Bmatrix} S_{iA} \\ S_{iA^*} \\ S_{iB} \\ S_{iB^*} \end{Bmatrix} + \begin{Bmatrix} R_{iA} \\ R_{iA^*} \\ R_{iB} \\ R_{iB^*} \end{Bmatrix} \quad (2)$$

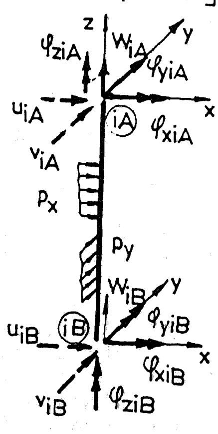

Fig.3 Characteristic Elements of Columns

in which the symmetric stiffness matrix is 12x12 and S_{iA}, S_{iA}^*, R_{iA} and R_{iA}^* are the stress and, respectively, response subvectors produced by the loads acting along the bar in the boundary iA and similary in the boundary iB on the B floor.

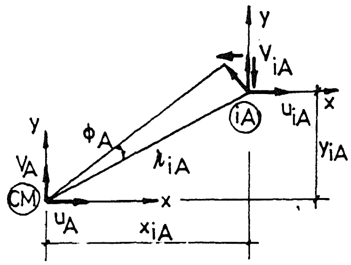

Fig.4. General and Local Axis

If the local reference system axes are parallel to those of the general reference system between the three dependent d.o.f., iA, and the three principal d.o.f. of the floor, there occur the transforms (fig.4):

$$d_{iA}^* = T_{iA^*} \cdot D_{MA} \; ; S_{iMA} = T^t_{iA^*} \, S_{iA^*} \; ; \quad R_{iMA} = T^t_{iA^*} \, R_{iA^*} \qquad (3)$$

where the transform matrix is:

$$T_{iA^*} = \begin{bmatrix} 1 & 0 & -Y_{iA} \\ 0 & 1 & X_{iA} \\ 0 & 0 & 1 \end{bmatrix} \qquad (4)$$

where t - the matricial transposition;

There are similar transform relations for the boundary iB on the lower floor B, so that the physical relationship (2) reduced to the mass centre of the two floorings becomes:

$$\begin{bmatrix} k_{iA,iA} & k^*_{iA,iA^*} & k_{iA,iB} & k^*_{iA,iB^*} \\ k^*_{iA^*,iA} & k^*_{iA^*,iA^*} & k^*_{iA^*,iB} & k^*_{iA^*,iB^*} \\ k_{iB,iA} & k^*_{iB,iA^*} & k_{iB,iB} & k^*_{iB,iB^*} \\ k^*_{iB^*,iA} & k^*_{iB^*,iA^*} & k^*_{iB,iB^*} & k^*_{iB^*,iB^*} \end{bmatrix} \begin{Bmatrix} d_{iA} \\ D_{MA} \\ d_{iB} \\ D_{MB} \end{Bmatrix} = \begin{Bmatrix} S_{iA} \\ S_{iMA} \\ S_{iB} \\ S_{iMB} \end{Bmatrix} + \begin{Bmatrix} R_{iA} \\ R_{iMA} \\ R_{iB} \\ R_{iMB} \end{Bmatrix} \qquad (5)$$

where

$$k^*_{iA,iA^*} = k_{iA,iA^*} T_{iA^*}$$
$$\vdots \qquad (6)$$
$$k^*_{iB^*,iB} = T^t_{iB^*} \cdot k_{iB^*,iB}$$

the stiffness matrix remaining symmetric.
If there are portions of rigid bars in order to define the theoretical nodes, iA and iB, the corresponding transform matrices have to be taken into account, as well.
For a first floor column supported by the foundation structure (girder grids, rafters or insulated foundations) no transform is performed in (5) for the boundary iB, only d_{iB}^*, S_{iB}^* and R_{iB}^* being left, while in (6) T_{iB^*} is considered to be the unitary matrix.

4. PHYSICAL RELATION OF THE SUBSTRUCTURE

For the whole substructure, made up of the floor A, the beams within it and the vertical support members, there will be introduced the subvectors d_A and d_B of all independent (semi-principal) d.o.f., of all nodes n_A and n_B on the two floorings and the corresponding vectors from stresses and responses to loads.
According to these subvectors, the physical assembled relation of the substructure has the form:

$$\begin{bmatrix} k_{A,A} & k_{A,MA} & k_{A,B} & k_{A,MB} \\ k_{MA,A} & k_{MA,MA} & k_{MA,B} & k_{MA,MB} \\ k_{B,A} & k_{B,MA} & k_{B,B} & k_{B,MB} \\ k_{MB,A} & k_{MB,MA} & k_{MB,B} & k_{MB,MB} \end{bmatrix} \begin{Bmatrix} d_A \\ D_{MA} \\ d_B \\ D_{MB} \end{Bmatrix} = \begin{Bmatrix} S_A \\ S_{MA} \\ S_B \\ S_{MB} \end{Bmatrix} + \begin{Bmatrix} R_A \\ R_{MA} \\ R_B \\ R_{MB} \end{Bmatrix} \qquad (7)$$

thus being obtained:
- the physical relation (1) of each beam on level A is expanded to subvector d_A value and is boundary marked by zeroes, corresponding to subvectors D_{MA}, d_B and D_{MB}
- the transformed physical relation (5) of each vertical structural element is expanded to the subvector d_A, D_{MA} d_B and D_{MB} dimensions;
- these expanded physical relation are added thus yielding (7)

In the first floor substructure the subvector D_{MB} is replaced in (7) by subvector d_B^* of all dependent d.o.f. of the nodes iB and with the corresponding subvector of stress and response to loads.
If the subvectors made up of subvectors d_A and D_{MA} is noted by D_A, then relation (7) may be written under the form:

$$\begin{bmatrix} k_{A,A} & k_{A,B} \\ k_{B,A} & k_{B,B} \end{bmatrix} \begin{Bmatrix} D_A \\ D_B \end{Bmatrix} = \begin{Bmatrix} \mathcal{S}_A \\ \mathcal{S}_B \end{Bmatrix} + \begin{Bmatrix} \mathcal{R}_A \\ \mathcal{R}_B \end{Bmatrix} \qquad (8)$$

The same form can be arrived at even if the subvectors d_A and d_B are eliminated, but in such a case subvector D_A becomes D_{MA} and the matrix is 6x6 in dimension corresponding only to the principal d.o.f. of the two floorings.
For the first floor substructure, by eliminating the subvector D_B, corresponding to the supporting nodes of the foundation substructures with null or known (support failures) displacements, the physical relation (8)

becomes:

$$k_{A,A} \cdot D_A = \mathcal{S}_A + \mathcal{R}_A \qquad (9)$$

and the response in the supporting nodes can be determined with the relationship:

$$\mathcal{R}_B = -\mathcal{R}_B + k_{B,A} \cdot \mathcal{D}_A + k_{B,B} \cdot \mathcal{D}_B \qquad (10)$$

5.CONDITIONING EQUATIONS

For the structural analysis at static forces there will be drawn the displacements vector D made of the substructure subvectors D_A from the top floor tawards the first floor(fig.5)corresponding to this vector the vector P of the exterior forces applied directly in the substructures nodes is drawn,as well.

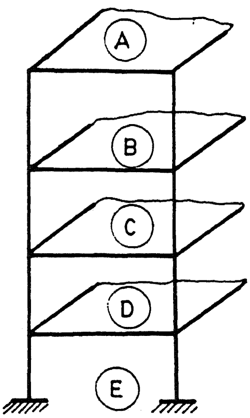

Fig.5.Levels Arrangement for the Vector of displacements (A - E)

The physical relationship of each substructure(8) and (9),respectively for the first floor substructure is expanded to the dimension of vector D and is summed up to obtain the algebraic equations:

$$k \cdot D + R = P \qquad (11)$$

in which the stiffness matrix k of the whole structure is symmetric and non singular;this matrix has a tridiagonal form corresponding to continuous structures if at each level there is only a floor portion on the main diagonal there are sums of principal sub-matrices relative to the floor which is part of two consecutive substructure ;on the diagonals parallel to the principal one there are interaction submatrices of the two floorings.

In this respect,eq.(11)can be solved by specific methods e.g.successive elimination ob subvectors D_A from the top floor substructure to the first floor one,or conversely.

After solving the equation and computing the displacements D,the stresses in the structure beams can be found from relations (8)(7)(2) and(1)

For the analysis of the structure at dynamic(seismic)forces,vector D is made up only of subvectors D_{MA} of the three principal d.o.f. of each flooring from the top floor to the first floor;according to this vector the diagonal inertia matrix M is performed.

The conditioning equation will be:

$$M \cdot \ddot{D} + C\dot{D} + kD = - M H \ddot{u}_t \qquad (12)$$

where C and k are the damping and stiffness matrices corresponding only to the principal d.o.f. $\ddot{u}_t$ is the ground acceleration during the seismic event while H= vector of seismic activity localization after d.o.f; the damping matrix can be considered a linear combination of the inertia and stiffness matrices.Eq (12)can be solved by:

-spectral analysis much used in design work;

-modal analysis and uncoupled equation integration;

-direct numeric integration to obtain the response in time of the structure.

6.CONCLUSIONS

The spatial analysis of structures subjected to static or dynamic(seismic)forces is carried out by substructuring which simplifies the writing of conditioning equations and lessens the dimensions of the algebraic problems to be solved.The substructure is made up of the structural members of a level-the stiff flooring in median plane,the inplane floor girders and the supporting columns. Programs for the automatic computation both of the behavioural range under the linear elastic boundary of the structure and beyond this are to be executed.

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