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A reduced degree of freedom approach in the pseudodynamic test method

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ABSTRACT: The pseudodynamic test method is a powerful procedure for seismic performance testing. However, the method has analytical limitations due to the discretization of the structure and to the numerical integration procedures used. The control of many degrees of freedom, as in multi-storey frames or in structures with distributed masses, may lead to an impractical test arrangement and to an excessive reduction of the time step of the integration algorithm. In this study a method in which a reduced number of degrees of freedom is controlled during the test is presented. The advantages of this approach are explored and practical guidelines concerning the minimum number of degrees of freedom to be controlled for different classes of structures are derived.

1 INTRODUCTION

The pseudodynamic test method is a hybrid procedure that uses computer calculation and control together with experimental measurement to provide a realistic simulation of the response of structures to dynamic loading. The equations of motion for a discrete model of the test structure are solved on-line using a step-by-step numerical integration method. The test is carried out quasi-statically, but the inertial and viscous damping forces are modelled analytically and the non-linear structural restoring force characteristics are measured experimentally.

The method has been proved to be an efficient tool for testing of structures which are too large or heavy to be tested on a shaking table. The major advantage of the pseudodynamic test method lies in the possibility of full scale testing of large structures without the need for high hydraulic power. The potential is greatly increased by the possibility of analytical simulations of multiple support excitations, soil structure interaction and geometric non-linearities. The further major advantages offered by the use of analytic substructuring concepts are also being explored.

As with any other testing procedure, the pseudodynamic test method suffers some disadvantages, arising both from the physical test arrangement and from the analytical treatment of the equations in the computer. Even though several major tests have been conducted around the world, a reliable implementation, able to take into account fully the nature of the specimen as well as the form of the equations representing the dynamic behaviour is still to come (Mahin et al., 1989).

A detailed description of the pseudodynamic test method and of its typical implementation can be found elsewhere (Shing and Mahin, 1984; Donea and Jones, 1991). This paper focuses on the possibility of establishing a general testing strategy, based on a "reduced degree of freedom approach", which could be adapted as appropriate to the particular structure to be tested.

2 THE TESTING STRATEGY

To perform a reliable pseudodynamic test, efforts must be made to minimize the errors arising both from the analytical integration method used and from the experimental setup. These are considered below.

2.1 Numerical aspects

Ideally, implicit numerical integration methods would be used for solving the time-discretized equation of motion in the pseudodynamic test method, because they are unconditionally stable. However, conventional implicit methods use an estimation of tangent stiffness (a reliable estimate of this for a real structure is very difficult to obtain), and an iterative procedure (due to the fact that the non linear behaviour of structures is usually path dependent such procedures should not be used experimentally). Consequently explicit numerical integration procedures have generally been used in the past. The two most common explicit methods in pseudodynamic test are the central difference and the Newmark methods; these have been proved to be numerically equivalent once started (Shing and Mahin, 1984). However, explicit methods are only conditionally stable, the maximum time step required for stability being dictated by the frequency of the highest mode which could possibly be excited during the test. Since in pseudodynamic tests errors tend to increase dramatically as the time interval decreases, the results can be rendered unreliable even though the integration method used is stable and accurate.

To overcome this difficulty, a hybrid implicit method has been suggested (Thewalt and Mahin, 1987). The hybrid approach finds a portion of the solution digitally, while the remainder is found in analogue form using directly the feedback voltages from the load cells attached to the actuators. In fact, even though at the end of the time step the restoring force is not yet available digitally to the computer, it is available continuously in analogue form. By modifying the feedback control loop it is thus possible to physically modify the command signal to take into account the restoring force. In this way the structure "finds" the solution to the implicit equations by itself. The algorithm has been proved to be superior to the explicit integration methods in terms of experimental error propagation characteristics (Peek and Yi, 1990; Shing and Manivannan, 1990). However, in spite of its apparent robustness, the method is far from being widely applied. A reason for this is thought to be the need to modify the hardware of the analogue controller to implement the technique; also the fact that the computer never "knows" the next displacement, but has to wait for the measurement of it after step completion, implies that there can be no direct checking of errors. However, here at the JRC Ispra, the use of digital controllers in place of analogue has made the implementation much simpler, and an error check at each 50 msec interval can now be achieved.

Another appealing approach has been suggested in the Nakashima operator splitting method (Nakashima et al., 1990). In this method the equilibrium equations are split into linear and nonlinear terms, so that the linear term can be treated implicitly, whilst the nonlinear term is treated explicitly using predictor and corrector displacement vectors. The method has been claimed to be unconditionally stable as long as the tangent stiffness is not greater than the initial stiffness (a condition fulfilled in most actual structures), and to be equivalent in accuracy to the Newmark method. The experimental set-up remains that of the simpler explicit methods. It has been also recognized that the method matches perfectly the scope of the pseudodynamic test method, in the sense that it relies on the experiment just for the information which is not possible to model analytically. However, the general robustness of the operator splitting method has not yet been claimed. The need has been expressed for further numerical experimentation and for practical tests to obtain a sound comprehension of error propagation as well as for practical criteria for its application.

Another unconditionally stable procedure using an implicit method has been proposed recently (Shing et al., 1991). In this approach numerical iteration is in fact used and this is based on the initial stiffness of the structure; however a displacement reduction factor is adopted in order to avoid problems related to path-dependency, together with a numerical error correction procedure. The method has the same practical advantages as the operator splitting method and the same restriction on stiffness changes but is claimed to be accurate, although it seems that the reduced displacements to be applied in the iteration phase may require excessively high resolution in the apparatus under some circumstances.

2.2 Equivalent SDOF system

An early attempt to overcome numerical stability-related problems, as well as to make the experimental set-up simple and effective, was shown by the methodology adopted in the full-scale tests conducted in Japan as a part of the U.S.-Japan Cooperative Earthquake Research Program. An exhaustive description of the activities can be found elsewhere (Wight, 1984). In the test of a full-scale reinforced concrete seven-storey structure, it was decided to treat it as a single degree of freedom system with external force distributed in inverted triangular mode. Any structure can be reduced to an equivalent single degree of freedom system if the corresponding mode of deflection is assumed. With the assumption of external force distributed in the inverted triangular mode, a static analysis was carried out to find the appropriate mode of deflection. It turned out that this did not change significantly regardless of the magnitude of the force, being almost identical in the elastic, inelastic and "mechanism" ranges. Using the deflection mode corresponding to the prescribed force distribution pattern, the governing equations could be transformed into an equivalent single degree of freedom equation in terms of a scalar generalized displacement. During the equivalent SDOF pseudodynamic test, the computer controlled the displacement of the actuators at the roof level. By measuring the force at the roof level, the forces in all the other actuators were imposed independently from the on-line system so that the distribution of external forces was held inverted-triangular. The higher vibrational modes are absent in the control equation, so that a standard explicit integration algorithm can be used without having to deal with excessively small time steps.

The extension of the above to a general reduced degree of freedom method is straightforward; if we could include in the test just those independent force patterns we need to represent satisfactorily the behaviour of the structure, the numerical constraints and the test arrangement would be effectively tailored to the structure to be tested.

The idea of reducing the number of independent degrees of freedom by using a set of Ritz shape functions has been proposed (Thewalt and Mahin, 1987), but doubts were expressed about the ability of this approach to represent effects such as soft-storey formation.

In this paper a more general approach to a reduced degree of freedom testing technique is presented. The effectiveness of the method is explored, seeking the possible limitations arising from the assumption of the force distribution along the structure. Numerical simulations have been performed for different structures using the reduced degree of freedom approach with different numbers of independent linear force variation patterns; the results are compared with the solution obtained with all the degrees of freedom.

3 RDOF APPROACH IN PSEUDODYNAMIC TEST

In the reduced degree of freedom (RDOF) approach a set of equations in terms of a prescribed number — more than one, but not all — of the independent degrees of freedom is established. The aim is to use the minimum number of degrees of freedom necessary to achieve a satisfactory approximation of the structural behaviour, whilst avoiding the need for a time step which would be excessively small when using a standard explicit algorithm.

For simplicity, let us first refer to the case of a RDOF approach controlling just two degrees of freedom in a multi-degree of freedom system. Associated with the two degrees of freedom, we select two master nodes, i.e. the two nodes in which we control the displacement during the test. We assume two suitable independent linear force patterns, for instance the ones depicted in Fig. 1, and for each force pattern we obtain, via static analysis, the corresponding deformed shape ψ_1, ψ_2. Since the deformed shapes will play the role of Ritz functions, the space they span will be the same if we assume either the deformed shapes resulting from linear analysis, or the ones from nonlinear analysis.

Using the same procedure as in a size-reduction via Ritz functions, we transform the discretized equation of motion,

$$\mathbf{m}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{P} = -\mathbf{m}\mathbf{I}\ddot{u}_g$$

where $\mathbf{m}$ and $\mathbf{C}$ are the mass and damping matrices, $\mathbf{P}$ is the restoring force vector, $\ddot{u}_g$ is the ground acceleration and $\mathbf{I}$ is the influence vector (column of ones) by introducing the relation:

$$\mathbf{x} = v_1\psi_1 + v_2\psi_2 = \psi\mathbf{v}\,,$$

where $\mathbf{v}$ is a vector in the reduced-size space (i.e. a 2x1 vector in the case at hand), containing the generalized displacements. Thus we find

$$\mathbf{m}\psi\ddot{\mathbf{v}} + \mathbf{C}\psi\dot{\mathbf{v}} + \mathbf{P} = -\mathbf{m}\mathbf{I}\ddot{u}_g$$

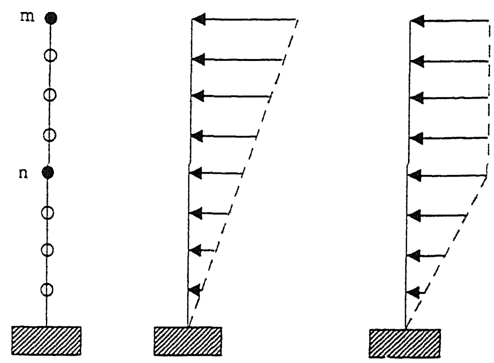

Figure 1. Assumed linear force patterns.

$$\psi^T\mathbf{m}\psi\ddot{\mathbf{v}} + \psi^T\mathbf{C}\psi\dot{\mathbf{v}} + \psi^T\mathbf{P} = -\psi^T\mathbf{m}\mathbf{I}\ddot{u}_g$$

$$\tilde{\mathbf{m}}\ddot{\mathbf{v}} + \tilde{\mathbf{C}}\dot{\mathbf{v}} + \tilde{\mathbf{P}} = -\tilde{\mathbf{I}}\ddot{\mathbf{u}}_g\,,\ where:$$

$$\tilde{\mathbf{m}} = \begin{vmatrix} \sum_i m_i\psi_{1i}^2 & \sum_i m_i\psi_{1i}\psi_{2i} \\ \sum_i m_i\psi_{1i}\psi_{2i} & \sum_i m_i\psi_{2i}^2 \end{vmatrix}$$

$$\tilde{\mathbf{C}} = \begin{vmatrix} \sum_i \psi_{1i}\sum_j C_{ij}\psi_{1j} & \sum_i \psi_{1i}\sum_j C_{ij}\psi_{2j} \\ \sum_i \psi_{2i}\sum_j C_{ij}\psi_{1j} & \sum_i \psi_{2i}\sum_j C_{ij}\psi_{2j} \end{vmatrix}$$

$$\tilde{\mathbf{P}} = \begin{vmatrix} \sum_i \psi_{1i}P_i \\ \sum_i \psi_{2i}P_i \end{vmatrix}$$

$$\tilde{\mathbf{I}} = \begin{vmatrix} \sum_i \psi_{1i}m_i \\ \sum_i \psi_{2i}m_i \end{vmatrix}$$

Both $\tilde{\mathbf{m}}$ and $\tilde{\mathbf{C}}$ are full matrices because the Ritz vectors are in general not orthogonal; however since they contain constant terms, they can be efficiently handled by the on-line computer.

At each time step the equilibrium equation is solved for the generalized displacements $\mathbf{v}$. The requirements for the numerical stability of the scheme are now based on the eigenvalues of the reduced-size system.

After the generalized displacements have been computed, the corresponding value of the real displacements is obtained as $\mathbf{x} = \psi\mathbf{v}$ and it is imposed by the actuators at the two selected master nodes. The displacements computed for the other nodes are not used, and forces corresponding to a piece-wise linear variation between the two master nodes are instead imposed by the actuators.

When the target displacements at the two master nodes are achieved, the forces are read, so that the equations can be solved for the next time step.

4 NUMERICAL SIMULATIONS

As a result of the RDOF approach the effective loads (resultant of inertial and damping forces, and forces due to base acceleration) are evaluated at the master nodes only, while a piece-wise linear pattern is imposed at the remaining nodes.

This analytical constraint has been implemented in the general nonlinear computer code ANSR-I (Mondkar and Powell, 1975). The results, in terms of displacements and shear forces, have been compared with the "reference" ones, i.e. those obtained taking into account all the degrees of freedom.

Two different types of structure have been considered, namely a reinforced concrete frame and a reinforced concrete shear wall. Different seismic inputs, representing the two cases of short and long focal distance earthquakes were used and structures with non-homogeneous mass distribution, as well as structures with different distribution of strength and stiffness along the height (including the case of a soft-storey) were included.

4.1 R/C Frame

As an example of a frame structure, an 8-storey, 3-bay R/C frame was considered. The structure is part of a building designed according to the Eurocode 8 (Pinto and Jones, 1991).

A linear dynamic analysis was first carried out for the case of one, two and three degrees of freedom control, as well as for the reference case (control of all the degrees of freedom). Various base motions were used in the analysis; these included the 1976 Friuli earthquake (as an example of short focal distance earthquakes), the 1940 El Centro earthquake (as a representative of earthquakes with distributed spectral power) and the 1968 Tokachi-Oki earthquake. This last earthquake was included because it had been applied to the specimen tested in Japan as a part of the U.S.-Japan Cooperative Research (Okamoto et al., 1984).

As an example, the results of the linear analysis for the case of the Tokachi-Oki earthquake are reported in Fig. 2. The results show the improvement which can be achieved by including more degrees of freedom; those obtained with three degree of freedom control (dotted line) compare fairly well with the reference response (solid line), while single degree of freedom control (broken line) gives comparatively poor results.

The considerations made for the linear analysis also apply to the nonlinear case (Fig. 3), except that the single degree of freedom solution now follows the other solutions much more closely. However, this seems to be due to the particular frequency content of the Tokachi-Oki signal, because a much greater deviation of the single degree of freedom control response from the reference curve was found for the other signals.

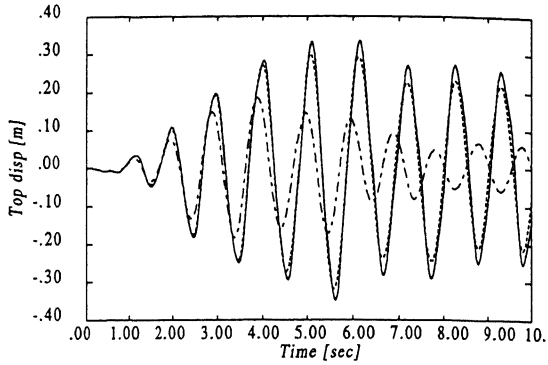

Figure 2. R/C frame: linear analysis with Tokachi-Oki earthquake, top displacement.

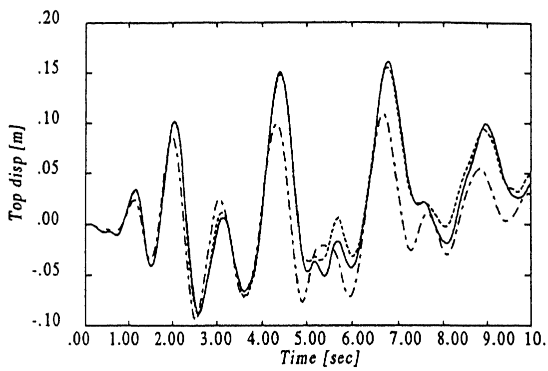

Figure 3. R/C frame: nonlinear analysis with Tokachi-Oki earthquake, top displacement.

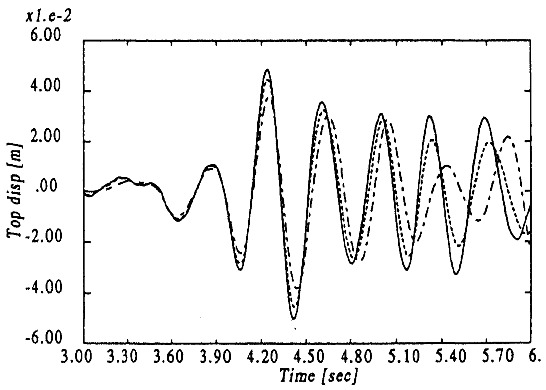

Figure 4. R/C shear wall: linear analysis with Friuli earthquake, top displacement.

4.2 R/C Shear wall

As a representative of a stiff shear wall structure, an 8-storey R/C shear wall was considered which is a crude idealization of the main shear wall of a build-

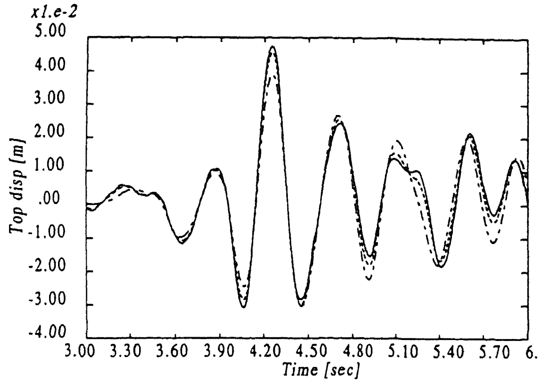

Figure 5. R/C shear wall: nonlinear analysis with Friuli earthquake, top displacement.

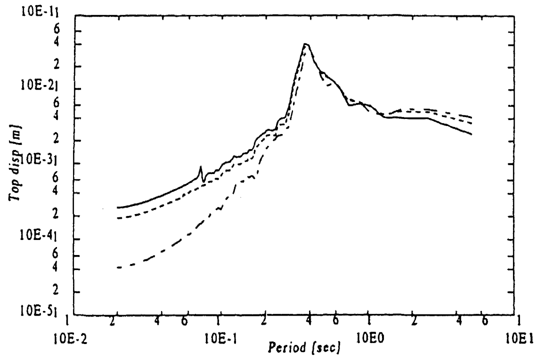

Figure 6. R/C shear wall: linear analysis with El Centro earthquake, FFT of top displacement.

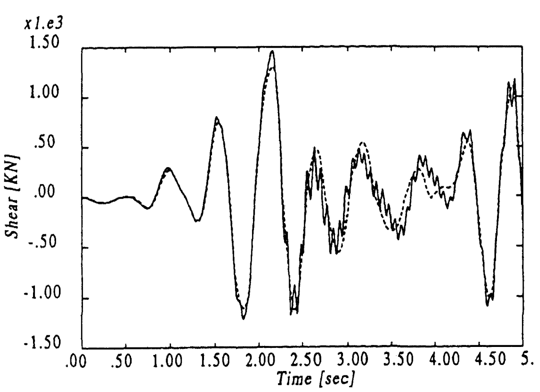

Figure 7. Soft-storey frame: base shear with single and double dof control.

ing proposed for testing in the Reaction Wall Facility of the Joint Research Centre in Ispra.

The results of a linear analysis are depicted in Fig. 4. In this case the Friuli earthquake was found to be the most significant, since the frequencies of the structure are considerably higher than those of the R/C frame. For this structure the analysis with two degree of freedom control (dotted line) compares reasonably well with the theoretical solution (solid line). Also the solution obtained with single degree of freedom control (broken line), is now seen to be much better as regards the displacement values, although a shift in the overall frequency of the response can still be observed.

In the nonlinear analysis (see Fig. 5), the reduced degree of freedom solutions were again found to be much improved compared to the linear case, although an improvement can still be obtained by controlling two degrees of freedom instead of one. The above considerations were also found to apply to the other earthquake excitations.

In Fig. 6 the Fourier transform of the top displacement for the elastic analysis for the 1940 El Centro earthquake, which is typical of such plots, is shown. This allows an understanding of the method; including a reduced number of degrees of freedom in the analysis is equivalent to filtering the seismic input with respect to the higher frequencies. By selecting the number of independent degrees of freedom to be controlled during the test we actually fix the contribution of the higher modes to the response.

4.3 Non-homogeneous strength-stiffness distribution

As an extreme situation, a soft-storey structure was analyzed. Both the stiffness and the strength of the 4th storey of the shear wall were set equal to one half of those of the rest of the structure.

Again, the nonlinear analysis with two degree of freedom control captures the dynamic behaviour well, while the solution with single degree of freedom is less good mainly in the small amplitude cycles. As a rule, base shear is more sensitive to higher mode contributions than the displacements. This can be seen in Fig. 7, which represents the base shear plots for the cases of single degree of freedom (dotted line) and two degree of freedom control (solid line). Once more, including more independent degrees of freedom in the test is equivalent to allowing a greater contribution of the higher modes to be seen but it does not significantly change the maximum values generated.

5 CONCLUSIONS AND FUTURE WORK

The reduced degree of freedom approach for the pseudodynamic test method is a testing strategy that simplifies the testing setup and allows the use of conditionally stable explicit numerical integration methods with an experimentally convenient integration time step. Using this method it is possible to tailor the test strategy to the structure and earthquake of interest. Only the vibration modes which are expected to contribute to the dynamic behaviour need be considered in the test, reaching the desired level of accuracy without worrying about the effects of higher frequencies on the numerical algorithm.

Numerical simulations have demonstrated the effectiveness of a reduced degree of freedom control for different types of structure. In general, the lower the dominant frequencies in the earthquake, the stiffer the structure and the more nonlinear the response, the lower is the number of degrees of freedom which need to be displacement controlled. Two or three are usually sufficient to capture well the true dynamic behaviour; one will only rarely suffice. The efficacy of the method for stiff structures is particularly noteworthy since these are normally the most problematic.

Particular behaviour such as soft-storey deformations, initially thought to be a major problem for the application of the RDOF method, does not represent a major difficulty if the master nodes are carefully chosen.

Real tests using this approach will now be conducted on simple multi-degree of freedom structures to validate the above conclusions. Further work on the possible extension of the method to structures having continuous mass distribution is also planned.

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