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Estimation on response of hysteretic non-linear system using momentary energy spectrum

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ABSTRACT: In this paper, a simplified response prediction method of single degree of freedom (SDOF) hysteretic non-linear system is presented, which is based on momentary energy balance. The proposed method evaluates the maximum displacement by newly-defined response spectra, named momentary energy input spectra. Comparison to non-linear simulations showed that the proposed method can easily predict the ultimate response of non-linear systems as like base isolated buildings, without complicated modeling of hysteretic rules in load-displacement relations.

1 INTRODUCTION

Though it is one of essential issue in seismic design to obtain hysteretic non-linear systems during strong seismic motions, sometimes it is difficult to determine hysteretic models of restoring forces for numerical non-linear simulation.

Since Housner noticed the importance of energy input from seismic motion in seismic damage of structures and non-linear response prediction, a lot of researchers, including Newmark and Akiyama, proposed simplified methods for non-linear response prediction. Most these methods are based on balance between energy input and energy absorption.

These preceding research showed strong relations between total energy input during earthquakes and maximum response. However, maximum response of non-linear systems often occurs in limited short duration of seismic motions, and only a few cycles would contribute the maximum response. This well-known fact suggests that momentary energy balance could be more essential in the determination of maximum responses of hysteretic systems. Nishzawa and Kaneta evaluated the response of SDOF systems with proposed momentary energy input.

In this paper, a simplified prediction method of response of SDOF non-linear systems is presented, which is based on momentary energy balance. The proposed method evaluates the maximum displacement by newly-defined response spectra, named momentary energy input spectra. The applicability of the proposed method is validated through comparisons to non-linear simulations of a base isolated building model.

2 SIMPLIFIED RESPONSE PREDICTION METHOD USING MOMENTARY ENERGY INPUT SPECTRUM

2.1 MOMENTARY ENERGY INPUT SPECTRUM

Total energy input E of a ground motion to a SDOF system is defined as follows :

$$E = \int_0^{t_o} M\ddot{x}_o\,\dot{x}\,dx \tag{1}$$

where $\ddot{x}_o$, $\dot{x}$, M and t_o are ground acceleration, relative response velocity, the mass of the system and the duration time of the ground motion.

In this study, 1-cycle momentary energy input E_m and 1-cycle momentary energy input spectrum S_{ve} are defined as follows:

$$E_m(T,h) = \int_t^{t+T} M\ddot{x}_o\,\dot{x}\,dx \quad (0 \leq t \leq t_o - T) \tag{2}$$

$$S_{ve}(T,h) = \sqrt{2\max(E_m)/M} \tag{3}$$

where T and h are equivalent natural period and equivalent damping factor.

Similarly, 1/4-cycle momentary energy input E_m' and 1/4 cycle momentary energy input spectrum S_{ve}' can be defined to change the range of integration. They are converted to momentary energy input during 1 cycle.

$$E_m'(T,h) = 4\int_t^{t+T/4} M\ddot{x}_o\,\dot{x}\,dx \quad (0 \leq t \leq t_o - T/4) \tag{4}$$

$$S_{ve}'(T,h) = \sqrt{2\max(E_m')/M} \tag{5}$$

These momentary energy input concept

shows the momentary intensity of ground motions. Momentary energy input spectrum varies according to the range of integration. As the range of integration gets smaller, momentary energy input spectrum gets larger. Therefore, 1-cycle momentary energy input spectrum during S_{ve} is smaller than 1/4 cycle-one S_{ve}'

2.2 MOMENTARY ENERGY ABSORPTION OF STRUCTURAL ELEMENTS

In order to predict non-linear response, it is necessary to determine energy absorption of a structural element. In this study, 1-cycle momentary energy absorption of a structural element is defined the sum of the 1 cycle hysteretic (plastic) energy and elastic strain energy as shown in Fig.1.

Similarly, 1/4 cycle momentary energy absorption can be defined as shown in Fig.2. Converting it to 1 cycle, it is multiplied by 4.

2.3 SIMPLIFIED RESPONSE PREDICTION

This study proposes a simplified response prediction method based on the momentary energy balance. It assumes that maximum displacement occurs when the momentary energy input and the momentary energy absorption are equal.

In order to predict non-linear response by the proposed method, the equivalent response period and the momentary energy absorption are determined at first, then they can be plotted on the momentary energy input spectrum. The crossing point of them can be regarded as the maximum displacement of the system subjected to a ground motion. Damping factor of momentary energy spectrum is determined to be nearly equal with equivalent damping factor of the system.

In this paper, two types of momentary energy input spectra and the momentary energy absorption are used so that two values are estimated. One is the estimated value during 1 cycle and the other is one during 1/4 cycle. They are compared each other.

3 VERIFICATION OF THE PROPOSED METHOD

3.1 STRUCTURAL MODEL

To verify the proposed method, a numerical model of a base isolated building was used (Fig.3). Tables 1, 2, 3 and 4 show the characteristics of the building and the ground model. The upper structure of the model was assumed to be elastic. The energy absorption of base isolated buildings during earthquakes is concentrated to the isolation layer so that the model can be regarded as SDOF systems.

The isolators of the model consist of laminated rubber bearings and dampers. Fig.4 shows the non-linear restoring force characteristics of the isolators. The parameters of the hysteresis characteristics were varied to verify the applicability of the simplified prediction method in the range of realistic design. The varied parameters were pre-yield natural period T_d, post-yield natural period T_1 and yield load Q_y. T_d and T_1 are expressed as follow:

$$T_d = 2\pi\sqrt{M/(k_1 + k_d)} \qquad (6)$$

$$T_1 = 2\pi\sqrt{M/k_1} \qquad (7)$$

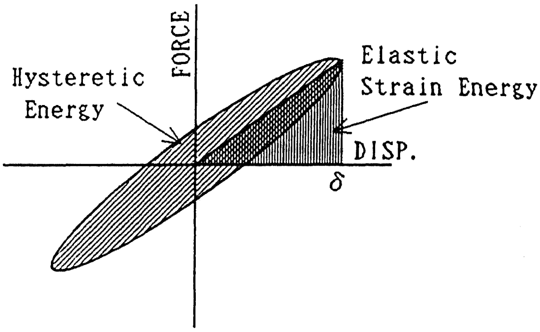

Fig.1 1-cycle momentary energy absorption

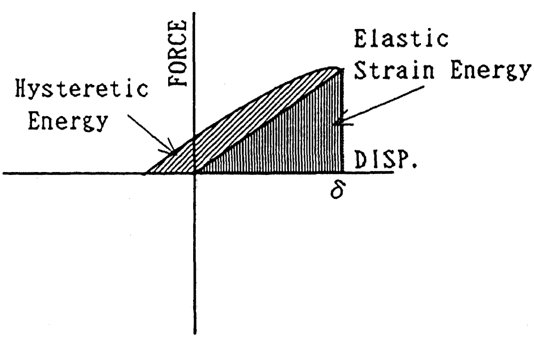

Fig.2 1/4-cycle momentary energy absorption

where M, k_1 and k_d are the mass of the upper-structure, the post-yield and the pre-yield stiffness respectively. The displacement where the hardening begin, δ_H, was assumed 50 cm and the stiffness of the hardening model k_2, k_3 and k_4 were assumed to be proportional to k_1.

The response of the building was simulated by direct integration method.

3.2 GROUND MOTIONS

For the purpose of the simulation, three ground motions were prepared, which is fit to the tentative design spectrum for seismic isolation systems of fast breeder reactors. The observed phase spectra of La Union (Mexico in 1985), Shiranuka and Furoufushi, (Japan in 1983) were used to generated the time histories of the ground motions. Three ground motions were named MEX, SRN, FRF after their original phase spectra. Fig.5 shows time histories of the input ground motions. The amplitude of the ground motions in the figure is correspond to S1 design earthquake.

3.3 SIMULATION CONDITION

Table 5 shows the cases of simulation. The amplitudes of ground motions were 1.5S1, 3.0S1 and 4.0S1. Total number of the cases were 162 for the combination of the parameters of the isolators and the ground motions with various phases and amplitudes.

4 COMPARISON

Fig.6 shows the maximum displacements of the isolators by the numerical simulation. The hardening of the isolators occurred in 3.0S1 and 4.0S1 cases, not in 1.5S1. The simulated results were scattered, especially in 1.5S1 input level.

An example of prediction by the proposed method is shown in Fig.7. The momentary energy input spectra are plotted by solid lines and the momentary energy absorption for every 1 cm of response displacement is plotted by a circle marker. The crossing point of them is estimated value.

The Error Δ of prediction by the proposed method were evaluated by following equation.

$$\Delta = (d_e - d_s)/d_s \qquad (8)$$

where d_e and d_s are maximum displacements evaluated by the proposed method and the numerical simulation, respectively. Figs.8, 9 and 10 show the error to each amplitude of ground motions. The mean of absolute values of the errors is shown in the figures.

In the case of 1.5S1, the errors were larger than the other cases as shown in Fig.8, especially the cases of pre-yield period 0.82 sec. In the numerical simulation, the response in the pre-yield stage may be influential. And it might be a major cause of the randomness of the responses. The distribution of the errors of the proposed method shows that the method gives too much conservative prediction in some cases.

In the case of 3.0S1 and 4.0S1, the mean of the errors was 0.12 $\sim$ 0.16. Most of the estimated values nearly agreed with the simulated values. The proposed method can predict the ultimate behavior of the base isolated structures.

The effect of the range in the integration, 1-cycle or 1/4-cycle was not important and there was no significant difference in the distribution of the errors.

5 CONCLUSIONS

A simplified prediction method of maximum displacement of SDOF hysteretic systems wase proosed. This method is based on momentary energy input spectra and momentary energy absorption of structures. This new simplified prediction needs no complicated models for restoring force characteristics, but only energy absorption capacity during 1-cycle of vibration, which is obtained from a structural element test.

The applicability of the proposed method was validated by the comparison to non-linear simulation with various ground motions and base isolated building models. Though the proposed method gave too much conservative prediction in some cases, the ultimate response of the base isolated building models could be evaluated.

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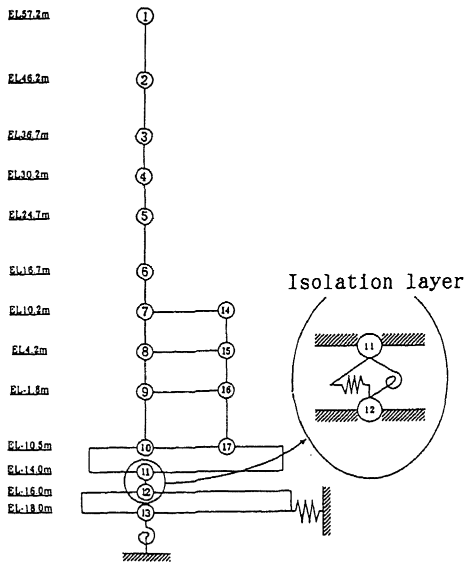


Fig.3 Model of a base isolation building

Table 1 Characteristics of the upper-structure

Level (m)	No. of Mass	Mass ($\times 10^6$ kg)	Shear Section Area (m^2)	Moment of Scond Order ($\times 10^4 m^4$)	Moment of Inertia ($\times 10^6 m^2$)
EL57.2	1	2.565			0.187
			27.7	0.338	
EL46.2	2	2.271			0.167
			27.7	0.338	
EL36.7	3	2.941			0.565
			64.3	1.45	
EL30.2	4	5.400			1.41
			81.2	3.06	
EL24.7	5	11.330			2.96
			107.	5.43	
EL16.7	6	14.080			3.68
			130.	6.25	
EL10.2	7	16.220			6.33
			175.	8.03	
EL4.2	8	16.220			4.96
			190.	8.03	
EL-1.8	9	14.570			4.51
			194.	8.03	
EL-10.5	10	22.300			5.84
			3584.	93.7	
EL-14.0	11	15.050			3.94
			-	-	
EL-16.0	12	10.420			3.34
			4340.	139.	
EL-18.0	13	10.420			3.34
EL10.2	14	7.950			0.390
			66.3	0.699	
EL 4.2	15	2.330			0.108
			66.3	0.699	
EL-1.8	16	2.660			0.124
			66.3	0.699	
EL-10.5	17	-			-

Table 2 Characteristics of ground model

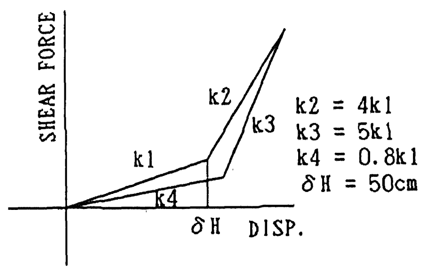
	Spring Constant	Coefficient of Damping
Horizontal Spring	N/m $K_h = 8.98 \times 10^7$	N·sec/m $C_h = 1.22 \times 10^{10}$
Rotational Spring	N·m/rad $K_\theta = 9.67 \times 10^{18}$	N·m·sec/rad $C_\theta = 1.91 \times 10^{16}$

Table 3 Rotational characteristics of the isolators

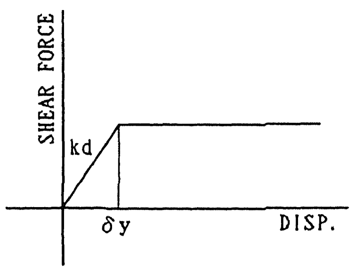
Rotational Spring Conatant	Dmping Ratio
$K_\theta = 6.61 \times 10^{14}$ N·m/rad	0.02

Table 4 Characteristics of material

	Elastic Modulus (N/m^2)	Shear Elastic Modulus (N/m^2)	Mass per Unit Volume (kg/m^3)	Damping Ratio
Concrete	2.3×10^2	9.66×10	2.44×10^3	0.05



(a) Rubber bearing model



(b) Damper model

Fig.4 Restoring force model of the isolators

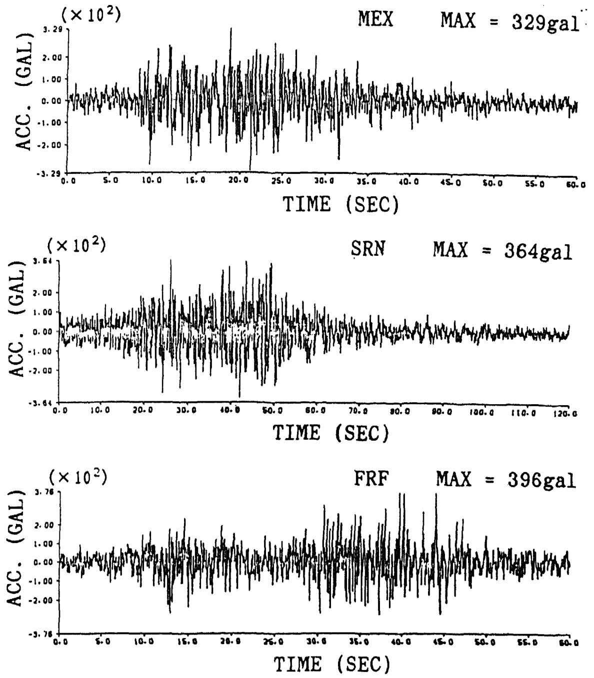


Fig.5 Ground motions (S1 design earthquake)

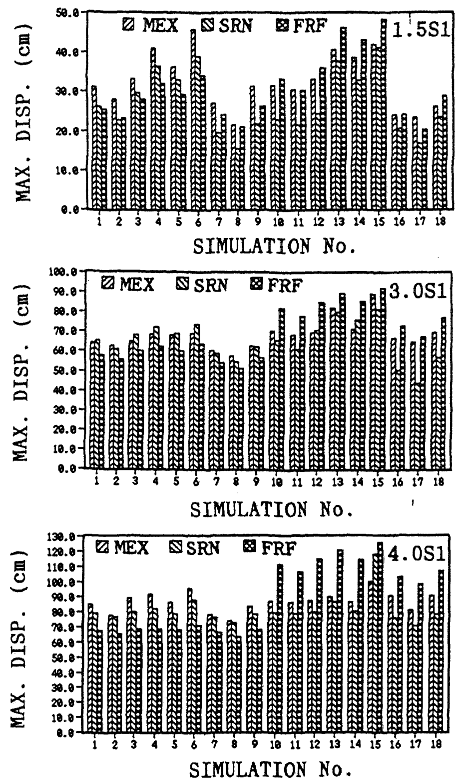


Fig.6 Maximum displacements simulated by numerical model

Table 5 Simulated cases

No. of Simulation	Post-yield Natural Period (sec)	Pre-yield Natural Period (sec)	Yield Load	Ground Motion	Magnification of input
M - 1.5 - 1 (S) (3.0) (F) (4.0)	2.0	1.0	0.050W	MEX SRN FRF	1.5 (3.0) (4.0)
2	2.0	0.82	0.050W		
3	2.0	1.15	0.050W		
4	2.0	1.0	0.025W		
5	2.0	0.82	0.025W		
6	2.0	1.15	0.025W		
7	2.0	1.0	0.075W		
8	2.0	0.82	0.075W		
9	2.0	1.15	0.075W		
10	3.0	1.0	0.050W		
11	3.0	0.82	0.050W		
12	3.0	1.15	0.050W		
13	3.0	1.0	0.025W		
14	3.0	0.82	0.025W		
15	3.0	1.15	0.025W		
16	3.0	1.0	0.075W		
17	3.0	0.82	0.075W		
18	3.0	1.15	0.075W		

W : Weight of the Upper-structure

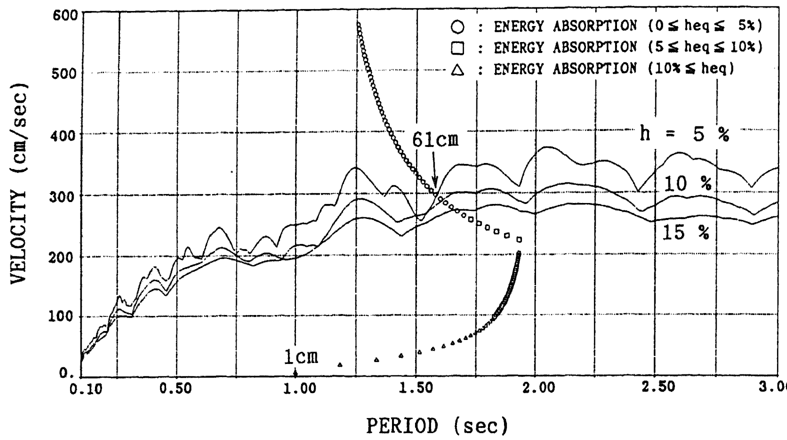
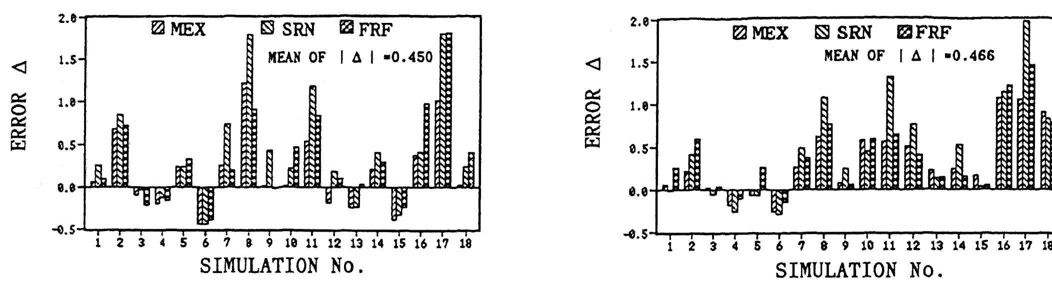
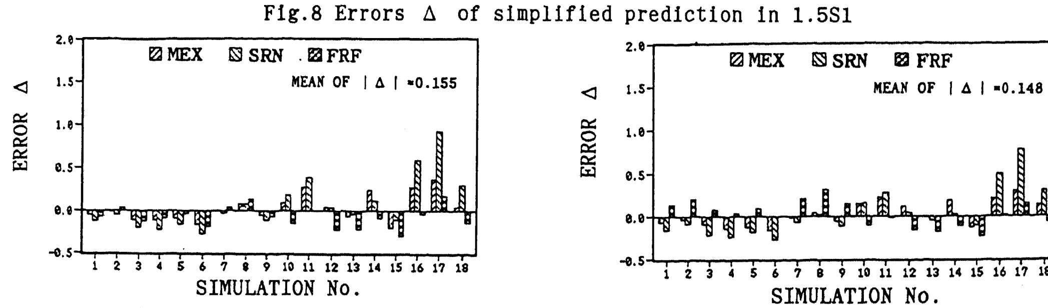


Fig.7 Simplified response prediction with 1-cycle momentary energy input spectra and 1-cycle energy absorption (Simulation No. : M-3.0- 1)



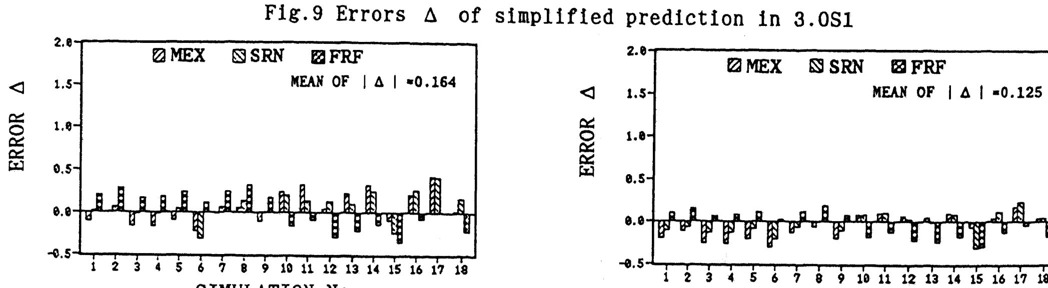
(a) 1-cycle prediction (b) 1/4-cycle prediction

Fig.8 Errors Δ of simplified prediction in 1.5S1



(a) 1-cycle prediction (b) 1/4-cycle prediction

Fig.9 Errors Δ of simplified prediction in 3.0S1



(a) 1-cycle prediction (b) 1/4-cycle prediction

Fig.10 Errors Δ of simplified prediction in 4.0S1