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On the importance of band limited excitation in direct integration analysis of structures

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ABSTRACT: Seismic analysis of structures, done using direct integration of equations of motion, often requires a time step which is much smaller than the sampling interval at which the accelerogram is available. This necessitates the need for interpolating the digital accelerogram, which is conventionally done by linear interpolation between samples. Linear interpolation modifies the frequency content of the data and inserts spurious high frequency components, thereby changing the band limited nature of the original accelerogram and increasing the high frequency components at the cost of reducing power in the low frequency range. Such an accelerogram when used as base acceleration for analysis shows high frequency jitters. The paper highlights the problems caused by linear interpolation. An interpolation scheme that maintains the band limited property and, thus, avoids spurious high frequency components in the structural response is proposed.

1 INTRODUCTION

Seismic analysis of structures is often done using direct integration methods in time domain, wherin the seismic input is provided in the form of an acceleration time history. This input accelerogram is a digital record of accelerations provided at a constant sampling interval (say 0.02 sec) to the analyst. Direct integration of equations of motion may require a time step which is much smaller than the sampling interval at which the accelerogram has been provided. This necessitates the need for interpolating the digital accelerogram, which is conventionally done by linear interpolation between samples. However, as the original digital accelerogram is essentially a band limited signal, linear interpolation modifies the frequency content of the data and inserts spurious high frequency components at the cost of reducing power in the low frequency range. High frequency insertion in input acceleration history, excites high frequency modes of the structure, thereby yielding a jittery response. This paper highlights the inadequacy of linear interpolation and suggests the use of an interpolation technique by virtue of which the band limited property is maintained.

The interpolation is done by zero packing the data to an extent required for analysis. This zero packed accelerogram is low-passed to recover the base band signal of interest and eliminate the unwanted image of components generated by sampling rate expander. Thus the proposed interpolation technique maintains the band limited property in the interpolated data. Simple structural systems are analysed for illustrating the usefulness of the scheme. These are analysed using both conventional linear interpolation and with the aid of the proposed band limited interpolation technique.

2 THE NEED TO INTERPOLATE

Seismic analyses using direct integration schemes employ a ground acceleration history that is either recorded or is synthetically generated. In either case these are digital values at equally spaced discrete time intervals. Since majority of the accelerographs are analog in nature the history obtained from them requires digitization. The instrument and the digitization process introduces noise that has to be removed by band pass filtering. The upper cut-off frequency of this band is generally 25 to 27 Hz. Moreover, most available corrected recorded accelerograms are available at a sampling interval of 0.02 sec, which amounts to a Nyquist frequency of 25 Hz. In other words the highest frequency content of the accelerogram also gets decided by the sampling interval.

Synthetic accelerograms are often generated at 100 samples per *sec*, but are normally provided to the analyst (generally the group deciding upon the seismic loading history is different from the group doing structural analysis) at a sampling interval of 0.02 *sec*. This again implies a Nyquist frequency of 25 *Hz*. Perhaps the reason for this is that design engineers normally use acceleration response spectra, in which 0.04 *sec* is the smallest period at which the acceleration response spectrum value is provided (UBC 1988).

Let us now briefly discuss the time step requirements for direct integration schemes. For conditionally stable schemes, stability considerations may require a time step $\Delta t \leq T_{min}/\pi$, where T_{min} is the smallest natural period of the structure (Owen and Hinton 1980). Unconditionally stable implicit schemes on the other hand may require a small Δt from the point of view of accuracy. It has been suggested that results are accurate when the time step is limited as $\Delta t/T \leq 0.01$ (Bathe and Wilson 1978; Owen and Hinton 1980), where T is the fundamental period of the structure. For the Newmark method ($\beta = 0.25$ and $\gamma = 0.5$) it has been shown (Bathe and Wilson 1978) that the period elongation for an undamped single degree freedom system is less than 3% for $\Delta t/T \leq 0.1$ and the method does not decay response amplitudes.

For nonlinear problems it becomes necessary to iterate within a time step to obtain a converged solution. It has been felt that it is better to reduce the time step rather than pushing iteration of nonlinear quantities within a time increment (Zienkiewicz *et al.* 1984).

The point that emerges from the above discussion is that the sampling interval at which the input accelerogram is available, may, in fact, be too large for direct integration. In such cases it has been considered reasonable to assume that ground accelration varies linearly in the time interval, while ecognising simultaneously that this may result in loss of accuracy (Zienkiewicz *et al.* 1984). Our ontention is that linear interpolation introdudes a igh frequency content which is absent in the original record provided.

3 INTERPOLATION SCHEMES

Let the earthquake accelerogram data, which is a band limited sequence of samples be available at interval T_1. The fundamental frequency interval of this band is $f_s = 1/T_1$. Let for direct integration it is required to interpolate the data to a smaller sampling interval $T_2 < T_1$.

Let us now consider linear and band limited interpolation schemes in some detail.

3.1 Linear interpolation

The impulse response function $h(n)$ of a linear interpolation filter can be written as

$$h(n) = \begin{cases} 1 - \frac{|n|}{R} & ; \quad n \in [-R, R] \\ 0 & ; \quad \text{otherwise} \end{cases} \qquad (1)$$

where R is a factor by which sampling rate has been increased. This function has a triangular shape and the frequency response can be obtained through Fourier transformation as

$$H(e^{j\omega}) = \frac{1}{R}\left(\frac{\sin\frac{\omega R}{2}}{\sin\frac{\omega}{2}}\right)^2 \qquad (2)$$

'he log magnitude of the frequency response for $= 10$ is shown in Figure 1. It can be seen that peak side lobe attenuates only to 26 dB relative the pass band. Consequently unless the original cord is highly band limited (*i.e.* to a band $\ll$ '$2RT_1$) it will fail to attenuate replicates of the ›ectrum and thus introduce jitters in the solution.

3.2 Band limited interpolation

Let the original accelerogram sampled at T_1 be $x(\cdot)$ and the interpolated accelerogram, interpolated by a factor R, which shall be assumed to be an integer, be $y(\cdot)$. If $h(\cdot)$ is the finite impulse response of the filter required for interpolation then the interpolated signal

$$y(n) = \sum_k h((k + \delta_n)T_1)x([n/R] - k) \qquad (3)$$

where

$$[n/R] = int(n/R) \quad i.e. \text{ integer part of } n/R$$

and

$$\delta_n = n/R - [n/R] \quad ; \quad \delta_n \in [0, 1]$$

From equation 3, it is clear that evaluation of $y(n)$ requires filter $h(\cdot)$ sampled at fractional delays of $\delta_n T_1$ from the sample. This makes the band limited interpolating filter a time varying system. However, the required time varying $h(\cdot)$ filter can be realized by R time invariant filters that are subsets of $h(\cdot)$. These time invariant filters operate at a low sampling rate and are known as polyphase fil-

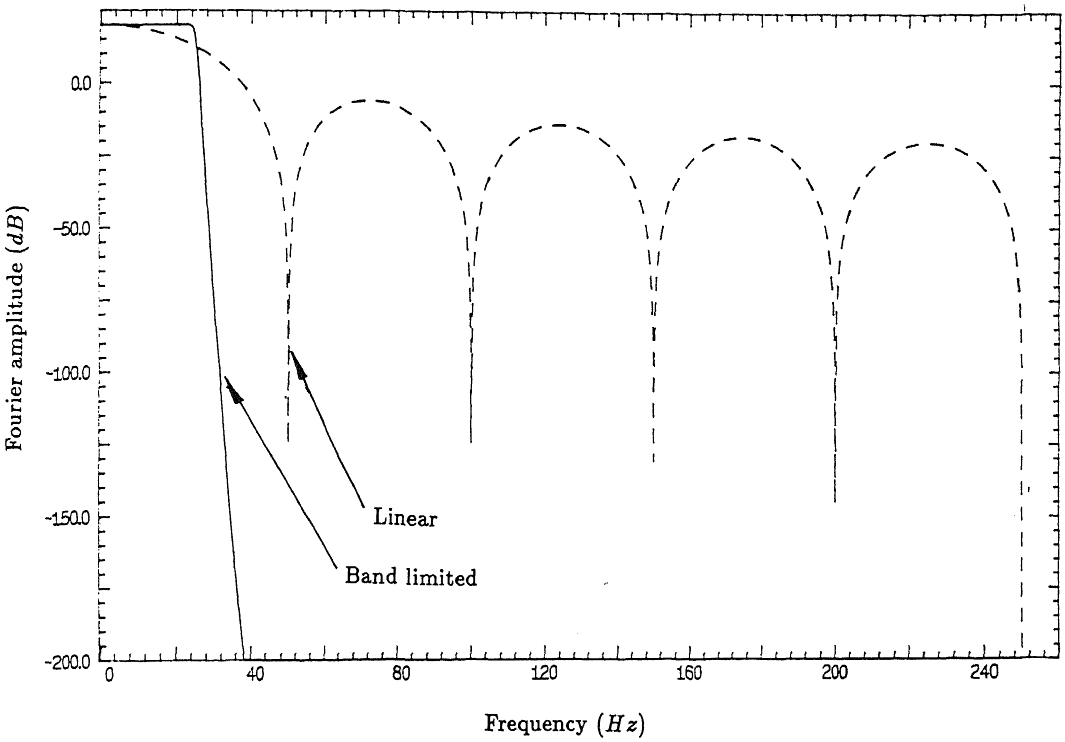

Figure 1: Frequency response of band limited and linear interpolation (initial sampling interval = 0.02 *sec*, interpolated sampling interval = .002 *sec*)

ters (Bellenger 1976). Equation 3 shows that the input sample is at time $[n/R] - R$ and the output at sample time n. The output can be written as

$$y(n) = \sum_k p_n(k)x([n/R] - k) \qquad (4)$$

where $p_n(k) = h((k + \delta_n)T_1)$. Note $p_n(k)$ is periodic in n with period R. The term $[n/R]$ increases by one for every R samples. The $x(\cdot)$ enters at low sample rate f_s and $y(\cdot)$ is evaluated at sample rate Rf_s. Equation 4 can be written in polyphase structure as

$$y(nR + m) = \sum_k p_m(k)x(n - k) \qquad (5)$$

where

$$p_m(k) = h(kR + m) \quad m = 0, 1, \cdots, R - 1$$

If the lowpass filter $h(\cdot)$ has N taps and N is divisible by R, then each of the R polyphase subfilters will be identical in structure with N/R taps. Each input $x(\cdot)$ will generate one output for each of the polyphase subfilters as indicated by equation 5.

The interpolation is done by inserting $R-1$ zeros in between two existing data points. This results in the desired decrease of sampling interval. This redefining of sample rate introduces R replicates of the spectrum in the band of frequencies defined at this higher rate. However, the spectrum remains periodic in $1/T_1$. The resulting data is low passed to eliminate the replicated copies. The task of this lowpass filter is to reject the spectral copies that occur at integer multiples ($< R$) of the input sampling frequency. Figure 1 compares frequency response of band limited interpolation and linear interpolation. The length N of the Finite Impulse Response (FIR) lowpass filter can be chosen using the relation for transition band width of FIR antialiasing (Elliot 1987) filter as

$$\Delta f = K(A)\frac{f_s}{N} \qquad (6)$$

where A is the minimum stopband attenuation and $K(A)$ is an attenuation related scale factor. $K(A)$ is bounded by

$$\frac{-20\log(A)}{25} < K(A) < \frac{-20\log(A)}{20} \qquad (7)$$

From experience of filter design, the new two sided analysis band width is

$$2f_p = \frac{\alpha f_s}{R} \quad (8)$$

where α is a factor for alias free band width. The sampling rate, f_s required to obtain an alias free pass band down to a level $1/A$ satisfies the Nyquist theorem given by

$$f_s/R = 2f_p + \Delta f \quad (9)$$

By using equations 6 and 8 into 9 gives

$$N = R\frac{K(A)}{(1-\alpha)} \quad (10)$$

Note that for every R samples of zero packed data passed through the filter only one sample is nonzero. This reduces computation to N/R multiplications and additions per output data point.

Oetken *et al.* (1975) give design of an optimal filter for interpolation.

4 ILLUSTRATIVE EXAMPLES

Let us now consider some examples to illustrate the points that we have been making. Here we limit ourself to elastic damped and undamped single degree freedom (*SDF*)systems subjected to base excitation in the form of an acceleration time history. Direct integration is carried out using Newmark method (Bathe and Wilson 1978) with constants $\beta = 0.25$ and $\gamma = 0.5$.

4.1 Sinusoidal excitation

An undamped *SDF* system with natural frequency of 34.8 Hz is subjected to a base acceleration of the form

$$\ddot{y}(t) = a_1 \sin 2\pi f_1 t + a_2 \sin 2\pi f_2 t + a_3 \sin 2\pi f_3 t$$

with

$$\begin{aligned} a_1 &= 10.0 \\ a_2 &= 5.0 \\ a_3 &= 3.0 \\ f_1 &= 23.19 \\ f_2 &= 13.43 \\ f_3 &= 5.49 \end{aligned}$$

The excitation is generated in a digital form at an interval 0.02 *sec*. This is then interpolated using linear and band limited interpolation with $R = 10$, which gives a record at 0.002 *sec*. These interpolated accelerograms are used as base acceleration ($\Delta t/T \approx 1/14$). Subsequent to direct integration from which the absolute acceleration response of the *SDF* system is obtained a Fourier analysis is carried out. Figure 2 shows the variation of Fourier amplitudes expressed in dB with frequency for the two interpolation schemes. For such an excitation it is also possible to find a closed form solution which is also shown in Figure 2. Clearly the response at excitation frequencies is accurately predicted by both interpolation schemes. However in the high frequency range the response due to linear interpolation is jittery showing a presence of high frequencies that did not exist in the original signal. At the structural frequency too linear interpolation overestimates the response.

4.2 Parkfield earthquake

An actual corrected earthquake accelerogram obtained from Parkfield earthquake of June 27, 1966 recorded in Chalome, Shandon (California Institute of Technology record no. IIB034, component N85E) was used as input excitation. The above record was available at a sampling interval of 0.02 *sec* and was interpolated by both the methods to obtain acceleration values at 0.002 *sec* ($R = 10$). Figure 3 shows the variation of Fourier amplitudes of the interpolated accelerograms. It can be seen that the linearly interpolated record contains considerable high frequency components.

A *SDF* system with frequency 40 Hz and damping of 2% of critical was subjected to these "interpolated earthquakes" ($\Delta t/T = 1/12.5$). Fourier spectra of the absolute acceleration response is shown in Figure 4. Once again it is seen that the linearly interpolated record overestimates the response in the high frequency range. Moreover, linear interpolation yields a jittery and thereby unreliable response as compared to band limited interpolation.

In order to test how the two interpolated accelerograms would perform on a structure with a small natural frequency these were used as input motions on a soft structure (T=0.4 *sec* and damping 2% of critical). This however also implied using a time step much smaller than strictly required ($\Delta t/T = 1/200$). The result of the analysis is shown in Figure 5, and it can be seen that the two interpolation techniques perform identically well in this case. However, whether this is due to a low natural frequency or due to a very small Δt needs to be examined.

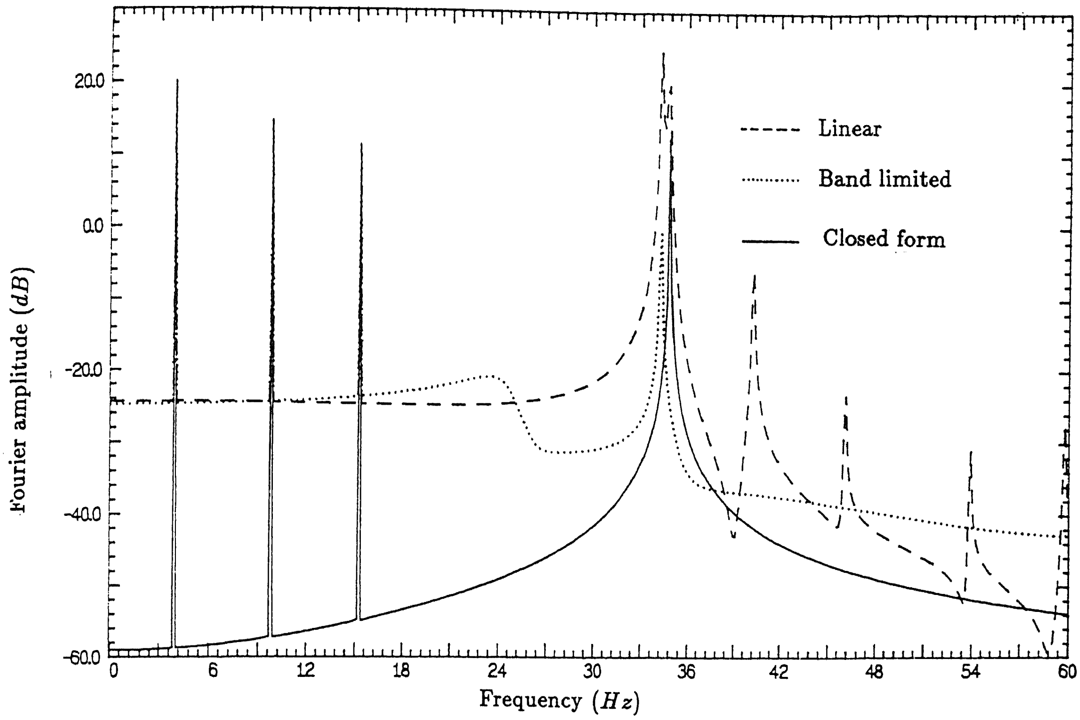

Figure 2: Fourier spectra of response of an undamped SDF system (natural frequency = 34.8 Hz) subjected to a sinusoidal input consisting of three frequencies

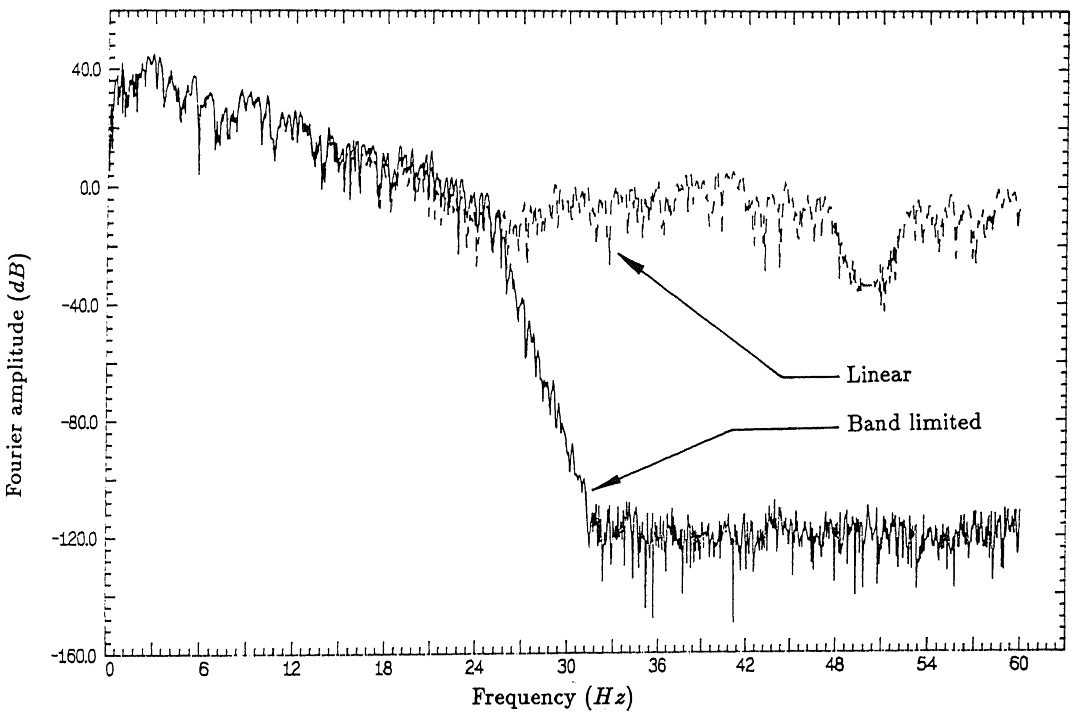

Figure 3: Fourier spectra of interpolated Parkfield accelerogram using band limited and linear interpolation

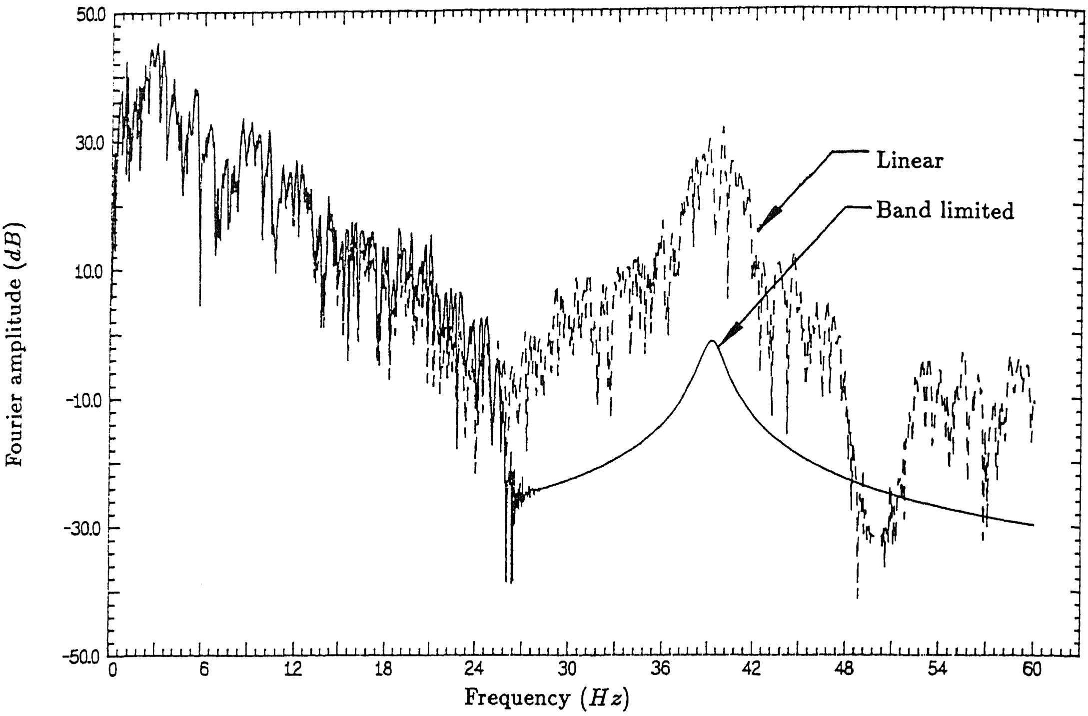

Figure 4: Fourier spectra of response of a SDF system (natural frequency = 40 Hz, damping = 2%) subjected to Parkfield earthquake

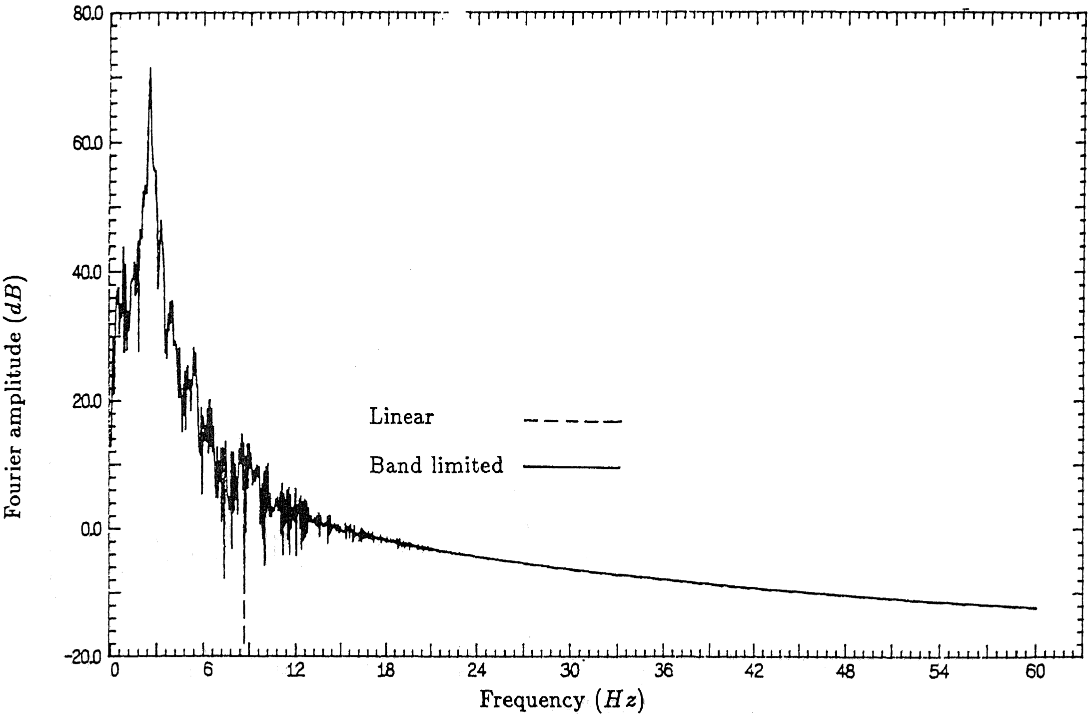

Figure 5: Fourier spectra of response of a SDF system (natural frequency = 2.5 Hz, damping = 2%) subjected to Parkfield earthquake

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