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Earthquake response analysis by numerical spline approximation

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ABSTRACT: A numerical spline approximation with higher order polynomial functions is used to study the earthquake response of structures. A fourth order polynomial function is utilized to represent the structural displacement. The corresponding acceleration is described by a second order polynomial function. This approximation is shown to yield excellent results with relatively little computational effort. A computer code has been developed by the authors specifically for this investigation.

1 INTRODUCTION

Among the various methods available for solving the nonlinear equation of motion, one of the most effective is step-by-step integration method. Two of the most popular methods are the constant and linear acceleration methods (Clough et al. (1975)). However, the high accuracy is demanded in the analysis of earthquake response of structures. In this paper, a fourth order polynomial function is used to represent the structural displacement. The corresponding acceleration is described by a second order polynomial function with time. This method is shown to yield excellent results with relatively little computational effort. The result has been examined to be unconditionally stable. The current result, in comparison with those obtained using the constant and linear acceleration methods, is higher accurate. The authors have developed an efficient computer code specifically for this investigation.

2 FOURTH ORDER POLYNOMIAL SPLINE FUNCTION APPROXIMATION

For convenience, a structure having only a single degree of freedom is considered, as shown in Fig. 1, which is assumed to be excited by an earthquake loading. In this case, the dynamic equilibrium equation can be represented as

$$M\ddot{u}+C\dot{u}+Ku=F(t) \tag{1}$$

in which u is the displacement relative to the ground motion, and M, C, and K are structural mass, damping, and stiffness, respectively. In (1), F(t) = -M×ä(t) with ä(t) representing an earthquake acceleration record.

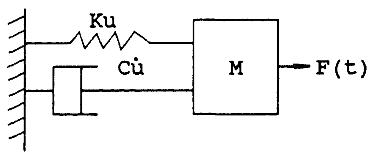

Fig.1 A single degree of freedom system.

The dynamic response of a nonlinear system over a time domain can be discretized into many segments. The displacement of each segment corresponding to each time interval Δt may be represented by a fourth order polynomial function D_4 (Cheney et al. (1980)).

$$D_4(\tau)=a_0+\frac{a_1\tau}{1!}+\frac{a_2\tau^2}{2!}+\frac{a_3\tau^3}{3!}+\frac{a_4\tau^4}{4!} =\{1,\frac{\tau}{1!},\frac{\tau^2}{2!},\frac{\tau^3}{3!},\frac{\tau^4}{4!}\}\{A\} \tag{2}$$

in which

$$\{A\}=\{a_0,a_1,a_2,a_3,a_4\}^T \tag{3}$$

in these expressions, $\tau = \theta\Delta t$ is a specified time parameter counted from the current time t, θ is a non-dimensional variable, Δt is the time step size, D_4 is an approximating displacement value with fourth order polynomial, and {A} is a fifth order coefficient vector which needs to be determined. The shape of the polynomial function D_4 is shown in Fig. 2.

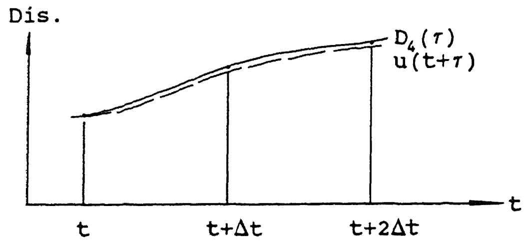

Fig.2 Representation of displacement by a fourth order polynomial.

In order to determine the coefficient vector {A}, we assume $\theta=1$ and 2. The initial conditions at time t can be given as follows:

$$D_4(0)=a_0=u(t) \qquad (4)$$

$$\dot{D}_4(0)=a_1=\dot{u}(t) \qquad (5)$$

$$\ddot{D}_4(0)=a_2=\ddot{u}(t) \qquad (6)$$

The dynamic structural response at time $t+\tau$ is described approximately by

$$\ddot{u}(t+\tau) = \ddot{D}_4(\tau) \qquad (7)$$

$$\dot{u}(t+\tau) = \dot{D}_4(\tau) \qquad (8)$$

$$u(t+\tau) = D_4(\tau) \qquad (9)$$

then, the dynamic equilibrium conditions at time $t+\Delta t$ and $t+2\Delta t$ can be established as

$$M\ddot{D}_4(\Delta t)+C\dot{D}_4(\Delta t)+KD_4(\Delta t)=F(t+\Delta t) \qquad (10)$$

$$M\ddot{D}_4(2\Delta t)+C\dot{D}_4(2\Delta t)+KD_4(2\Delta t) =F(t+2\Delta t) \qquad (11)$$

(10) and (11) can be combined to solve for the unknown coefficients as

$$a_3=(c_5c_1-c_2c_4)^{-1}(c_3c_5-c_6c_2) \qquad)$$

$$a_4=(c_5c_1-c_2c_4)^{-1}(c_6c_1-c_3c_4) \qquad (1.$$

in which

$$c_1=\Delta tM+\frac{\Delta t^2}{2}C+\frac{\Delta t^3}{6}K \qquad (14$$

$$c_2=\frac{\Delta t^2}{2}M+\frac{\Delta t^3}{6}C+\frac{\Delta t^4}{24}K \qquad (15)$$

$$c_3=F(t+\Delta t)-\{K,C+\Delta tK,M+\Delta tC+\frac{\Delta t^2}{2}K\}\{a_0,a_1,a_2\}^T \qquad (16)$$

$$c_4=2M\Delta t+2\Delta t^2C+\frac{4}{3}\Delta t^3K \qquad (17)$$

$$c_5=2\Delta t^2M+\frac{4}{3}\Delta t^3C+\frac{2}{3}\Delta t^4K \qquad (18)$$

$$c_6=F(t+2\Delta t)-\{K,C+2\Delta tK,M+2\Delta tC+2\Delta t^2K\}\{a_0,a_1,a_2\}^T \qquad (19)$$

the structural responses for the next time step is

$$u(t+\Delta t)=D_4(\Delta t) \qquad (20)$$

$$\dot{u}(t+\Delta t)=\dot{D}_4(\Delta t) \qquad (21)$$

$$\ddot{u}(t+\Delta t)=\ddot{D}_4(\Delta T) \qquad (22)$$

A vector form of (20), (21), and (22) is written as

$$\{V(t+\Delta t)\}=\{u(t+\Delta t),\dot{u}(t+\Delta t),\ddot{u}(t+\Delta t)\}^T =[R]\{a_0,a_1,a_2\}^T \qquad (23)$$

3 NUMERICAL STABILITY

Assume $\delta\{V(t)\}$ to be the accumulative error at time t introduced by computer round-off. The errors so induced would propagate to the next time step as the following quantity

$$\delta\{V(t+\Delta t)\}=[R]\delta\{V(t)\} \qquad (24)$$

The error at time t+NΔt is given by

$$\delta\{V(t+N\Delta t)\}=[R]^N\delta\{V(t)\} \qquad (25)$$

For a stable integration scheme, the error $\delta\{V(t+N\Delta t)\}$ should be bounded as N approaches infinity. This condition occurs if and only if all the absolute eigenvalues of matrix [R] are equal to or less than one. Then, the stability criterion is

$$\lambda = Max(\lambda_i([R]))\leq 1 \qquad (26)$$

The stability analysis is conventionally studied by using an undamped single degree of freedom system without a loss generality. An undamped system is chosen because it is less stable than a damped system. The numerical results of λ([R]) are plotted against Δt/T in Fig. 3, and a comparison is made with those obtained from constant and linear acceleration methods. T is the natural period, which represents the dynamic characteristics of an undamped single degree of freedom system. The range of Δt/T used in computer analysis is between 10^{-4} and 10^{4}. Fig. 3 illustrates that the proposed fourth order polynomial approximation is unconditionally stable (Yener, Shen, and Gong (1991)).

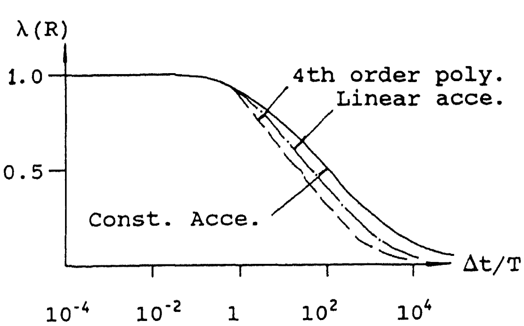

Fig.3 Spectral radii λ(R) with Δt/T for stability checking

4 ACCURACY

The accuracy of numerical integration algorithms can be evaluated by considering amplitude decay and period elongation of a dynamic response. Fig. 4 shows a plot of the amplitude decay and period elongation versus Δt/T compared with both constant and linear acceleration methods. The results indicate that the proposed approach exhibits considerably less amplitude decay in comparison with the amplitude decay exhibited by both constant and linear acceleration methods. Furthermore, the results indicate that the proposed method also shows less period elongation than the constant and linear acceleration methods (Yener, Shen, and Gong (1991)).

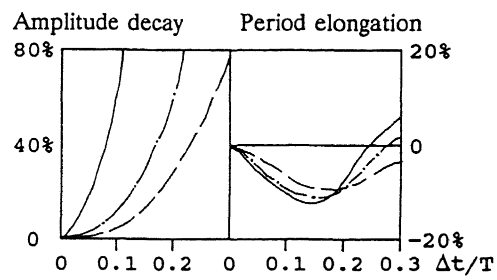

Fig.4 Amplitude decay and period elongation

5 CONCLUSIONS

A fourth order polynomial spline approximation is proposed to increase the accuracy of numerical analysis of earthquake structural resistant response. The numerical stability and accuracy of the proposed method has been systematically investigated. In the basis of numerical results obtained by using the computer code developed by the authors, it is concluded that this method is unconditionally stable and highly accurate.

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