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Structural reliability analysis under earthquake loading

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ABSTRACT: In this paper, a method to estimate the structural seismic reliability is presented. The nonlinear structural behavior under earthquake loading is described by a smooth nonlinear hysteretic model. An equivalent linearization method is used to simplify the structural analysis. The statistical characteristics of structural response are obtained analytically. An example problem is solved to determine the structural seismic reliability of a one story building.

1 INTRODUCTION

The concept of structural seismic reliability has been gaining more attention during the past thirty years. Structural seismic reliability requires an analytical method that evaluates the factors contributing to reliability. The first factor to be considered is the distribution of seismic parameters. For low seismic activity regions, it is appropriate to use site intensity and peak ground acceleration as seismic parameters. The second consideration is the probability of structural failure based on seismic parameters. Therefore, structural seismic reliability is written as

$$R = 1 - P(D>d/S,A)\,P(S,A) \qquad (1)$$

in which R is the structural seismic reliability, P(D>d/S,A) is the conditional probability of structural failure, and P(S,A) is the probability of seismic parameters. The probability of site intensity and that of peak ground acceleration have been discussed systematically by Yener and Shen (1991).

2 SEISMIC RESPONSE ANALYSIS OF STRUCTURES

2.1 *Spectrum of earthquake ground acceleration*

The frequency-spectral characteristics of ground acceleration can be described by the following stationary Kanai-Tajimi-type spectrum (Yang (1986)).

$$S_a(\omega) = S_0\left[\frac{\omega_g^4 + 4\omega_g^2\beta_g^2 + \omega^2}{(\omega^2 - \omega_g^2)^2 + 4\omega_g^2\beta_g^2\omega^2}\right] \qquad (2)$$

in which S_a is the spectrum of ground acceleration, S_0 is a constant power spectral density, ω is modal frequency, ω_g is soil frequency, and β_g is soil damping. S_0 in (2) can be computed by integrating both sides of this equation with respect to ω, and by equating the right hand side terms of (2) and the following expression.

$$\int_{-\infty}^{\infty} S_a\, d\omega = \left(\frac{A_{S_I}}{2.65}\right)^2 \qquad (3)$$

The nonstationary motion-time history is generally represented by a stationary time history, which is multiplied by an appropriate time factor $\phi(t)$ as

$$\phi(t) = \begin{cases} \left(\frac{t}{t_1}\right)^2 & for\ 0 \le t < t_1 \\ 1 & for\ t_1 \le t < t_2 \\ e^{c(t_2 - t)} & for\ t_2 \le t < t_d \end{cases} \qquad (4)$$

in (4) t_1, t_2, c are the statistical parameters based on soil type, and t_d is earthquake duration.

2.2 *Stochastic seismic analysis of nonlinear systems*

Structural response caused by an earthquake loading having a strong site intensity tends to be nonlinear. A simple model for a nonlinear structure having n DOF is shown in Fig. 1. In this case, the equation of motion is modified as (Wen 1980)

$$[M]\{\ddot{Y}\}+[C]\{\dot{Y}\}+\{\alpha\}[K]\{Y\}+(\{1\}-\{\alpha\})[K]\{Z\}=-[M]\{a\}_{s_i} \qquad (5)$$

in which an element α_i of vector α represents the ratio of the nonlinear stiffness to the linear stiffness for the ith level. Similarly, an element Z_i of vector **Z** is the hysteretic component of the ith level, which is a function of Y_i. Z_i is related to Y_i through the following first-order nonlinear differential equation (Wen 1980).

$$\dot{Z}_i=A_i\dot{Y}_i-\beta_i(|\dot{Y}_i|)Z_i(|Z_i|)^{r-1}+\gamma_i\dot{Y}_i|Z_i|^r \qquad (6)$$

in which β_i and γ_i are parameters which define the shape of the hysteretic loop for the ith level, A_i is the restoring force amplitude at the ith level, and r is smoothness parameter for the transition from elastic response.

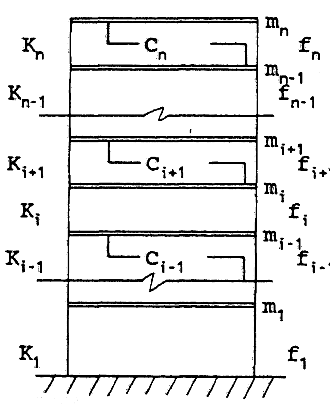

Fig.1 Single degree of freedom with smooth nonlinear hysteretic system

The equivalent linearized form of (6) is

$$\dot{Z}_i = C_{ei}\dot{Y}_i + K_{ei}Z_i \qquad (7)$$

in which C_{ei} and K_{ei} are equivalent coefficients. If a_{s_i} is represented by a zero-mean stationary Gaussian process, for the case r=1, these two coefficients can be evaluated in terms of the second moments of Y_i and Z_i as (Wen 1980)

$$C_{ei} = -A_i + \sqrt{\frac{2}{\pi}}[\gamma_i\frac{E(\dot{Y}_iZ_i)}{\sigma_{\dot{Y}_i}} + \beta_i\sigma_{Z_i}] \qquad (8)$$

$$K_{ei} = \sqrt{\frac{2}{\pi}}[\gamma_i\sigma_{\dot{Y}_i} + \beta_i\frac{E(\dot{Y}_iZ_i)}{\sigma_{Z_i}}] \qquad (9)$$

On the basis of the extensive amount of data produced by Shen (1988) on reinforced concrete structures, the writers have induced the following relationships for parameters in (6).

$$A_i = 1, \quad r = 2, \quad \beta_i = -3\gamma_i \qquad (10)$$

in which

$$\gamma_i = -\frac{1}{2}[\frac{(1-\alpha_i)k_i}{d_i}]^2 \qquad (11)$$

In (11), d_i represents an element of vector d, and is the ratio of the nonlinear ultimate strength to linear yield strength for the ith level. By substituting (10) and (11) into (6), the coefficients C_{ei} and K_{ei} in the equivalent linearized equation (7) can be determined as

$$C_{ei}=1-\frac{2}{\pi}\beta_i\sigma_{Z_i}^2[\frac{\pi}{2}-arctg(\frac{\sqrt{1-\rho_{\dot{Y}_iZ_i}^2}}{\rho_{\dot{Y}_iZ_i}})+\rho_{\dot{Y}_iZ_i}\sqrt{1-\rho_{\dot{Y}_iZ_i}} - \delta_i\sigma_{Z_i}^2] \qquad (12)$$

$$K_{ei}=-\beta_i-\frac{4}{\pi}\sigma_{\dot{Y}_i}\sigma_{Z_i}\{(1-\rho_{\dot{Y}_iZ_i}^2)^{\frac{3}{2}}+\rho_{\dot{Y}_iZ_i}[\frac{\pi}{2}arctg(\frac{\sqrt{1-\rho_{\dot{Y}_iZ_i}^2}}{\rho_{\dot{Y}_iZ_i}})] + \rho_{\dot{Y}_iZ_i}\sqrt{1-\rho_{\dot{Y}_iZ_i}^2}]\} - 2\sigma_iM[\dot{Y}_iZ_i] \qquad (13)$$

in which

$$\rho_{\dot{Y}_iZ_i} = \frac{M[\dot{Y}_iZ_i]}{\sigma_{\dot{Y}_i}\sigma_{Z_i}} \qquad (14)$$

Representing the restoring force f_i at level i by

$$f_i = C_{ei}\dot{Y}_i + \alpha_iK_{ei}Y_i + (1-\alpha_i)K_{ei}Z_i \qquad (15)$$

from Fig. 1, the equation of motion for each level from 2 to n can be written as

$$\ddot{Y}_i-\frac{f_{i-1}-f_i}{m_{i-1}}+\frac{f_i-f_{i+1}}{m_i}=0,\quad (i=2,3,\dots,n) \qquad (16)$$

In a similar manner, the equation governing the response of the first level becomes

$$\ddot{Y}_1+\frac{f_1-f_2}{m_1}=2\xi_g\omega_g\dot{Y}_g+\omega_g^2Y_g \qquad (17)$$

the equation of motion of the ground level is a function of a(t), and can be written as

$$\ddot{Y}_g+2\xi_g\omega_g\dot{Y}_g+\omega_g^2Y_g=-a(t) \qquad (18)$$

the acceleration record measured under bedrock a(t) is assumed to be described by the write-noise process.

In a matrix form, the system equation of motion takes the form

$$\{\dot{u}\}+[F]\{u\}=-\{v\}a(t) \qquad (19)$$

in which {u} is structural response vector, [F] is restoring force matrix, {v} is coefficient vector.

In order to obtain the statistical governing equation, we first postmultiply (19) by $\{u\}^T$, and take the mean of the resulting expression. We then add this expression to its transpose to obtain the following,

$$[\dot{S}(t)]+[F(t)][S(t)]+[S(t)][F(t)]^T=[P] \qquad (20)$$

in which

$$[S(t)]=[M(u(t)u(t)^T)] \qquad (21)$$

$$[P]=\{v\}\{v\}^T2\pi S_0 \qquad (22)$$

In a similar manner, we postmultiply (19) by $\{u(t_2)\}^T$, and take the mean of the resulting expression to obtain

$$\frac{\partial}{\partial t_1}[S(t_1,t_2)]=-[F(t_1)][S(t_1,t_2)]+[M(-\{v\}\alpha(t_1)\{u(t_2)\}^T)] \qquad (23)$$

in which

$$[S(t_1,t_2)]=[M(\{u(t_1)\}\{u(t_2)\}^T)] \qquad (24)$$

It should be noted that for $t_1 > t_2$, $[M(-\{v\}\alpha(t_1)\{u(t_2)\}^T)] = 0$. Hence (23) reduces to

$$\frac{\partial}{\partial t_1}[S(t_1,t_2)]=-[F(t_1,t_2)][S(t_1,t_2)] \qquad (25)$$

3 SEISMIC RELIABILITY OF STRUCTURES

3.1 *Conditional failure probability of structures*

In general, the probability that the structural response D_i exceeds the limit of structural level failure B_i is small. Hence, we assume that the response D_i can be represented by a continuous Poisson distribution. The above structural response statistics are to be used in determining the conditional failure probability of the ith level, for a specified site intensity S_i and ground peak acceleration A_{S_i}. This probability is given as (Ang 1975)

$$P_i(D_i>B_i/S=S_i,A=A_{S_i})=$$
$$1-\exp[-\int_0^T\frac{\sigma_{\dot{D}_{S_iA S_i}}}{\pi\sigma_{D_{S_iA S_i}}}\exp[-\frac{B_i^2}{2\sigma^2_{D_{S_iA S_i}}}]dt] \qquad (26)$$

Then, the probability of structural conditional failure is expressed as

$$P_{S_iA_{S_i}f}=1-\prod_{i=1}^{n}(1-P_i) \qquad (27)$$

3.2 *Seismic reliability of structures*

The determination of structural seismic reliability in a specified region should include the effects of the probabilities of site intensity index. Furthermore, based on past earthquake records, it can be concluded that site intensity index does not necessarily indicate the highest intensity earthquake which may have occurred in that region. Hence, the determination of structural seismic reliability should be based on the results of sprobability of seismic parameters by using several different site intensities. On this basis, considering the distribution of site intensity, peak ground acceleration, and the structural conditional failure probability, we propose the use of the following equation for estimating the structural seismic reliability.

$$R = 1 - \sum_i \int_0^\infty P_{S_i A_{s_i}} P_{A_{s_i}} f_{S_i 1} dA_{S_i} \qquad (28)$$

For the purpose of implementation, (28) can be put into the following discrete form

$$R=1-\sum_{i=5}^{12} P_{S_i} = 1-\sum_{i=5}^{12} P_{S_i A_{S_i}} P_{A_{S_i}} f_{S_i 1} \qquad (29)$$

4 ILLUSTRATIVE EXAMPLE

A one story frame structure shown in Fig. 2 is analyzed for seismic reliability. To carry out the analysis, the structural parameters (Table 1) and soil characteristics (Table 2) need to be specified. The information listed in Table 1 is obtained by testing a prototype model of the one story structure being considered (Shen 1988).

Based on the test data, the nonlinear structural hysteretic parameters are computed, and listed in Table 3. The test is also used to determine the structural failure limit. The parameters given in Table 2 are representative of Type III soil. By the method mentioned, the standard deviation of the relative displacement response is normalized with respect to a reference yield displacement. Using the experimentally determined failure level displacement, (1/150)H, in which H is the height of structure, as the structural failure limit, the structural seismic reliability is analytically determined to be 0.935. In this analysis, the structural failure rule is represented by the both bound mechanism (Yang 1986).

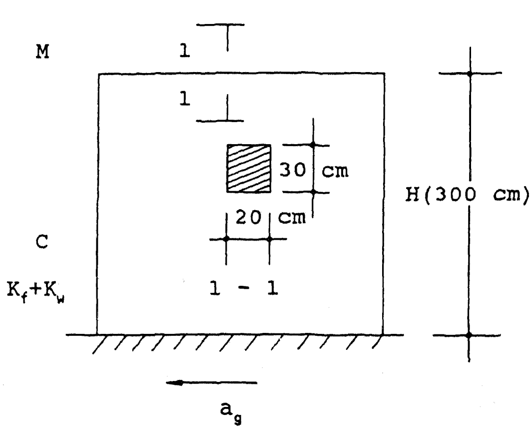

Fig. 2. Nonlinear hysteretic SDOF Structure

Table 1. Structural prameters of one story building

Col. net (cm)	Col. sec. (cm)	Beam sec. (cm)	Window hole (cm)
450×350	30×30	30×20	90×120

Ini. K_{frame} (T/cm)	Ini. K_{wall} (T/cm)	Yield load P_y(T)
9.824	28.29	18.32

Yield disp. (cm)	Ult. load P_u(T)	Ult. disp. U_u(cm)
0.971	26.10	1.693

Table 2. Parameters for type III soil

t_1	t_2	c	t_d	ω_g	β_g	σ_A^2
1.0	8.0	0.6	9.82	16.5	0.80	$100.4S_0$

Table 3. Structural hysteretic parameters

Hyst. parameters	A	r	α	K	β
Brick wall frame	1.0	2	0.219	20.963	1.016
Frame	1.0	2	0.349	9.824	0.938

5 CONCLUSION

A method is presented to estimate seismic reliability of structures based on site intensity and ground peak acceleration. Structural nonlinear property is represented by a smooth nonlinear model. As indicated by the illustrative example, the procedure can be used to analyze various structures in low and intermediate seismic activity regions. The computations required for the seismic reliability analysis developed in this paper can be carried out without undue complexity.

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