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Optimal nonlinear analysis by bayesian methods

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ABSTRACT: The paper presents a Bayesian method for i) the evaluation of the knowledge about the failure probability of a structure resulting from nonlinear computations and for ii) the selection of the earthquake intensities to be used as input in those computations; the probability dostributions of the earthquake intensity and of the available ductility in the structure are assumed known. The failure probability is determined by an input-space formulation where the probability distributions of the available ductility in the different structural variables are projected on the space of the variables that describe the earthquake action through the vulnerability function. The Bayesian method presented operates on the probability distributions that represent the state of knowledge about the vulnerability function. The earthquake intensity to be used in the nonlinear computations is selected through a preposterior analysis i.e. the probable results of the analysis are evaluated and the one corresponding to the greater improvement in knowledge is selected.

1 INTRODUCTION

The development of Earthquake Engineering in the last years have shown so large a multiplicity of new models and numerical techniques that it seems now more worthwhile to try to organize the existing ones than to develope new ones. In this paper a first step in this direction is presented under the form of a Bayesian method to orientate the application and to interpret the results of nonlinear analysis. In effect, the availability of sophisticated models for the dynamic analysis of structures in the nonlinear range needs a reassessment of the use of analytical methods in earthquake engineering. A sophisticated model can give very detailed information about structural behaviour while a coarser model can give information which may suffice in given circunstances or, more precisely, the coarse model furnishes a restricted description of the results obtained from the sophisticated model (Muncaster, 1983; Segev, 1990). From a pragmatic perspective the differentiation between coarse and sophisticated models results from the large difference in the amount and quality of data needed in their quantification and in the even larger difference in the amount of the results of the analyses. While the quantification of parameters in coarse models is generally straigthforward, the quantification of at least some parameters in sophisticated models may be very difficult without specific experimental studies (e.g. the quantification of the parameters controlling the strength degradation); in consequence, a designer may feel justifiably confident in attributing values to the parameters of coarse models but should feel much more keenly the uncertainties and deficiencies in information when selecting the values of the parameters in a sophisticated model. On the other hand, the output of a coarse model may be easily interpreted in terms of design operations (e.g. the results of a response spectrum analysis may be 'processed' as the results of an analysis for static loads) while the information contained in the output of a sophisticated model (e.g. the time-histories of the structural response for several realizations of the earthquake action) will surely be much more difficult to integrate into the design procedure in a clear and complete manner. In consequence, the passage from coarse to sophisticated models will give a greater emphasys to the informational aspects of earthquake engineering.

The informational aspects of engineering comprise the construction of methodologies and the processing of information. The construction of methodologies involve the identification of variables, the evaluation of models, the selection of representative values for the variables and the definition of 'strategies' (Duarte and Campos-Costa, 1988) that determine the decisions to be made in consequence of the result of the analyses. The processing of information comprehends the different techniques that can be utilized and the calibration of the 'strategies' (Duarte and Campos-Costa, 1989). The object of the informational viewpoint is to establish the relationships between the different models rather than to value each one, which cannot be done without considering some instance of application. The present paper briefly presents the approach that is being developed in the National Laboratory for Civil Engineering, Lisbon, to those informational aspects of structural and earthquake engineering (Duarte, 1991).

The developments of the informational aspects are also becoming more important due to the evolution of earthquake resistance codes from documents where the consensual knowledge of engineers and scientists is compiled to documents where some objectives are identified and the means to attain those objectives are formulated in a logical way. This rational structure of the recent earthquake resistance codes is clearly in evidence in Eurocode 8.

However, the informational aspects of earthquake engineering may only be fully developed if the earthquake engineering itself is "formalized" in a rational and coherent structure, where the informational content of each part can be effectively defined. A possible "formalization" was attempted by Duarte, 1990. The relevant parts of this formalization are here included for easiness and completeness of reading.

2 DESCRIPTIVE FUNCTIONALS

The earthquake action will be considered to have several components and a finite duration T_1; hence, every possible earthquake action will be assumed to belong to the space A_s of vector valued functions Lebesgue integrable in absolute value (L_V). Conversely, every function belonging to A_s will be considered to be a generalized earthquake action.

The response of the structure is assumed to be described by a finite number r of variables (displacements, accelerations, internal forces ...); the time histories of the responses to all generalized actions constitute a 'space' $\mathcal{R}_r$; the duration of interest T_2 of the response time histories will be assumed to be greater than T_1.

The relationship between an element $\mathrm{a}(t)$ of $\mathcal{A}_s$ and the corresponding element $\mathrm{r}(t)$ of $\mathcal{R}_r$ defines the structure operator $\mathcal{E}$:

$$\mathrm{r}(t_2) = \mathcal{E}(\mathrm{a}(t_1)) \quad t_1 \in [O, T_1] \quad t_2 \in [0, T_2] \qquad (1)$$

The operator $\mathcal{E}$ is assumed to be sufficiently continuous i.e. for two action histories $\mathrm{a}_1(t_1)$ and $\mathrm{a}_2(t_2)$ sufficiently close to one another, the corresponding response histories will also be close to one another. The equation (1), which is an equation of motion, must be understood in the sense that it establishes a correspondence between the complete time history of the action $\mathrm{a}(t_1)$ and the complete time history of the response $\mathrm{r}(t_2)$.

The description of the response of the structure by an equation of motion may be too detailed for engineering purposes. A possibility to concentrate on the essentials of the response and of the action is to define a small number of "descriptive functionals". If a large enough number of descriptive functionals is defined a perfect description of action and response may be obtained but no significative simplification is gained. On the other hand the identification of the best descriptive functionals is an open problem although the definition of their descriptive power (Duarte, 1990) furnishes a first step. In consequence, in this paper it is assumed, when that is necessary, that the peak values of action and response have adequate descriptive power and they will be represented by the supremum norm $||^{\infty}$; for generality, the descriptive functionals for the actions will be noted by $\mathbf{h}$ and the descriptive functionals for responses by $\mathbf{c}$ and will be called respectively the intensity measures and control variables;

3 THE VULNERABILITY FUNCTION

The vulnerability function represents structural behaviour, not as a mapping of the time histories of the earthquake input into the time histories of the structural response, but as a much simpler relation between the intensity measures $\mathbf{h}$ and the control variables $\mathbf{c}$. From the equation of motion it is possible to write:

$$c = c(\mathcal{E}(\mathrm{a}(t_1))) \qquad (2)$$

In this new and simplified form, the equation of motion represents a mapping from the space of the actions into the Euclidean vector space C of the control variables. The last step needed for the vulnerability function to appear is to substitute $\mathbf{a}(t)$ by $\mathbf{h}$. This substitution poses somes problems. In effect, it is possible to use the inverse image $\mathcal{F}(\mathbf{h})$ of $\mathbf{h}$ defined by

$$\mathcal{F}(\mathbf{h}) = \{\mathbf{x}(t) : \mathbf{h}(\mathbf{x}) = \mathbf{h}, \mathbf{x} \in \mathcal{A}_s\} \qquad (3)$$

In general, $\mathcal{F}(\mathbf{h})$ will be constituted by a large number of time histories $\mathbf{x}(t)$; for instance if the descriptive functional is the peak value of acceleration, $\mathcal{F}(\mathbf{h})$ will be all acceleration time histories with peak value $\mathbf{h}$. Thus if the vulnerability function $\mathbf{V}(\mathbf{h})$ is defined as $\mathbf{V}(\mathbf{h}) = \mathbf{c}\ (\epsilon\ (\mathcal{F}\ (\mathbf{h})))$ for a given vector $\mathbf{h}$ a large number of possible values for the control variables would result. The effective way to side-step this indetermination is to define as the value of the vulnerability function a weighted average of all the possible $\mathbf{c}$ values. This can be done in a relatively straightforward manner when the earthquake action is idealized as a stochastic process (Duarte, 1990). A stochastic process is an ensemble of functions where a probability measure μ_a is defined. The advantage of introducing stochastic processes is that the expected values of the descriptive functionals can be considered to represent the whole space of actions or structural responses. Let $\mu_a(i), i \in I$ be a family of probability measures such that $E(\mathbf{h}, \mu_a(i)) \neq E(\mathbf{h}, \mu_a(j))$ for $i \neq j$, where $\mathrm{E}(*,*)$ means expected value in terms of the indicated probability measure.

Let the mapping $\mathcal{F}$ be the inverse of the mapping $\mathbf{h} = \int_{\mathcal{A}_s} \mathbf{h}\,(\mathbf{a}(t_1))d\mu_a(i)$. Then the vulnerability function is defined as

$$c = \mathbf{V}(\mathbf{h}) = \int_{\mathcal{A}_s} c(\mathcal{E}(\mathbf{a}(t_1)))\, d\mu_a(\mathcal{F}(\mathbf{h})) \tag{4}$$

For the present purposes, a stochastic process is a weighted ensemble of functions of the time. In mathematical terms and considering a L_v space as the ensemble of functions, the definition of a measure in L_v needs the definition of a σ-field S of subsets of L_v; this σ-field is the σ-field generated by the cylinder subsets of L_v i.e. subsets comprising all functions $\mathbf{x}(t)$ for which $\mathbf{x}(t_1) \in \mathbf{X}_1$, $\mathbf{x}(t_2) \in \mathbf{X}_2, \dots \mathbf{x}(t_N) \in \mathbf{X}_N$, where $t_1, t_2, \dots t_N \in (0, T_1)$ and $(\mathbf{X}_1, \mathbf{X}_2, \dots \mathbf{X}_N)$ are Borel sets in R^{vN}. Now a stochastic process is defined as a triplet (L_v, S, μ) with $\mu(L_v) = 1$; μ is a measure defined on the σ-field S. The specification of the measure μ defines the process; in general, a large number of functions may have a zero probability density of occurrence for a given process; however for Gaussian processes all functions $\mathbf{x}(t) \in L_v$ have nonzero probability density (Ibrahimov and Rozanov, 1974). A function with a nonzero probability density is called a realization of the process.

The measure defined in a L_v space may be used for the definition of functionals which will be descriptive of the complete space and not of a single realization. For instance it is now possible to define the mean peak value of a function $\mathbf{x}(t)$ as $mpv(\mathbf{x}) = \int_{L_v} \| \mathbf{x}(t) \|^{\infty}\, d\mu$ i.e. the mean peak value is the ensemble average of the greatest maximum (in absolute value) of each function in the ensemble. In a similar way the mean value of any functional $g(\mathbf{x}(t))$ is $E(g(\mathbf{x}(t))) = \int_{L_v} g(\mathbf{x}(t))d\mu$. The stochastic analysis of a structure signifies the characterization, in terms of probability concepts, of its response from the knowledge of the probability measure defined on the vibration space $\mathcal{A}_s$. In a general way, a stochastic model of the vibrations is a triplet $(\mathcal{A}_s, S_a, \mu_a)$ where $\mathcal{A}_s$ is the vibration space, S_a is a σ-field defined on $\mathcal{A}$ and μ_a is its probability measure; the corresponding stochastic model of the responses is a triplet $(\mathcal{R}_r, S_r, \mu_r)$ where $\mathcal{R}_r$ is the response space $\mathcal{R}_r = \mathcal{E}(\mathcal{A}_s)$, S_r is the σ-field generated by the $\mathcal{E}$-images of the subsets in S_a, and μ_r is the measure induced in $\mathcal{R}_r$ by $\mathcal{E}$ i.e. to each measurable set in $\mathcal{R}_r$ is attributed the measure of its inverse $\mathcal{E}$-image in $\mathcal{A}_s$.

In general it is only needed or useful to know some functionals of the responses, as for instance the mean peak value:

$$\begin{aligned} \mathrm{mpv}(\mathrm{r}) &= \int_{\mathcal{R}_r} \| \mathrm{r}(t_2) \|^{\infty}\, d\mu_r = \\ &= \int_{\mathcal{A}_s} \| \mathcal{E}(\mathbf{a}(t_1)) \|^{\infty}\, d\mu_a \end{aligned} \tag{5}$$

This expression emphasizes that the responses may be characterized by performing the ensemble averaging with the probability measure defined on the vibration space.

It is now possible to define rigorously and in general terms a vulnerability function. Taking the expected value of equation (2):

$$\mathrm{E}(c) \equiv \int_{\mathcal{R}_r} c\, d\mu_r = \int_{\mathcal{A}_s} c(\mathcal{E}(\mathbf{a}(t_1)))d\mu_a = \mathrm{E}(c, \mu_a) \tag{6}$$

The expected value of a descriptive functional $\mathbf{h}(\mathbf{a}(t))$ of the actions is

$$E(\mathbf{h}) \equiv \int_{\mathcal{A}_s} \mathbf{h}(\mathbf{a}(t_1))d\mu_a = E(\mathbf{h}, \mu_a) \tag{7}$$

Both expected values depend on the measure μ_a; consequently the notation $E(\mathrm{c}, \mu_a)$ and $E(\mathbf{h}, \mu_a)$ is used. Let μ_{ai}, $i \in I$ be a family of probability measures, with I being a suitable index set; this family is constructed in a such way that

$$E(\mathbf{h}, \mu_{ai}) \neq E(\mathbf{h}; \mu_{aj}) \quad i \neq j \tag{8}$$

Hence the mapping

$$E(\mathbf{h}) = E(\mathbf{h}, \mu_{ai}) = \int_{\mathcal{A}_s} \mathbf{h}(\mathbf{a}(t_1))d\mu_{ai} \tag{9}$$

is a one-to-one mapping between i and $E(\mathbf{h})$ with domain I and range $\mathcal{H}$ defined by

$$\mathcal{H} = \{\ \mathrm{x} : \mathrm{x} = \int_{\mathcal{A}_s} \mathbf{h}(\mathbf{a}(t_1))d\mu_{ai}, \mathrm{i} \in I\} \tag{10}$$

The family of probability measures is supposed to be sufficiently diversified so that $\mathcal{H}$ is large enough for the applications; in general, the family of probability measures will depend continuously on a given number k of independent parameters and thus $I \equiv \mathcal{R}^k$ and in consequence $\mathcal{H}$ will be homeomorphic to $\mathcal{R}^k$; let $\mathcal{F}$ be the inverse mapping of the mapping defined by expression (9) i.e. $i = \mathcal{F}(E(\mathbf{h}, \mu_{ai}))$. Then the vulnerability function is defined by the composition of the mapping $\mathcal{F}$ and the mapping defined by equation (6):

$$\mathrm{c} = E(\mathrm{c}, \mu_{ai}) \quad i = \mathcal{F}(\mathbf{h}) \quad \mathbf{h} \in \mathcal{H} \tag{11}$$

where c and **h** are vectors of numerical values (and not values of the descriptive functionals); emphasizing the dependence of the probability measure on the index i by the notation $\mu_a(i) = \mu_{ai}$, the vulnerability function is written as:

$$\mathrm{c} = \mathrm{V}(\mathbf{h}) = E(\mathrm{c}, \mu_a(\mathcal{F}(\mathbf{h}))) \tag{12}$$

Probabilities used in this section are completely artificial in the sense that they do not pretend to represent any 'random' physical process. On the other hand they are the basis for the evaluation process of the state of knowledge described below, and they may be considered to be 'constructed' for that purpose.

4 PROBABILITY OF FAILURE AND LIMIT STATES

4.1 Hazard probability

Let μ^s be the probability measure defined in $\mathcal{H}$ by the hazard corresponding to the seismicity of the site. This probability defines two probability distributions of interest, one on the time histories of the response and other on the values of the control variables. The probability distribution of the time histories of the response is defined by the composition of the mapping defined by the structure operator $\mathcal{E}$ (equation(1)) with the inverse of the mapping between a(t) and h; this last mapping is a probabilistic mapping, since to every value of $\mathrm{h} \in \mathcal{H}$ it associates a probability measure $\mu_a(i)$ in $\mathcal{A}_s$, with $i = \mathcal{F}(\mathrm{h})$(expression (11)). Let S_r be the σ-field generated in $\mathcal{R}_r$ by the $\mathcal{E}$-images of the Borel sets in $\mathcal{A}_s$ and $\mu_r(i)$ be the measure induced in $\mathcal{R}_r$ by $\mu_a(i)$; then for every set $X \in S_r$ is associated a probability:

$$P(X) = \int_{\mathcal{H}} \int_X d\mu_r(\mathcal{F}(\mathrm{h}))d\mu_h \equiv \int_X d\mu_S \qquad (13)$$

where μ_h is the probability measure defined in the space of the actions by the hazard model and μ_S is the probability measure in the space of time histories defined by the seismicity model and the stochastic model of the actions.

In consequence the consideration of the earthquake hazard mathematically translates only as a reorganization of the probability distribution in $\mathcal{A}_s$.

4.2 Probability of failure

The probabilistic idealization of structural failure may be carried out in two ways. In the more rigorous approach, the structural behaviour is dependent on the actual values of the limit states; hence the vulnerability function is dependent on the probabilistic definition of the limit states and either elementary or averaged vulnerability functions may be considered; an elementary vulnerability function is the vulnerability function for a given realization of the limit states; the averaged vulnerability function is the probability-weighted average of the elementary vulnerability functions for all limit states realizations. In the simplified approach limit states and structural behaviour are mutually independent i.e. the vulnerability function is not dependent on the limit states values, and the limit states may be defined without reference to the vulnerability function; only this approach will be considered in the present paper and only with deterministic limit states.

It is assumed that the structure may have independent collapse mechanisms (Campos-Costa, 1990); each collapse mechanism is characterized by a distribution of plastic hinges; when each plastic hinge belongs to just one collapse mechanism, the collapse mechanisms are independent. Failure occurs when the ductility demand in all plastic hinges of a collapse mechanism is greater than their ductile capacity. Let $F_{ij}(\mu)$ be the cumulative distribution of the ductile capacity of the j-th hinge on the i-th mechanism; assume the earthquake hazard is represented by the probability density $p(\overline{a})$ of the mean peak value of the acceleration; then the cumulative distribution of failure $F(\overline{a})$ in terms of the mean peak acceleration is

$$F(\overline{a}) = 1 - \Pi_i(1 - \Pi_j F_{ij}(V_{ij}(\overline{a}))) \qquad (14)$$

where $V_{ij}(\bar{a})$ is the vulnerability function for hinge j in mechanism i. The probability of failure is given by $p = \int_o^\infty p(\overline{a})F(\overline{a})d\overline{a}$. Assuming that actions $(p(\bar{a}))$ and resistances $(F_{ij}(\mu))$ are known, only the vulnerability functions must be determined, by analysis or testing, in order that the probability of failure may be computed.

5 THE BAYESIAN METHOD

5.1 Probabilistic Representation of Knowledge

Assuming that the probability distributions of the earthquake vibrations and resistances are known, the evaluation of the probability of failure is dependent on the knowledge about the vulnerability function. The Bayesian method here presented has the purpose of estimating this function by a process which is optimal in regard to errors in the probability of failure. The estimation of the vulnerability function involves a probabilistic representation whose mean value will converge to the true value of the vulnerability function and whose variability will tend to zero as the analysis advances.

Presuming total absence of information about structural behaviour, the vulnerability function may be any nondecreasing function, since it is reasonable to expect that an increase of the load would not correspond to a decrease of the load effect. Hence a complete ensemble of nondecreasing functions is a first step to the representation of our knowledge (or its absence) about structural behaviour. Completness means that given the vulnerability function of any structure, there should be in the ensemble a function which is equal (under an appropriate norm) to that vulnerability function. From a practical viewpoint, it is not necessary, nor makes sense, to find the value of the probability of failure with great accuracy. Thus, it is not needed to identify the vulnerability function with much sharpness. Hence, completeness may be understood to be attained if in the ensemble there is always a function which is sufficiently close to any possible vulnerability function. Under this approximation and assuming that the domain and the range of the vulnerability functions are finite (since they represent the values of physical quantities) it follows that the number of ensemble functions that must be considered is finite. This fact suggests that a not very large number of ensemble functions V_i may be used (i<1000) if during the analysis it is investigated if this number should be increased.

This investigation is carried out by verifying how many ensemble finctions are near the average vulnerability function; if this number is small (< 10) more ensemble functions should be generated.

The robustness of the final estimates depends on the qualities of the functions in the ensemble. To control the robustness two partial ensembles are used. The first one is composed of analytical functions of the form

$$c = \alpha h + \beta h^2 + \gamma h^3 \tag{15}$$

appropriate values of α, β and γ being chosen to generate the selected number of functions. The second partial ensemble is constituted by realizations of discrete Markov processes where the value c_i of the control variable, corresponding to a value h_i of the load intensity, suitably discretized is given by

$$c_i = (1 + \delta x)c_{i-1} \tag{16}$$

where δ is a constant and x is a random variable with an uniforme distribution in the interval (0,1). This discrete function is transformed into a continuous function by a linear interpolation rule applied on a bilogarithmic plot. The values of δ are chosen to generate a partial ensemble with the desired characteristics. Results obtained by considering separately the partial ensembles allow to evaluate the robustness.

The ensemble of functions is probabilized by associating to each function V_i a probability value p_i. Each set of values $\mathbf{p} = \{p_1, p_2 ...\}^T$ represents a state of knowledge. There is a a good state of knowledge when all the ensemble functions are associated to very small probability values with the exception of those which are close to the true vulnerability function, which have much larger probability values.

Before any nonlinear analysis is performed there is no knowledge, i.e. the state of knowledge is non-informative. Absence of information must necessarily be understood in relation to some quantity, which in this case is considered to be the probability of failure. A non-informative state of knowledge is represented by a constant probability density in a logarithmic scale (Berger, 1985). This means e.g. that the probability of the probability of failure lying in the interval $(10^{-3}, 10^{-4})$ is equal to the probability of it being comprised in the interval $(10^{-4}, 10^{-5})$. It should be noted that the total probability is theoretically infinite, as frequently happens with non-informative distributions. Probability values p_i may be easily computed to secure approximately a non-informative distribution. The uncertainty associated to a state of knowledge may be quantified by the difference between the 5% and 95% fractiles of the probability distribution of the probability of failure.

5.2 Bayesian Analysis

The value of the vulnerability function is the mean value of the control variable. When a nonlinear computation is performed, necessarily using a realization of the stochastic process representing the earthquake action, the control variable value obtained is only an estimate of the true value of the vulnerability function. However, several realizations may be used and, as a result, a sample of control variables values are obtained. The sample mean value is obviously a better estimate than any individual value. If the number of sample elements is not small, the probability distribution of the sample mean value is approximately a Gaussian distribution (by the central limit theorem) with a mean value equal to the mean value of the response to one realization and with a variance equal to the variance of the response to one realization divided by the number n of sample elements.

In the case of structural response to earthquakes it may be assumed that variance values correspond to a coefficient of variation with a value of 0.3.

This permits the definition of the conditional probability $P(r \mid V_i(h))$ of obtaining average response r for earthquake intensity h if $V_i(h)$ is the true probability function by

$$P(r \mid V_i(h)) = G(V_i(h), 0.09V_i(h)^2/n) \tag{17}$$

where $G(\mu, \sigma^2)$ represents a Gaussian distribution with mean value μ and variance σ^2. Hence, after performing n computations and obtaining an average value r the *a posteriori* probabilities $P(V_i \mid r)$ may be computed by Bayes theorem:

$$P(V_i(h) \mid r) = p_i P(r \mid V_i(h)) / \textstyle\sum_i p_i P(r \mid V_i(h)) \tag{18}$$

These probabilities represent the new state of knowledge. If the variability of the conditional probability $(P(r \mid V_i(h))$ is underestimated (due to either an underestimation of the value of the coefficient of variation or to the true distribution being not sufficiently close to a Gaussian distribution) the Bayesian analysis may present instabilities; should they be detected, a value higher than 0.3 for the coefficient of variation should be used. If the variability of $P(r \mid V_i(h))$ is overestimated the convergence is more slower than it could be but no other undesirable effects result.

5.3 Optimal earthquake intensities for nonlinear computations

The value h of the intensity of the earthquake vibration to be used in the nonlinear computations may be selected to provide an optimal increase in knowledge through a *preposterior* analysis. The idea behind this analysis is to evaluate for a large number of values of h what change in knowledge may be expected if the computations are performed. This evaluation is carried out by considering the probability distribution of obtaining a value r of the response, as may be computed from the probabilities associated with the ensemble functions, for each intensity (Duarte, Ritto-Corrêa, Vaz and Campos-Costa, 1990). It is not necessary to evaluate very rigorously this probability distribution and experience has shown that if a discrete probability distribution of 0.1 probability values coincident with the 5%, 15%... 95% fractiles of the true distribution is adopted, satisfactory results are obtained. Then, the probability of the increase in knowledge assuming that probability distribution is computed and the intensity corresponding to the larger expected increase in knowledge is selected.

6 FINAL REMARKS

The Bayesian method presented in this paper has been proved very useful in the earthquake analysis of structures in the nonlinear range, namely of reinforced concrete bridges whose nonlinear characteristics are idealized by filament models (Duarte, Vaz and Ritto-Corrêa, 1990 and 1991; Vaz, 1991). It seems also worthwhile to indicate the main directions the present authors intend to develop this line of research.

One advacement is the Bayesian evaluation of the conditional probabilities (expression (17)). This evaluation could only take in account, in a first phase, the uncertainties about the coefficient of variation, even if its estimates are not robust. In a second phase more approximate conditional distribution could be postulated, as function of the number of sample elements. The purpose of this advancement is the optimization of the number of sample elements i.e. the number of nonlinear compitations that must be performed.

Another advancement is a more significative quantification of the state of knowledge. It is possibly more important to know accurately the upper bound of the range of the probability of failure than the 90% confidence interval. The only reason this was not already done is because of the difficult semantics involved: in effect, the result would be a probability distribution of a lower fractile of the probability distribution of the probability of failure.

It is also the author's opinion that the application of this Bayesian method to definition of the optimal testing conditions of expensive models (e.g. the selection of the best earthquake for a shaking table test) could be very fruitful. This application would naturally involve extensive numerical simulation.

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