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Numerical study on experimental error propagation of systems with complex hysteretic properties in pseudo-dynamic testing

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ABSTRACT: The pseudo-dynamic testing is an efficient method to investigate dynamic behaviors of structure. It is known ,however, that various experimental errors propagate and exert significant influence on responses of structure if an explicit numerical integration method to predict response was adopted. The authors et al. have verified that satisfactory responses can be predicted by applying the error compensation method in the case of linear or elasto-plastic system with simple hysteretic properties. In the present paper, the error propagation properties for system with complex hysteretic properties are rigorously studied.

1. *INTRODUCTION.*

The pseudo-dynamic testing is a very efficient method to investigate dynamic response characteristics for such members or structures that reveal complex hysteretic behaviors. In the testing process, however, the error are inevitably introduced into the measured restoring forces and displacements. Furthermore, in the computational process using the explicit numerical integration, the restoring forces including the errors are used to predict the response at the subsequent time steps. Consequently, the errors introduced propagate and under some circumstances unrealistic spurious responses, such as dissipation or unbounded growth of responses, may be predicted. The basic properties of error propagation on the elastic systems and the elasto-plastic systems with the simple hysteretic properties have been studied so far by the authors et al(Shirai, Kanda, Adachi et al. 1988). On the other hand, the error propagation properties for such system as reinforced concrete structures with complex hysteretic properties have not clarified yet. In the present paper, the error propagation properties for such systems are rigorously studied by making use of the analytical method developed by the author et al.(Kanda, Shirai, Adachi et al. 1988), which can simulate the complex hysteresis curves of reinforced concrete members accurately.

2. EXPERIMENTAL ERROR IN PSEUDO-DYNAMIC TESTING.

First of all, in this chapter, the concept of the pseudo-dynamic testing and the experimental errors are briefly described.

Figure 1 shows the general composition and process of the pseudo-dynamic testing schematically. The pseudo-dynamic testing is a hybrid method which combines the quasi-static loading test with the response analysis by the computer. In the computational process, therefore, the following equation of motion for the system has to be numerically solved just as for the standard earthquake response analysis.

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + \{R\} = \{F\} \quad (1)$$

Where, $[M]$ and $[C]$ indicate the mass and damping matrices, $\{\ddot{X}\}$ and $\{\dot{X}\}$ the vectors of acceleration and velocity, $\{F\}$ the vector of external excitation and $\{R\}$ the vector of restoring force. Furthermore, the testing is composed of the two processes; that is, the control process, that the computed displacements from Eq(1) are applied by the actuators, and the measuring process, that the displacements and the restoring forces are measured. Thus, the experimental errors may be introduced into either the measured displacements $\{X^m\}$ or the measured restoring forces $\{R^m\}$ as follows.

$$\{X^m\}_n = \{X^c\}_n + \{e^{dm}\}_n \quad (2.a)$$

$$\{R^m\}_n = \{R^c\}_n + \{e^{Rm}\}_n \quad (2.b)$$

in which, $\{X^c\}_n = \{X\}_n + \{e^{dc}\}_n$ and $\{R^m\}_n = \{R\}_n + \{e^{Rc}\}_n$

where, $\{X\}_n$ and $\{R\}_n$ indicate the computed response displacements and the restoring forces corresponding to $\{X\}_n$, $\{X^c\}_n$ and $\{R^c\}_n$ the controlled displacements and the corresponding restoring forces, $\{e^{dc}\}_n$ and $\{e^{Rc}\}_n$ the error displacements, at which the controlling is terminated, and the corresponding error of restoring forces, $\{e^{dm}\}_n$ and $\{e^{Rm}\}_n$ the measured errors of displacements and restoring forces and the subscript n the integration time step. Note that the errors $\{e^{dc}\}_n$, $\{e^{dm}\}_n$ and $\{e^{Rm}\}_n$ are induced mainly due to the sources such as the calibration limit of measuring apparatuses, the resolution limit of A/D converter, the control limit of actuator motion and the frictional forces induced in the loading apparatus(Shing and Mahin 1983,Nakanishi, Adachi,Shirai et al. 1988). Just as general errors, the experimental errors stated in the above can be classified into the systematically generating errors (refers to as "the systematic errors") and randomly generating errors(refers to as "the random errors"). It is known that the systematic errors have either an energy adding or dissipating effect and thus their effect on responses is rather significant than that of the random errors. The undershoot error, in which the control displacements do not arrive at the target displacements, and the overshoot error, in which the control displacements go beyond the target displacements, are the representative of the systematic errors. Furthermore, if a specific loading apparatus is adopted, the frictional forces are often generated systematically against the movement of actuators.In this case, the measured restoring forces are overestimated and thus referred to as "the frictional error". The undershoot error gives an apparent variation of stiffnesses and is the energy adding type.If the errors of the energy adding type are induced, an erroneous growth and phase shift of responses may occur. On the other hand,the overshoot and frictional errors also give an apparent variation of stiffness but is the energy dissipating type. If the errors of the energy dissipating type are induced, an erroneous dissipation and phase shift of responses may occur. In view of generation mechanism of these errors, it is inevitable to eliminate the errors even if an accuracy of measuring apparatus is highly improved. In this paper,the propagation properties of the errors; ① the undershoot error, ② the overshoot error and ③ the frictional error, shall be investigated.

3.ANALYTICAL PROCEDURE

3.1 Analytical method

The experimental errors are induced not only in the pseudo-dynamic testing but also in the static loading test. In the case of the static loading test, the errors do not propagate and thus do not have a significant effect on the results. In the case of the pseudo-dynamic test, on the other hand, the errors do propagate and have a significant effect on responses even if an amount of errors is little. Therefore, it is very difficult to obtain the true or approximate elasto-plastic responses of reinforced concrete structures and to evaluate the error propagation properties experimentally. An alternative to this is an utilization of analytical approach to determine the restoring forces of structures. However,it is required that the analytical method to be applied has to be accurate enough to simulate complex elasto-plastic hysteretic behaviors of reinforced concrete structures including such phenomena as the cracking and crushing in concrete,the steel yielding and the opening and closing of cracks. Fortunately, the authors et al. developed a reliable anal-

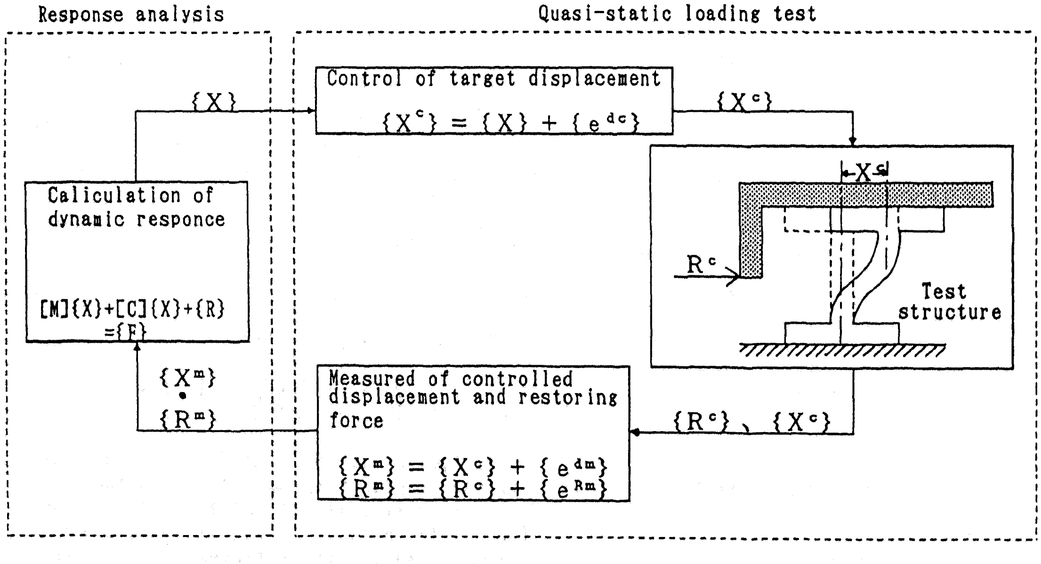


Figure 1. Flow chart and experimental errors in pseudo-dynamic testing

ytical method, which is able to simulate the cyclic hysteretic behaviors of reinforced concrete members, on the basis of the concept of fiber method(Kanda,Shirai,Adachi et al. 1988). Therefore, this fiber method shall be applied to evaluate the restoring forces of reinforced *concrete members. The fiber method is a kind* of finite element method and thus the members to be analyzed are discretized into the segments along the member axis and the slices along the depth. Since this method is based on the hypotheses that the plane section remains plane after deformation and each sliced element is only able to resist the uniaxial stresses acting along the member axis, it can simulate only flexural behaviors of members. The characteristics of the method developed are the constitutive laws used for concrete and reinforcement. The endochronic theory (Bazant et al. 1976) was used for concrete and the Ciampi's model(Ciampi et al. 1982) was used for reinforcing bars.

3.2 Generation of errors

In this study, the experimental errors are artificially generated on the basis of Eqs.(2.a) and (2.b). For the undershoot or the overshoot error, the error vectors in Eq.(2), $\{e^{dc}\}_n$, $\{e^{dm}\}_n$ and $\{e^{Rm}\}_n$ shall be definded as follows.

$$\{e^{dc}\}_n = \{\overline{e}^A * \mathrm{sgn}(\Delta x^c)\}_n + \{\widetilde{e}^A\} \quad (3.a)$$

$$\{e^{dm}\}_n = \{0\} \quad (3.b)$$

$$\{e^{Rm}\}_n = \{0\} \quad (3.c)$$

in which, $\{\Delta x^c\}_n = \{x^c\}_n - \{x^c\}_{n-1}$

Where , $\{\overline{e}^A\}_n$ indicates the average error vector of either undershoot displacements or overshoot displacements. If $\{e^{dc}\}_n$ is the undershoot error, each component of $\{\overline{e}^A\}_n$ becomes a negative constant value.On the other hand, if $\{e^{dc}\}_n$ is the overshoot error, each component of $\{\overline{e}^A\}_n$ becomes a positive constant value. It has been experimentally shown that the components of $\{e^{dc}\}_n$ induced over the total time duration of responses are not constant but deviate almost according to the Gaussian distribution. $\{\widetilde{e}^A\}_n$ indicates the deviation vector of the errors stated in the above. For the frictional error, the error vectors in Eq.(2), $\{e^{dc}\}_n$, $\{e^{dm}\}_n$ and $\{e^{Rm}\}_n$ shall be defined as follows.

$$\{e^{dc}\}_n = \{0\} \quad (4.a)$$

$$\{e^{dm}\}_n = \{0\} \quad (4.b)$$

$$\{e^{Rm}\}_n = \{\overline{e}^f * \mathrm{sgn}(\Delta x^c)\}_n + \{\widetilde{e}^f\}_n \quad (4.c)$$

where, $\{\overline{e}^f\}_n$ indicates the average error vector of frictional forces and each component of $\{\overline{e}^f\}_n$ is assumed to be a positive constant value. $\{\widetilde{e}^f\}_n$ is the deviation vector of frictional error. In the present study,each error of the undershoot, overshoot or frictional errors was independently generated to investigate its error propagation properties. Furthermore, an effect of $\{\widetilde{e}^A\}$ and $\{\widetilde{e}^f\}$ on *responses was not considered.*

4. ANALYTICAL RESULTS AND DISCUSSION

4.1 Structural models for analysis

Three models were prepared as the structural model for the pseudo-dynamic analysis. The model 1 and the model 3 are the reinforced concrete columns with the same dimension and bar arrangement as shown in Fig.2(a) and the model 2 is the reinforced concrete shear wall as shown in Fig.2(b). The reinforced concrete columns and shear wall were modeled as a single degree of freedom system.The parameters for each model such as material properties, mass, damping and input excitations are listed in Table 1. The explicit Newmark method was adopted for a numerical integration. Note that an excessive axial force, which corresponds to the radio of the average axial stress σ_0 to the compressive strength of concrete f'_c, $\eta = \sigma_0 / f'_c = 0.50$, was applied in the model 3. The analysis was carried out twice for each model.The first trial is the analysis in which the error is not introduced. The authors refer to this as "the true response". The second trial is the analysis in which the error is introduced. The authors refer to this as "the error response".

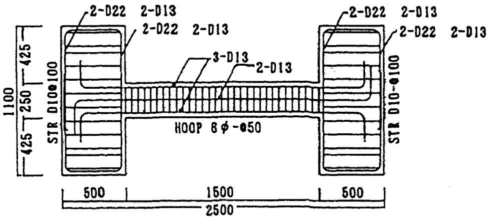


Figure 2-(a). Detail of reinforced concrete column specimen

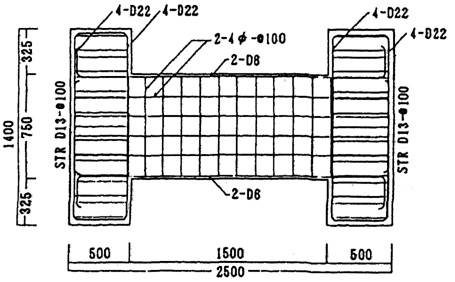
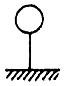


Figure 2-(b). Detail of reinforced concrete wall specimen

·Table 1. Information for numerical experiment.

	model1	model2	model3
Type of structure	column (Fig. 2(a))	wall (Fig. 2(b))	column (Fig. 2(a))
Analysis model	Single degree of freedom system 		
Mass (M)	1973.7 kg		
Damping (C)	3%		
Axial force ratio	0.13	0.18	0.50
Integration method	Explicit Newmark method (β=0)		
Time interval	0.01 sec		
Input exitation	N-S component of 1940 EL-CENTRO earthquake Max acc.= 3.42m/s^2		
Amplification factor of accerelation	1.46	0.8	1.46
Compresive strength of concrete	22.6MPa		
Yield strength of reinforcing bar	475.6MPa		

4.2 Results and discussion

First, the pseudo-dynamic analysises of the model 1 were carried out for either the undershoot error of $\bar{e}^A$=-0.02 mm, the overshoot error $\bar{e}^A$=+0.02 mm or the frictional error of $\bar{e}^f$=1962 N. These errors were assumed with reference to the past test results of pseudo-dynamic testing(Nakanishi, Adachi, Shirai et al. 1988). The time histories of response displacements including the errors are compared with the true response in Figs.3,4 and 5, respectively. The response hysteresis curve including the frictional error is compared with the true hysteresis curve in Fig.6. As far as the time histories of response displacements are concerned, no difference between the error and true responses is observed. On the other hand, some minor difference is observed between the hysteresis curves. This may be attributable to the hysteretic properties of the column member analyzed. That is, this member has the hysteresis curve of spindle shape with a sufficient capacity of energy absorption. Thus, it is considered that the hysteresis damping was dominant in comparison with the energy effect by the error.

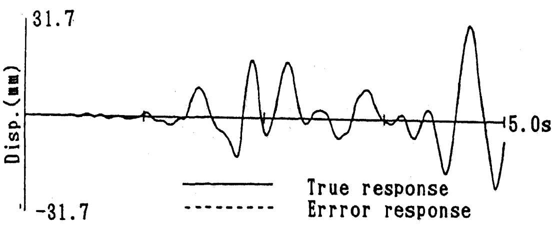


Figure 3. Time histories of response displasements (Undershoot error, $\bar{e}^A$ = -20μ)

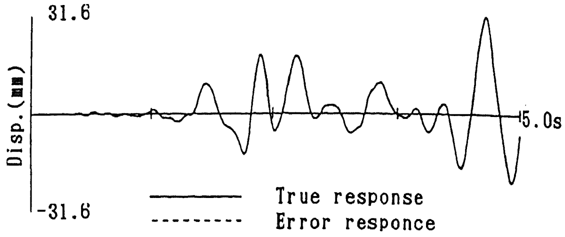


Figure 4. Time histories of response displasements(Overshoot error, $\bar{e}^A$ = $+20\mu$)

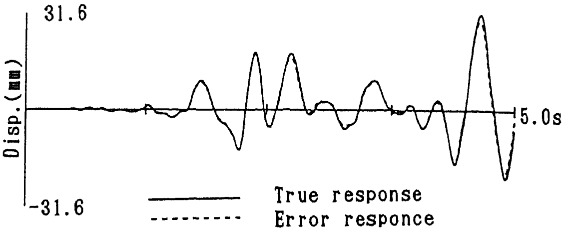


Figure 5. Time histories of response displasements(Frictional error, $\bar{e}^f$ = 1926N)

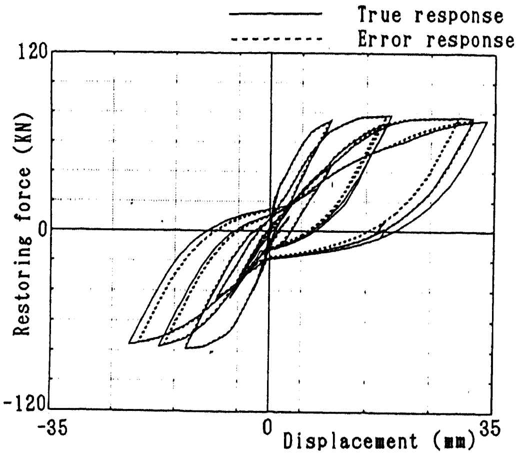


Figure 6. Comparison of hysteresis curves (Friction error, $\bar{e}^f$ = 1926N)

In the next place, the pseudo-dynamic analysis of the model 2 were carried out for either the undershoot error of $\bar{e}^A$=-0.02 mm, the overshoot error of $\bar{e}^A$=+0.02 mm and the frictional error of $\bar{e}^f$=1962 N. The time histories of response displacements including the error are compared with the true response in Figs.7,8 and 9, respectively. The response hysteresis curve including the frictional error is compared with the true hysteresis curve in Fig.10. The response displacements including the undershoot error shown in Fig.7 are growing erroneously in the early stage but this tendency is deteriorating as the time goes by.

This erroneous growth of response displacements may be due to the hysteretic properties of the wall member analyzed. That is, this member has the slender hysteresis curve of S-shape with less energy absorption,especially in the early stage of response. Thus,the added energy by the undershoot error cloud not be sufficiently dissipated. From that time on, however, the energy dissipation slightly activates as the hysteresis area of the member begins to increase. Furthermore, the phase shift over the time duration of responses is observed. This may be due to the reason that the apparent variation of stiffness was caused by the undershoot error. The responses including the overshoot or the frictional error as shown in Figs.8 and 9 are dissipating and this tendency is becoming notable as the time goes by. It is seen that the overshoot and frictional errors have the energy dissipating effect. The energy dissipation by the errors works just as the hysteresis damping. Thus, it is considered that the erroneous dissipation of responses was caused by the damping effect of the errors. If the hysteresis area was sufficiently large, the damping effect of the errors would be cancelled. Furthermore, the phase shift was also observed over the time demotion of responses.

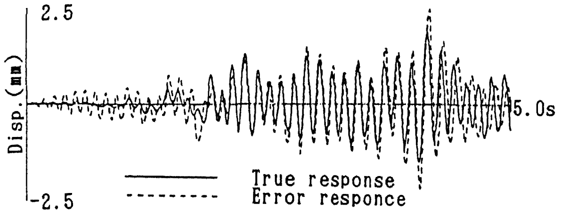


Figure 7. Time histories of response displasement (Undershoot error, $\overline{e}^A = -20\mu$)

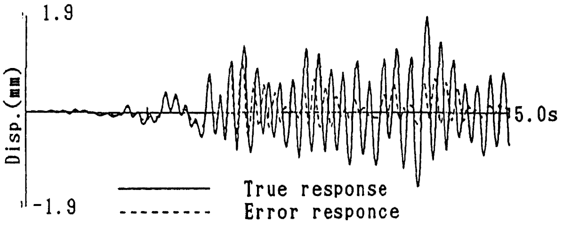


Figure 8. Time histories of response displasement (Over shoot error, $\overline{e}^A = +20\mu$)

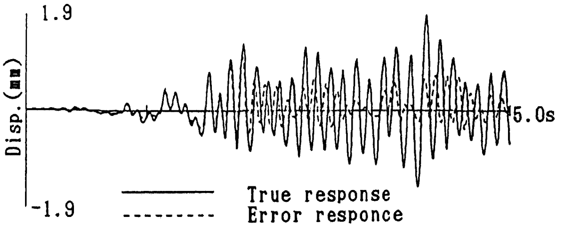


Figure 9. Time histories of response displasement (Friction error, $\overline{e}^f$ = 1926N)

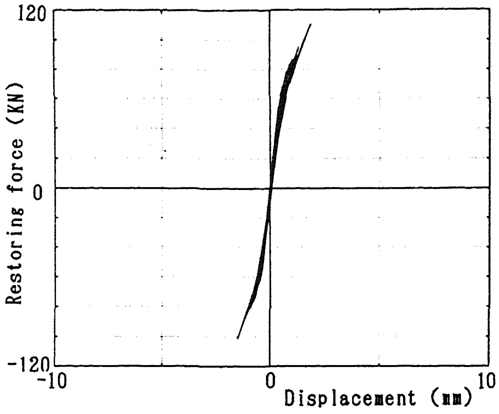


Figure 10-(a). True hysteresis curve

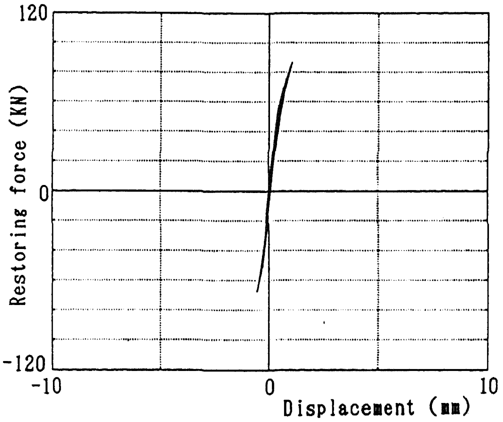


Figure 10-(b). Hysteresis curve including error (Frictional error, $\overline{e}^f$=1926N)

Finally, the pseudo-dynamic analysis of the model 3 was carried out for the frictional error of $\overline{e}^f$=2943 N. The time history of response displacements including the error is compared with the true response in Fig.11. The response hysteresis curve including the error is compared with the true hysteresis in Fig.12. As is seen from Figs.11 and 12,in the case of true responses, the hysteresis curve shows a significant strength reduction after the peak load and the response displacements are largely drifting in the positive direction near the time step of 5.3 sec. In the case of the responses including the error,on the other hand,although a significant strength reduction after the peak load is also observed, a drifting phenomenon of the response displacements is not predicted.It should be noted that the pseudo-dynamic test may not evaluate a sudden failure phenomenon of reinforced concrete members correctly if the experimental errors are induced.

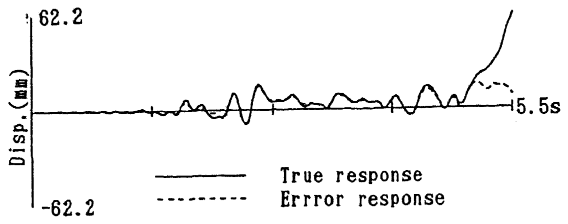


Figure 11. Time histories of response displasement (Frictional error, $\overline{e}^{f}$=2943N)

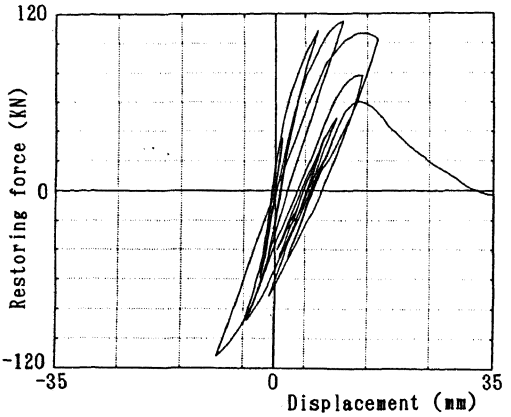


Figure 12-(a). True hysteresis curve

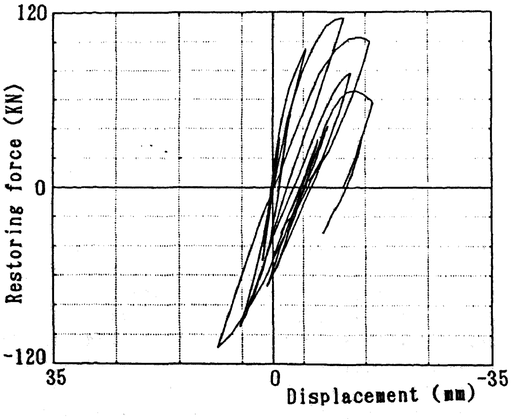


Figure 12-(b). Hysteresis curve including error (Frictional Error, $\overline{e}^{f}$=2943N)

5. CONCLUSIONS

In order to clariby the error propagation properties, the pseudo-dynamic analyses on the reinforced concrete structures with complex hysteretic properties were carried out.The following conclusions were obtained.

1.The effect of the errors of energy adding or dissipating type on the responses becomes notable for the structures with less energy absorption.

2.The pseudo-dynamic test may not evaluate the failure phenomenon of structures correctly if the errors with the energy effect are induced.

3.The analytical method presented in this paper is very effective to evaluate the error propagation properties for the structures with complex hysteretic properties.

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