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Extended Kalman filter – Weighted local iteration method for dynamic structural identification

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Abstract: A dynamic system identification has become increasingly important in area of structural engineering, and the extended kalman filter is to be effectively used in structural system identification method of static as well as dynamic phenomena. These systems are generally much larger in size and much more complex and manifest intricate behavior under seismic excitation so that accurate mathematical idealization is not easy. Therefore, for the purpose of effective structural engineering applications specialized techniques of system parameter identification need to be developed.

This paper studies a system parameter identification procedure based on the extended Kalman filter, which may accommdate the finite element method relevant to the identification on such complex systems. This procedure is named extended Kalman filter-weighted local iteration procedure. And numerical studies are carried out for demonstration.

1 Introduction

Recently, the extended Kalman filter has been applied on structural identification problems by many authors, which is essentially a method of sequential least squares estimation, and the vast applicability of the filter was clarified not only for dynamic parameter identification but also for many other, static or dynamic system identification problems. In the past, one of the authors studied the application of the extended Kalman filter in the identification of linear and nonlinear structural systems subject to sisemic excitations, and it was found that dynamic properites of systems were identified in a recursive way by filtering procedure.

In this study, a weighted local iteration procedure with the extended Kalman filter was newly developed and applied on system parameter identification using a linear single degree of freedom system as a pilot model. This procedure is named the EK-WLI procedure. In this procedure, we locally iterate the filtering process by purposely amplifing error covariance on each set of observation data so as to attain faster and stable convergency.

On the other hands, dynamic simulation procedure often uses finite element method, because these systems are generally much larger in size and much more complex. Besides, general structural systems either super structures or soil foundations are often represented by a finite element method modeling.

In order to solve problems under this situation, a procedure is developed to identify system parameters of a finite element model by effectively applying the EK-WLI procedure. In this procedure, emphasized is an independent treatment of the finite element method algorithm apart from the main flow of the Kalman filtering so that any finite element method code may be incorporated with the Kalman filtering without rearrangement.

In order to investigate the procedure, a linear single degree of freedom model is identified on the basis of simulated data under various noise conditions. Finally, numerical examples are demonstrated to show the usefulness of the proposed method.

2 EK-WLI Procedure

In this procedure, we locally iterate the extended Kalman filter algorithm by purposely amplifing error covariance matrix on each iteration , in order to obtain stable solutions as well as their fast convergency to optimal solutions.

2.1 Extended Klaman filter

The extended Kalman filter algorithm is a recursive procedure to estimate the optimal state vector $X(t_k/t_k)$ and the corresponding error covariance matrix $P(t_k/t_k)$ on the basis of nonlinear continuous state vector equation and nonlinear discrete observation vector equation as follows.

$$dXt/dt=f(Xt,t)+GtWt \quad (1)$$

$$Yt_k=h(Xt_k,t_k)+Vt_k \quad (2)$$

in which $X(t_k/t_k)$=state vector, Yt_k=observation vector at $t_k=k\Delta t$, Wt and Vt_k=system and observation noises respectively. They are vectors of zero mean white noise Gaussian processes with
$E[wt_k\ wt_s^T]=Qt_k\delta t_{ks}$, $E[vt_k\ vt_s^T]=Qt_k\delta t_{ks}$
and δt_{ks}=Kronecker delta.
If the initial state vector $X(t_0/t_0)=Xt_0$ and the error covariance $P(t_0/t_0)=Pt_0$ are given and then as the observation Yt_k are processed, it is possible to estimate the state vector $\hat{X}(t_k/t_k)$ and the error covariance $P(t_k/t_k)$ from the following extended Kalman filter algorithm.

$$\hat{X}(t_{k+1}/t_k)=\hat{X}(t_k/t_k)+\int_{t_k}^{t_{k+1}} f[\hat{X}(t/t_k),t]dt \quad (3)$$

$$P(t_{k+1}/t_k)=\phi[t_{k+1},t_k;\hat{X}(t_k/t_k)]P(t_k/t_k) *\phi^T[t_{k+1},t_k;\hat{X}(t_k/t_k)]+\Gamma t_k Qt_k \Gamma^T t_k \quad (4)$$

$$\hat{X}(t_{k+1}/t_{k+1})=\hat{X}(t_{k+1}/t_k)+K\{t_{k+1};\hat{X}(t_{k+1}/t_k)\} *[Yt_k-h\{\hat{X}(t_{k+1}/t_k),t_{k+1}\}] \quad (5)$$

$$P(t_{k+1}/t_{k+1})=[I-K\{t_{k+1};\hat{X}(t_{k+1}/t_k)\} *M\{t_{k+1};\hat{X}(t_{k+1}/t_k)\}]P(t_{k+1}/t_k) *[I-K\{t_{k+1};\hat{X}(t_{k+1}/t_k)\}M\{t_{k+1};\hat{X}(t_{k+1}/t_k)\}]^T +K\{t_{k+1};\hat{X}(t_{k+1}/t_k)\}Rt_{k+1}K^T\{t_{k+1};\hat{X}(t_{k+1}/t_k)\} \quad (6)$$

$$K\{t_{k+1};\hat{X}(t_{k+1}/t_k)\}=P(t_{k+1}/t_k) *M^T\{t_{k+1};\hat{X}(t_{k+1}/t_k)\}[M\{t_{k+1};\hat{X}(t_{k+1}/t_k)\} *P(t_{k+1}/t_k)M^T\{t_{k+1};\hat{X}(t_{k+1}/t_k)\}+Rt_{k+1}]^{-1} \quad (7)$$

in which $\hat{X}(t_k/t_k)$=state estimate at t_k given Yt_k, $P(t_k/t_k)$=covariance matrix of error in $\hat{X}(t_k/t_k)$, $\phi\ [t_{k+1},t_k;\hat{X}(t_k/t_k)]$=state transfer matrix from t_k to t_{k+1}, $K\{t_{k+1};\hat{X}(t_{k+1}/t_k)\}$ =Kalman gain matrix at t_{k+1}, $Yt_k=\{yt_0,\cdots yt_k\}$, yt_k=observation at t_k, I=a unit matrix and

$$M\{t_k;\hat{X}(t_k/t_{k-1})\}=\left[\frac{\partial h_i(Xt_k,t_k)}{\partial x_j}\right]_{Xt_k=\hat{X}(t_k/t_{k-1})} \quad (8)$$

in which h_i=the ith component of $h_i(Xt_k,t_k)$, and x_j = the jth component vector of Xt_k. The ϕ matrix in this algorithm is given from Taylor's expansion of first order as follows.

$$\phi[t_{k+1},t_k;\hat{X}(t_k/t_k)]=I+\Delta F\{t_k;\hat{X}(t_k/t_k)\} \quad (9)$$

$$F\{t_k;\hat{X}(t_k/t_k)\}=\left[\frac{\partial f_i(Xt_k,t_k)}{\partial x_j}\right]_{Xt_k=\hat{X}(t_k/t_k)} \quad (10)$$

in which Δ=sampling interval of observation waves.

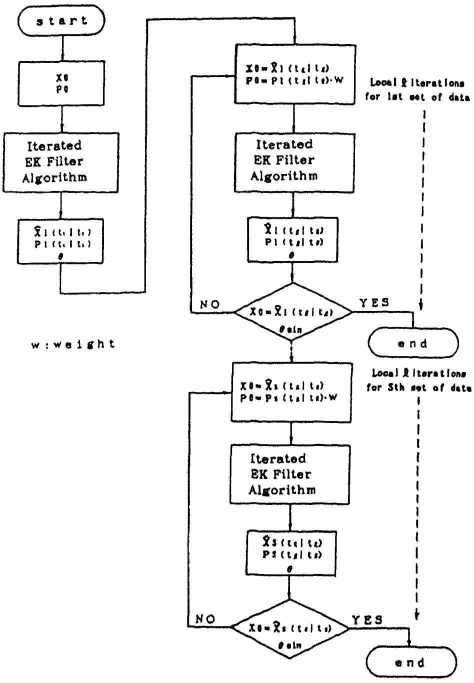

Fig.1. Algorithm of EK-WLI procedure

2.2 EK-WLI Procedure

Since a finite duration time of observation waves and arbitrarily assigned values for initial conditions are used and linearization of the state vector equation by Talor's expansion is employed in the analysis, it is doubtful that the estimated parameters are stable and convergent to exact ones. Therefore, it is necessary to assess the correct estimated parameters in terms of stability and convergency. A weigthted local iteration procedure, including an objective function for the stability, has been developed. This procedure is shown in Fig.1.
In order to release the linearization effect of the observation vector equation and/or the state vector equation on the estimation error, and to promote stable convergency to the optimal solution despite of small amount of observation data, a weighted local iteration procedure is proposed.
This procedure is to locally iterate the filtering process on the first set of data in order to assess the information inherent to the data. Here, at every iteration, the error covariance matrix of estimation is enlarged with a modification weight to faster the convergency.
Then, the second set of data is processed to the filtering similarly and the third one and so on. This procedure is named the extended

Kalman filter-weighted local iteration procedure.
Also in the weighted local iteration procedure of each set of data , applied is the iterated extended Klaman filter. In the iterated extended Kalman filter with ℓ_1 time iterations the state vector $\hat{X}(t_{k+1}/t_k)$ is replaced by the iterator η_1. The iteration starts with $\eta_1=\hat{X}(t_{k+1}/t_k)$, and terminates after ℓ_1 time iterations. After ℓ_1th iteration, the covariance matrix is then computed based on $\eta_{\ell_1}=\hat{X}(t_{k+1}/t_{k+1})$, and the last iterate η_{ℓ_1}, is taken for the estimate $\hat{X}(t_{k+1}/t_{k+1})$.
In this proceadure employed is double loop iteation on each set of data. This procedure of the extended Kalman filter may be effective under a finite sets of observation, and an efficient method to identify $\hat{X}(t_{k+1}/t_{k+1})$.
And then, the covariance matrix is multiplied by a weight, W. In order to attain faster and stable convergency of the state vector, again the extended Kalman filter algorithm is locally iterated. Covariance matrix which corresponds to the convergency of state vector, is actually distorted by a weight, W. So, for the confirmation of state vector convergency, an objective function is used.
However, it must be carefully examined to see whether or not the estimated parameters are stable and convergent to true ones, since only finite duration time of observation data and initial conditions are to be used in the analysis. The objective function is represented by

$$q_i t_k = Y_i t_k - h_i\{\hat{X}(t_k/t_k)\} \tag{11}$$

$$\theta = 1/M\{\sum_{i=1}^{M} q_i^2 t_k / \sum_{i=1}^{M} Y_i^2 t_k\} \tag{12}$$

in which M=number of sampling points of observation waves. θ =objective function. Therefore, θ_{min} indicates that the difference between each observation and corresponding estimate becomes minimum.

3 Numerical Example 1

Consider a linear single degree of freedom system of shear type shown in Fig.2.
This example has been already discussed and solved by many authors. But, in order to check our procedure, the algorithm in Fig.1 is used to solve this simplest problem.
The governing equation of motion is given by

$$\ddot{u}+2\beta\omega_0\dot{u}+\omega^2_0 u=-\ddot{u}_0 \tag{13}$$

in which u,$\dot{u}$ and $\ddot{u}$ are response displacement, velocity and acceleration.
$\ddot{u}_0$=input acceleration, $\beta=c/2(mk)^{1/2}$, $\omega^2_0=k/m$, β=coefficient of critical viscous damping, ω_0=

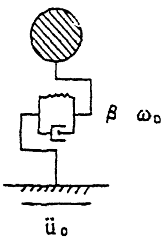

Fig.2. SDOF model

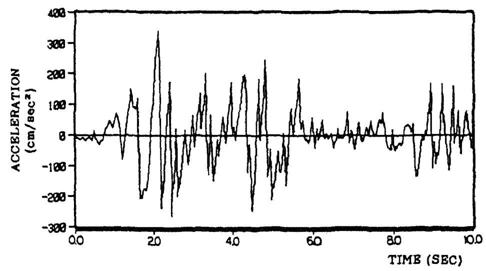

Fig.3. Input acceleration

natural circular frequency. Equation(13) may be reduced to the corresponding state equation by introducing the state variables, x_1=u and $x_2=\dot{u}$, and input $u=\ddot{u}_0$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}x_2\\ -2\beta\omega_0 x_2-\omega^2_0 x_1-u\end{bmatrix} \tag{14}$$

Then letting $x_3=\beta$ and $x_4=\omega_0$, and assuming that β and ω_0 are constant with respect to time, the nonlinear continuous state equation can be represented as follows.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}x_2\\ -2x_3x_4x_2-x_4^2x_1-u\\ 0.0\\ 0.0\end{bmatrix} \tag{15}$$

If the observation is structural response acceleration, then the nonlinear discrete observation equation (2) and equation of motion (13) are given by

$$yt_k=h(Xt_k,t_k)=-2x_3x_4x_2-x_4^2x_1+vt_k \tag{16-a}$$

and transfer matrix is given by

$$M\{t_k;\hat{X}(t_k/t_{k-1})\}$$
$$=[-x^2_4,\ -2x_3x_4,\ -2x_2x_4,\ -2(x_2x_3+x_1x_4)] \tag{16-b}$$

Simulated time history of the erthquake acceleration, $\ddot{u}_0(t)$ and the observation time history of a SDOF model are shown in Fig.3 and 4, where the system parameters β=0.1 and ω_0=7.07 (rad/sec) are employed.
The level of observational noise included in time history of the SDOF model is assumed to be

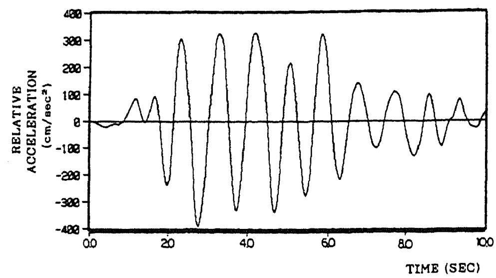

Fig.4. Response acceleration

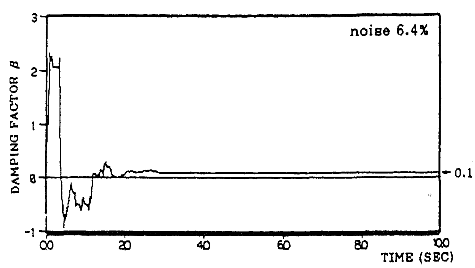

Fig.5. Identified Parameter β

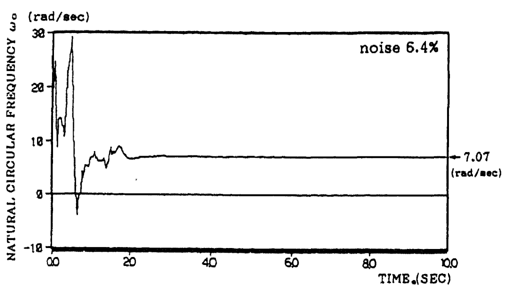

Fig.6. Identified Parameter ω_0

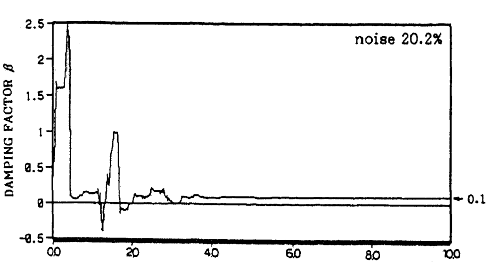

Fig.7. Identified Parameter β

6.4% and/or 20.2% of the structural response in the ratio of standard deviation.
The initial conditions for this analysis are given in Table 1.
The results are shown in Figs.5,6,7 and 8. The results give good estimations of parameters.

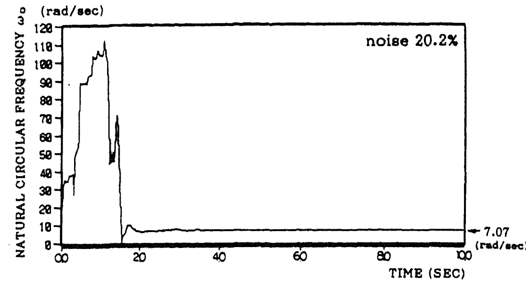

Fig.8. Identified Parameter ω_0

Table 1. Initial Conditions (1)

Paremeter	x_1	x_2	x_3	x_4
$X(t_0/t_0)$	0.0	0.0	0.5	20.0
$P(t_0/t_0)$	1.0	1.0	100	1000

Note : $R=1.0*10^{-2}$, $Q=0.0$, $W=1.2$
Iteration number=3

4 Finite Element Fomulation

In this section shown is the general formulation of the extended Kalman filter, which may accommdate the finite element method.
Then, taking into account the stationarity of system parameters, the state vector equation(1) is rewritten by

$$\hat{X}(t_{k+1}/t_k)=\hat{X}(t_k/t_k)+wt_k \tag{17}$$

On the other hand, a finite element equation of motion is given by

$$M\ddot{u}+C\dot{u}+Ku=f(t) \tag{18}$$

where u , $\dot{u}$ and $\ddot{u}$ are respectively displacement, velocity and acceleration response vectors. $f(t)$ is an input vector. M,C and K are mass, damping and stiffness matrices respectively.
The current identification problem is to estimate the unknown system parameters based on observation data. To make the problem unique the equation of motion is rewritten, as the response of system can be measured in time of displacement, velocity and acceleration.
If the acceleration at finite numbers of nodal points are observed, the equation (18) is rewritten by

$$\ddot{u}=-M^{-1}\{C\dot{u}+Ku-f(t)\} \tag{19}$$

The observation vector of these accelerations may be related with the acceleration vector in

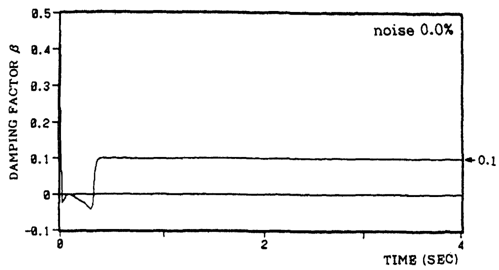

Fig.9. Identified Parameter β

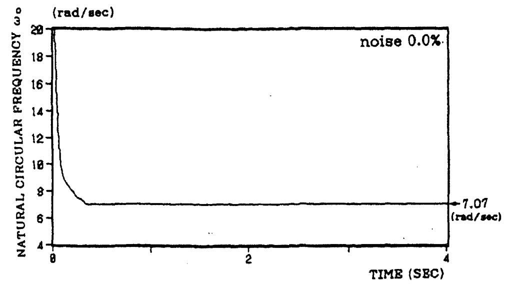

Fig.10. Identified Parameter ω_0

Table 2. Initial Conditions (2)

Paremeter	x_1	x_2
$X(t_0/t_0)$	0.5	20.0
$P(t_0/t_0)$	100	1000

Note:$R=1.0*10^{-3}$,$Q=0.0$,$W=1.2$
Iteration number=2
Objecive function
noise=0.0%($\theta=1.0*10^{-4}$)
noise=0.2%($\theta=1.0*10^{-3}$)

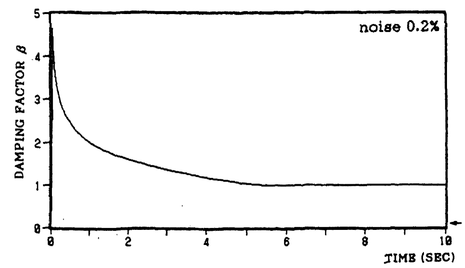

Fig.11. Identified Parameter β

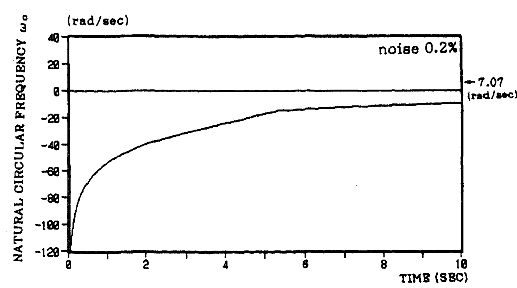

Fig.12. Identified Parameter ω_0

the following manner. The observation equation(2) is rewritten by

$$Yt_k = ut_k + Vt_k \qquad (20)$$

where Yt_k is an observation vector of the t_k th samples, and Vt_k is an obervation vector. with eq.(18), equation (20) is rewritten by

$$Yt_k = -M^{-1}\{C\dot{u}+Ku-f(t)\}+Vt_k \qquad (21)$$

It is indicated that the mass, damping and stiffness matrices consist of system parameters and in general M,C andK are nonlinear matrices of $\hat{X}(t_k/t_k)$. Thus, $\ddot{u}=-M^{-1}\{C\dot{u}+Ku-f(t)\}$ means a nonlinear function of $\hat{X}(t_k/t_k)$.
In general, equation (21) is expressed by

$$y_k = h(X_k)+v_k \qquad (22)$$

The state vector equation (17) and the observation vector equation (22) are the basic equations in the identification of the state vector $\hat{X}(t_k/t_k)$ by the extended Kalman filter.
In this procedure, the transfer matrix $M\{t_k;\hat{X}(t_k/t_{k-1})\}$ which stems from the linearization of the nonlinear observation vector equation (2) will be evaluated by the influence coefficient method. This evaluation may be carried out numerically by a finite element method computer code outside of the main flow .

The influence coefficient equation is represented by

$$\frac{\partial h_i(X_k)}{\partial x_{j,k}} = \frac{h_i(X_k+\Delta x_{j,k}e_j)-h_i(X_k)}{x_{j,k}} \qquad (23)$$

where $h_i(X_k)$ is the ith element of $h_i\{\hat{X}(t_k/t_k)\}$, $\Delta x_{j,k}$ is a finite increment of $x_j.t_k$. $x_j.t_k$ is the jth element of $\hat{X}(t_k/t_k)$. e_j is a vector whose jth element is unity and other elements are zero.
In other words, the transfer matrix is approximated by the right hand side of the equation (23) in which $h(X_k+\Delta x_{j,k}ej)$ and $h(X_k)$ are obtained numerically by the finite element method.
It should be advantageous that in the above procedure the finite element method code is not integrated into the Kalman filtering algorithm and the outputs of the finite element method code are called to the evaluation of eq (23). In

this way, without arrangement, we can use any available finite element method code.

5 Numerical Example 2

Consider a linear single degree of freedom system of shear type Fig.2, and the governing equation of motion is equation (13). For the system, it is assumed that β and ω_0 are constant with respect to time. Then, the state vector of equation(17) can be represented as follows.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \beta \\ \omega_0 \end{bmatrix} \qquad (24)$$

If the observation is structural response acceleration, then the nonlinear discrete observation equation (22) is given by

$$y_1 = -2x_1 x_2 \dot{u} - x^2_2 u - \ddot{u}_0 + v_1 \qquad (25)$$

Simulated time history of the earthquake acceleation, ü(t), and the observation time historiy of a SDOF model are shown in Fig.3 and 4 respectively.

The initial conditions for this analysis are given in Table 2.

The level of observational noise included in time history of a SDOF model is assumed to be 0.0% and/or 0.2% of the structural response in the ratio of standard deviation. The results are shown in Figs.9,10,11 and 12. This procedure gives good estimations of parameters. But in the case of using simulated datum, which is contaminated by measurment noise of 0.2%, this procedure does not give good estimations of system parameters.

6 Concluding Remarks

A Kalman filter - weighted local iteration procedure and the general formulation of the extended Kalman filter, which may accommdate the finite element method in parameter identification problems are presented and through numerical examples using a linear single degree of freedom pilot model, the following conclusions are drown;

(1) A weighted local iteration procedure of Kalman filter with an objective function is found to be effective for stable estimation of state vector.

(2) Numerical analyses are carried out to show the usefulness of this method in system parameters identification of the finite element method model in dynamic problems.

In the future, this procedure will be extended to accommdate the finite element method in which data are heavily contaminated by measurment noise.

Finally, this study is performed by the second author, incorporated with theoretical suggestion by the first author.

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