Development of SLP for identification of structural systems

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ABSTRACT: The purpose of this study is to present a new nonlinear optimization scheme, which is modification of Successive Linear Programming (SLP), for identification of a linear chain oscillator. The identification problem is formulated in the frequency domain, in order to estimate the stiffness and damping coefficients of the system with known mass distribution. In numerical analysis, the results of identification of 5-DOF system by SLP and modified SLP are compared. It is shown that the convergency of parameters is dramatically improved by applying the modified SLP.

1 INTRODUCTION

The development of dynamic models is necessary for predicting the response of structural systems during earthquake motions. In seismic design of structures, dynamic models can be theoretically synthesized from the properties of structural components and their interactions. Such dynamic models should be improved, because there still remain many uncertainties in the synthesis. Fortunately, a large number of records pertinent to structural response during earthquake motions have been accumulated, which offers an opportunity to study dynamic characteristics of structures. Using these records, dynamic properties of structures can be identified and then structural dynamic models are improved.

Identification problem has been studied by many researchers, and various approaches are proposed. Many of the identification techniques that have been employed in earthquake engineering can be classified as output-error approach. In this approach, the optimal estimates of parameter values of a dynamic model are obtained by achieving the least-squares match between the responses of structure and model subjected to nominally the same excitation. Then, the identification problem can be formulated as an optimization problem of structural parameters. In order to solve such optimization problem, we can use many schemes. Nonlinear optimization schemes can be classified into direct search method and descent method. The latter is the method that uses the partial derivatives of objective function and is often employed in identification of structural systems. McVerry (1980) applied Gauss-Newton method to identify modal parameters of a linear shear structure excited by earthquake ground motion in frequency domain analysis. Matsui and Kuriha (1989) also applied Gauss-Newton method to identify a structural system in time domain analysis. Tsujihara, Sawada and Hiro (1987) applied SLP (Successive Linear Programming) to identify stiffness and damping coefficients of a linear chain oscillator. Tsujihara, Sawada and Sugito (1990) applied SLP and DFP (Davidon-Fletcher-Powell) method to identify S-wave velocity and quality factor in the subsurface layered soil using vertical array records of earthquake ground motion.

Among these optimization schemes, SLP has the advantage whose algorithm is relatively simple. Particularly, in such optimization problem with no constraint equations as identification of dynamic structural systems, approximation in each stage can be carried out without using the scheme of LP (Linear Programming). Then, we need only the information about the sign of the first-order partial derivative with respect to each parameter. Moreover, SLP is very effective to identify simultaneously such parameters as stiffness and damping of a structural system whose size and sensitivity are considerably different from each other, because the linear-search subproblem does not exist in the algorithm. However, SLP has the serious defect that the convergency is not good in the neighborhood of an optimum point.

In this paper, we introduce a new optimization scheme based on SLP. The second-order partial derivatives are used to overcome the defect in the convergency near a optimum point. First, we show the algorithm for identification of a linear chain oscillator. Secondly, we show through numerical analysis that the convergency is dramatically improved by applying this scheme.
2 FORMULATION OF IDENTIFICATION PROBLEM

Fig.1 shows a linear chain oscillator with n-degree-of-freedom, in which mass is represented by \( m_j \), stiffness by \( k_j \), damping by \( c_j \), \( j=1,2,\ldots,n \). Denoting the absolute motion of masses by vector \( x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T \), the equation of motion can be expressed by

\[
M\ddot{x}(t)+C\dot{x}(t)+Kx(t)=f(t)
\]

(1)

where

\[
M = \begin{bmatrix}
m_1 \\
m_2 \\
\vdots \\
m_n 
\end{bmatrix}
\]

(2)

\[
C = \begin{bmatrix}
c_{12}+c_{23} & -c_{23} & \cdots & -c_{n1} \\
-c_{23} & c_{22}+c_{33} & \cdots & -c_{n2} \\
\vdots & \vdots & \ddots & \vdots \\
-c_{n1} & -c_{n2} & \cdots & c_{nn}
\end{bmatrix}
\]

(3)

\[
K = \begin{bmatrix}
k_{12}+k_{23} & -k_{23} & \cdots & -k_{2n} \\
-k_{23} & k_{22}+k_{33} & \cdots & -k_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
-k_{2n} & -k_{2n} & \cdots & k_{nn}
\end{bmatrix}
\]

(4)

and

\[
f(t)=\begin{bmatrix}c_1\dot{x}_1(t)+k_1x_1(t) \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

(5)

\(x_0(t)\) is the displacement of the base. Taking Fourier transform of both sides of Eq. (1), we have

\[
(-\omega^2 M+i\omega C+K)X(\omega)=\begin{bmatrix}i\omega c_{12}+k_1 \\
0 \\
\vdots \\
0\end{bmatrix}X_0(\omega)
\]

(6)

where \( \omega \) is circular frequency; \( X(\omega) \) and \( X_0(\omega) \) are the Fourier transforms of \( x(t) \) and \( x_0(t) \), respectively. Eq. (6) can be solved as follows.

\[
X(\omega)=A(\omega)^{-1}F(\omega)X_0(\omega)
\]

(7)

where

\[
A(\omega)=\begin{vmatrix}
-i\omega c_{12}+k_1 & 0 & \cdots & 0 \\
0 & \ddots & \vdots & \vdots \\
0 & \cdots & -i\omega c_{n-1,n}+k_n & 0 \\
0 & \cdots & 0 & -i\omega c_{nn}+k_n
\end{vmatrix}
\]

(8)

\[
F(\omega)=[i\omega c_{12},0,\ldots,0]^T
\]

(9)

Consider that the records of motion are available at the base and an arbitrary mass \( r \). Denote a measured record by \( \tilde{x}_r(t) \), and Fourier transforms of those records \( \tilde{x}_r(t) \) and \( \tilde{x}_0(t) \) by \( \tilde{X}_r(\omega) \) and \( \tilde{X}_0(\omega) \), respectively. The transfer function between the base and mass \( r \) is expressed by

\[
\tilde{H}(\omega)={\tilde{X}_r(\omega)}/\tilde{X}_0(\omega)
\]

(10)

While, the transfer function of the model, \( H(\omega;\alpha) \), is calculated from Eq. (7) as a function of system parameters \( \alpha \) \( (m_1, k_1, \ldots, c_1, \ldots) \). Therefore, the identification problem becomes to determine the optimal estimates of the parameters, so as to minimize the least-squares match between \( \tilde{H}(\omega) \) and \( H(\omega;\alpha) \).

We adopt the following objective function:

\[
S(\alpha)=\sum_{i=1}^{N_i} \left( \left| \tilde{H}(\omega_i;\alpha) \right| - \left| H(\omega_i;\alpha) \right| \right)^2 \rightarrow \min
\]

(11)

where \( \omega_i \) denotes \( i \)-th sampled circular frequency and \( N_i \) the total number of samplings.

Eq. (11) can be solved in an iterative manner with an optimization scheme.

Only the amplitude of the transfer function is used in Eq. (11). The analysis in the frequency domain, in general, corresponds to that in the time domain by considering the amplitude and phase. For a linear chain model, however, the information of the system dynamics can be extracted from only the amplitude of the transfer function because the unit impulse response function of the system has causality and its Laplace transform is minimum phase function (Papoulis, 1962). Therefore, the system under consideration is identifiable through the objective function as shown in Eq. (11).

3 ALGORITHM OF MSLP (MODIFIED SLP)

The optimum values of \( \alpha \) are to be found by means of iterative procedure. At the start of \( r \)-th iteration, we possess current values \( \alpha^{(r)} \), and we seek new values \( \alpha^{(r+1)} \) using the formula

\[
\]
\[ a^{(r+1)} = a^{(r)} + \Delta a^{(r)} \]  

where \( \Delta a^{(r)} \) are the step size in the \( r \)-th iteration.

Taking Taylor series expansion of Eq. (11) in the \( r \)-th iteration of optimization procedure, we have

\[
S^{(r+1)} = S^{(r)} + \sum_{i=0}^{\infty} \frac{\partial S^{(r)}}{\partial a_i} \Delta a_i^{(r)} + \sum_{i=0}^{\infty} \frac{\partial^2 S^{(r)}}{\partial a_i \partial a_j} \Delta a_i^{(r)} \Delta a_j^{(r)} + \ldots \]

where \( S^{(r+1)} \) and \( S^{(r)} \) are \( S(a^{(r+1)}) \) and \( S(a^{(r)}) \), respectively, and \( N \) is the total number of unknown parameters. Partial derivatives of the first and second order of objective function are expressed by

\[
\frac{\partial S}{\partial a_i} = 2 \sum_{i=1}^{N} [H(\omega_i; a) - \overline{H}(\omega_i)] \frac{\partial H(\omega_i; a)}{\partial a_i}
\]

\[
\frac{\partial^2 S}{\partial a_i \partial a_j} = 2 \sum_{i=1}^{N} \frac{\partial H(\omega_i; a)}{\partial a_i} \frac{\partial H(\omega_i; a)}{\partial a_j} + 2 \sum_{i=1}^{N} [H(\omega_i; a) - \overline{H}(\omega_i)] \frac{\partial^2 H(\omega_i; a)}{\partial a_i \partial a_j}
\]

Traditional SLP requires only the second term in Eq. (13) to find \( \Delta a^{(r)} \) under the following constraints of move limit.

\[-F a^{(r)} \leq \Delta a^{(r)} \leq F a^{(r)} \quad (j=1\sim N) \]

where \( F \) is coefficient that determine the move limit of \( \Delta a^{(r)} \), and \( 0.02 \sim 0.2 \) is often set as \( F \). \( \xi \) is to be set smaller in the neighborhood of an optimum point.

In the scheme which we call WSLP(modified SLP), by taking it into consideration that Eq. (13) is a equation of the second degree with respect to \( a^{(r)} \), we try to minimize Eq. (13) by SLP. The problem is that the second-order partial derivatives of transfer function are contained as shown in Eq. (15). In the system under consideration, however, they are easily obtained because the second-order partial derivatives of \( A(\omega) \) and \( F(\omega) \) in Eq. (8) and (9) are null matrix and null vector, respectively (see APPENDIX).

The algorithm of WSLP is as follows. We rewrite Eq. (13) for the sake of simplicity of explanation.

\[
T = \sum_{i=1}^{N} D_i \beta_i + \sum_{i=1}^{N} \sum_{k=1}^{N} E_{ik} \beta_i \beta_k
\]

where \( D_i = \left( \frac{\partial S}{\partial a_i} \right)^{(r)} \), \( E_{ik} = \left( \frac{\partial^2 S}{\partial a_i \partial a_k} \right)^{(r)} \), \( \beta_i = \Delta a_i \), and \( \beta_k = \Delta a_k \). We omit the first term \( S^{(r)} \) in Eq. (13) because minimization is carried out regardless of the value of \( S^{(r)} \). \( D_i \) and \( E_{ik} \) in Eq. (17) need to be calculated once in the \( r \)-th global iteration.

Linearizing Eq. (17) with respect to \( \beta \) by Taylor series expansion,

\[
T^{(r)} = T^{(r)} + \sum_{j=1}^{N} \left( \frac{\partial T}{\partial \beta_j} \right)^{(r)} \Delta \beta_j^{(r)}
\]

where

\[
\frac{\partial T}{\partial \beta_j} = D_j + \sum_{k=1}^{N} E_{jk} \beta_k^{(r)}
\]

Then, \( \Delta \beta_j^{(r)} \), \( j=1\sim N \), can be determined by using the scheme of LP under the following constraints of move limit.

\[-c \beta_j^{(r)} \leq \Delta \beta_j^{(r)} \leq c \beta_j^{(r)} \quad (j=1\sim N)
\]

where \( \beta_j = F a_j^{(r)} \) and \( c \) is coefficient to determine the move limit of \( \Delta a_j^{(r)} \). We may set 0 to the initial approximation \( \beta_j^{(r)} \), \( j=1\sim N \).

By the way, by taking it into account that the constraint equation is nothing but Eq. (20), we can determine \( \Delta \beta_j^{(r)} \) only from the information about the sign of \( \frac{\partial T}{\partial \beta_j} \), as follows.

\[
(\frac{\partial T}{\partial \beta_j})^{(r)} \geq 0 \quad \text{then} \quad \Delta \beta_j^{(r)} = -c \beta_j^{(r)}
\]

\[
(\frac{\partial T}{\partial \beta_j})^{(r)} < 0 \quad \text{then} \quad \Delta \beta_j^{(r)} = c \beta_j^{(r)}
\]

Thus, local iteration to minimize Eq. (17) can be carried out without using the scheme of LP. After such local iterations, the optimal estimate \( \beta_i^{*} \) of \( \Delta a_i^{(r)} \) is obtained. Then, the new approximation \( a_j^{(r+1)} \) in the global iteration is determined by

\[
a_j^{(r+1)} = a_j^{(r)} + \beta_j^{*} \quad (j=1\sim N)
\]

We adopt the total number of iteration such that 20 as the terminating criterion of local iteration, and Eq. (24) as the terminating criterion of global iteration.

\[
S \leq \varepsilon
\]

where \( S \) is the value of objective function and \( \varepsilon \) is positive small value such that \( 10^{-5} \).

4 NUMERICAL RESULTS

Since the purpose of this study is to examine the efficiency of WSLP compared with traditional SLP, simulated data for the analytical model is used instead of actually recorded data. The model to be identified is a linear chain oscillator, as shown in Fig.1, with 5 degree of freedom. The system parameters are shown in Table 1. In the analysis, transfer function between the base and lowest mass (mass 1) of the system is calculated at 100 frequency points of equal intervals between 0.1Hz - 10.0Hz which is used as \( H(\omega) \) in Eq. (11). Therefore, the transfer function used in this
Figure 2  Convergence process of stiffness and damping coefficients by SLP

Figure 3  Value of objective function by SLP

Figure 4  Convergence process of stiffness and damping coefficients by SLP & MSLP

Figure 5  Value of objective function by SLP & MSLP
paper is of noise free. In the following, the stiffness and damping coefficients are identified, assuming known mass. All of the initial approximation of stiffness and damping coefficients are 1.5 multiplied by exact values. We adopt $10^{-4}$ as $e$ in Eq. (24).

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<th>$c_i$ [t/s/cm]</th>
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<tr>
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<td>3.750</td>
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</tr>
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Table 1: Exact values of parameters

4.1 Application of SLP

The results of identification by SLP is shown in Fig. 2, Fig. 3 and Table 2. Move limit is gradually smaller when the solution fluctuates in the process of iteration. Fig. 2 is the convergence process of stiffness and damping coefficients. In the figure, horizontal axis is the number of iteration and vertical one is value of each parameter, and broken line corresponds to exact value of each parameter. Stiffness coefficients almost converge to the exact values around 40 iterations. Damping coefficients, however, have not converged by 100 iterations yet. In Table 2, initial values, estimated values and ratios of the former to latter are shown. Damping coefficients contain maximum error of 27% in estimated values.

Table 2: Results of identification by SLP

4.2 Application of MSLP

The results of identification by MSLP is shown in Fig. 4, Fig. 5 and Table 3. In order, to save computational time, SLP is used in the early stage of the process, namely, first 15 iterations are carried out by SLP. As to the terminating criterion of local iteration in MSLP, we adopt total number of iteration of 20. Stiffness coefficients almost converge to the exact values about 20 iterations, and another 12 iterations are enough to satisfy the terminating criterion of Eq. (24). Stiffness and Damping coefficients are estimated within the error of 0.1% and 2%, respectively.

Table 3: Results of identification by SLP & MSLP

Fig. 6 shows the transfer function calculated by exact, initial and estimated parameters. Estimated transfer function is in good agreement with exact one.

Thus, it is demonstrated that the convergence of unknown parameters are dramatically improved by using MSLP compared with using only SLP.

5 CONCLUSIONS

A simple and efficient optimization scheme based on SLP (Successive Linear Programming) has been developed for identification of stiffness and damping coefficients of a linear chain oscillator. Near a optimum point, the second-order terms in the Taylor series expansion dominate. In this scheme, objective function, which are approximated by up to the second-order terms in the Taylor series expansion, is minimized by SLP in iterations. The features and results in this paper are as follows;

1. Only the information about the sign of the first-order and second-order partial derivative of each parameter is required to perform
identification by SLP and MSLP, in such problem that does not have any constraints but move limit of parameters to be identified.

2. The second-order partial derivatives can easily be derived from the first-order ones in a linear chain oscillator.

3. The convergency is dramatically improved by using MSLP.

REFERENCES


APPENDIX

Partial derivatives of transfer function

(1) First-order partial derivatives

Vector of transfer functions between masses and the base is expressed by

\[ H(\omega) = A(\omega)^{-1} F(\omega) \]  \hspace{1cm} (A1)

where \( H(\omega) = \{x_1(\omega)/x_b(\omega), x_2(\omega)/x_b(\omega), \ldots, x_n(\omega)/x_b(\omega) \} \). In the following, we abbreviate \( \omega \) for the sake of simplicity in expression.

The first-order partial derivatives of \( H \) with respect to a parameter \( \alpha \), is

\[ \frac{\partial H}{\partial \alpha} = A^{-1} \left( \frac{\partial F}{\partial \alpha} - \frac{\partial A}{\partial \alpha} \right) \]  \hspace{1cm} (A2)

where \( \frac{\partial F}{\partial \alpha} \) and \( \frac{\partial A}{\partial \alpha} \), are obtained as follows.

1) \( \frac{\partial F}{\partial \alpha} \),

\[ \frac{\partial F}{\partial \alpha} = \{ x_0, 0, \ldots, 0 \} \]

2) \( \frac{\partial F}{\partial k_j} \),

\[ \frac{\partial F}{\partial k_j} = \{ 0, 0, \ldots, 0 \} \hspace{1cm} j = 2, 3, \ldots, n \]

3) \( \frac{\partial F}{\partial c_j} = i \omega \cdot \frac{\partial F}{\partial k_j} \),

\[ j = 1, 2, \ldots, n \]  \hspace{1cm} (A3)

(2) Second-order partial derivatives

The second-order partial derivatives of \( H \) with respect to parameters \( \alpha_i, \alpha_k \) is

\[ \frac{\partial^2 H}{\partial \alpha_i \partial \alpha_k} = A^{-1} \left( \frac{\partial^2 F}{\partial \alpha_i \partial \alpha_k} - \frac{\partial^2 A}{\partial \alpha_i \partial \alpha_k} \right) \]  \hspace{1cm} (A4)

\[ - \frac{\partial A}{\partial \alpha_i} \frac{\partial F}{\partial \alpha_k} - \frac{\partial A}{\partial \alpha_k} \frac{\partial F}{\partial \alpha_i} \]  \hspace{1cm} \( j = 1, 2, \ldots, n \) \( k = 1, 2, \ldots, n \) \hspace{1cm} (A5)

where \( \alpha_i \) is \( k \) or \( c \). The second-order partial derivatives of \( A \) and \( F \) are null matrix and null vector, respectively, taking account of Eq. (A4). Therefore, Eq. (A5) reduces to

\[ \frac{\partial^2 H}{\partial \alpha_i \partial \alpha_k} = - A^{-1} \left( \frac{\partial A}{\partial \alpha_i} \frac{\partial F}{\partial \alpha_k} + \frac{\partial A}{\partial \alpha_k} \frac{\partial F}{\partial \alpha_i} \right) \]  \hspace{1cm} (A6)

Thus, the second-order partial derivatives are calculated by first-order partial derivatives of \( A \) and \( F \) which are shown in Eq. (A3) and (A4).