Development of SLP for identification of structural systems

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ABSTRACT: The purpose of this study is to present a new nonlinear optimization scheme, which is modification of Successive Linear Programming(SLP), for identification of a linear chain oscillator. The identification problem is formulated in the frequency domain, in order to estimate the stiffness and damping coefficients of the system with known mass distribution. In numerical analysis, the results of identification of 5-DOF system by SLP and modified SLP are compared. It is shown that the convergency of parameters is dramatically improved by applying the modified SLP.

1 INTRODUCTION

The development of dynamic models is necessary for predicting the response of structural systems during earthquake motions. In aseismic design of structures, dynamic models can be theoretically synthesized from the properties of structural components and their interactions. Such dynamic models should be improved, because there still remain many uncertainties in the synthesis. Fortunately, a large number of records pertinent to structural response during earthquake motions have been accumulated, which offers an opportunity to study dynamic characteristics of structures. Using these records, dynamic properties of structures can be identified and then structural dynamic models are improved.

Identification problem has been studied by many researchers, and various approaches are proposed. Many of the identification techniques that have been employed in earthquake engineering can be classified as output-error approach. In this approach, the optimal estimates of parameter values of a dynamic model are determined by achieving the least-squares match between the responses of structure and model subjected to nominally the same excitation. Then, the identification problem can be formulated as a optimization problem of structural parameters. In order to solve such optimization problem, we can use many schemes.

Nonlinear optimization schemes can be classified into direct search method and descent method. The latter is the method that uses the partial derivatives of objective function and is often employed in identification of structural systems. McVerry(1980) applied Gauss-Newton method to identify modal parameters of a linear shear structure excited by earthquake ground motion in frequency domain

analysis. Matsui and Kurita(1989) also applied Gauss-Newton method to identify a structural system in time domain analysis. Tsujihara, Sawada and Hirao(1987) applied SLP(Successive Linear Programming) to identify stiffness and damping coefficients of a linear chain oscillator. Tsujihara, Sawada and Sugito(1990) applied SLP and DFP(Davidon-Fletcher-Powell) method to identify S-wave velocity and quality factor in the subsurface layered soil using vertical array records of earthquake ground motion.

Among these optimization schemes, SLP has the advantage whose algorithm is relatively simple. Particularly, in such optimization problem with no constraint equations as identification of dynamic structural systems, approximation in each stage can be carried out without using the scheme of LP(Linear Programming). Then, we need only the information about the sign of the first-order partial derivative with respect to each parameter. Moreover, SLP is very effective to identify simultaneously such parameters as stiffness and damping of a structural system whose size and sensitivity are considerably different from each other, because the linear-search subproblem does not exist in the algorithm. However, SLP has the serious defect that the convergency is not good in the neighborhood of a optimum point.

In this paper, we introduce a new optimization scheme based on SLP. The second-order partial derivatives are used to overcome the defect in the convergency near a optimum point. First, we show the algorithm for identification of a linear chain oscillator. Secondly, we show through numerical analysis that the convergency is dramatically improved by applying this scheme.

2 FORMULATION OF IDENTIFICATION PROBLEM

Fig.1 shows a linear chain oscillator with n-degree-of-freedom, in which mass is represented by m_j, stiffness by k_j, damping by c_j ; $j=1, 2, \cdots, n$. Denoting the absolute motion of masses by vector $x(t) (= \{x_1(t), x_2(t), \cdots, x_n(t)\}^T)$, the equation of motion can be expressed by

$$M\ddot{x}(t)+C\dot{x}(t)+Kx(t)=f(t) \qquad (1)$$

where

$$M = \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_n \end{bmatrix} \qquad (2)$$

$$C = \begin{bmatrix} c_1+c_2 & -c_2 & & \\ -c_2 & c_2+c_3 & -c_3 & \\ & \ddots & \ddots & \\ & & \ddots & c_n \\ & & -c_n & c_n \end{bmatrix} \qquad (3)$$

$$K = \begin{bmatrix} k_1+k_2 & -k_2 & & \\ -k_2 & k_2+k_3 & -k_3 & \\ & \ddots & \ddots & \\ & & \ddots & k_n \\ & & -k_n & k_n \end{bmatrix} \qquad (4)$$

and

$$f(t) = \begin{Bmatrix} c_1\dot{x}_0(t)+k_1x_0(t) \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \qquad (5)$$

$x_0(t)$ is the displacement of the base. Taking Fourier transform of both sides of Eq.(1), we have

$$[-\omega^2 M+i\omega C+K]X(\omega) = \begin{Bmatrix} i\omega c_1+k_1 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} X_0(\omega) \qquad (6)$$

where ω is circular frequency; $X(\omega)$ and $X_0(\omega)$ are the Fourier transforms of $x(t)$ and $x_0(t)$, respectively. Eq.(6) can be solved as follows,

$$X(\omega)=A(\omega)^{-1}F(\omega)X_0(\omega) \qquad (7)$$

where

$$A(\omega)=[-\omega^2 M+i\omega C+K] \qquad (8)$$

$$F(\omega)=\{i\omega c_1+k_1, 0, \cdots, 0\}^T \qquad (9)$$

Consider that the records of motion are available at the base and an arbitrary mass r. Denote a measured record by $\sim$, and Fourier transforms of those records $\tilde{x}_0(t)$ and $\tilde{x}_r(t)$ by $\tilde{X}_0(\omega)$ and $\tilde{X}_r(\omega)$, respectively. The transfer
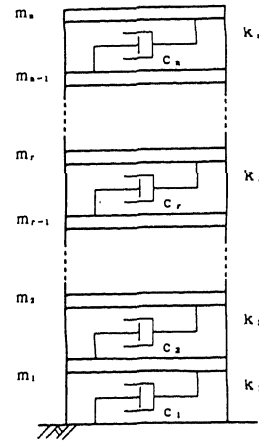

Figure 1 System model

function between the base and mass r is expressed by

$$\tilde{H}(\omega)=\tilde{X}_r(\omega)/\tilde{X}_0(\omega) \qquad (10)$$

While, the transfer function of the model, $H(\omega; \alpha)$, is calcurated from Eq.(7) as a function of system parameters α (m_j, k_j and c_j ; $j=1, 2, \cdots, n$). Therefore, the identification problem becomes to determine the optimal estimates of the parameters, so as to minimize the least-squares match between $\tilde{H}(\omega)$ and $H(\omega; \alpha)$.
  We adopt the following objective function;

$$S(\alpha) = \sum_{i=1}^{Nf} [|H(\omega_i; \alpha)| - |\tilde{H}(\omega_i)|]^2 \rightarrow min \qquad (11)$$

where ω_i denotes i-th sampled circular frequency and Nf the total number of samplings. Eq.(11) can be solved in an iterative manner with an optimazation scheme.
  Only the amplitude of the transfer function is used in Eq.(11). The analysis in the frequency domain, in general, corresponds to that in the time domain by considering the amplitude and phase. For a linear chain model, however, the information of the system dynamics can be extracted from only the amplitude of the transfer function because the unit impulse response function of the system has causality and its Laplace transform is minimum phase function(Papoulis, 1962). Therefore, the system under consideration is identifiable through the objective function as shown in Eq.(11).

3 ALGORITHM OF MSLP(MODIFIED SLP)

The optimum values of α are to be found by means of iterative procedure. At the start of r-th iteration, we possess current values $\alpha^{(r)}$, and we seek new values $\alpha^{(r+1)}$ using the formula

$$\alpha^{(r+1)} = \alpha^{(r)} + \triangle \alpha^{(r)} \qquad (12)$$

where $\triangle \alpha^{(r)}$ are the step size in the r-th iteration.

Taking Taylor series expansion of Eq. (11) in the r-th iteration of optimization procedure, we have

$$S^{(r+1)} \simeq S^{(r)} + \sum_{j=1}^{N} \frac{\partial S^{(r)}}{\partial \alpha_j} \triangle \alpha_j^{(r)}$$

$$+ \frac{1}{2} \sum_{j=1}^{N} \sum_{k=1}^{N} \frac{\partial^2 S^{(r)}}{\partial \alpha_j \partial \alpha_k} \triangle \alpha_j^{(r)} \triangle \alpha_k^{(r)} \qquad (13)$$

where $S^{(r+1)}$ and $S^{(r)}$ are $S(\alpha^{(r+1)})$ and $S(\alpha^{(r)})$, respectively, and N is the total number of unknown parameters. Partial derivatives of the first and second order of objective fuction are expressed by

$$\frac{\partial S}{\partial \alpha_j} = 2 \sum_{i=1}^{Nf} [H(\omega_i; \alpha) - \widetilde{H}(\omega_i)] \frac{\partial H(\omega_i; \alpha)}{\partial \alpha_j} \qquad (14)$$

$$\frac{\partial^2 S}{\partial \alpha_j \partial \alpha_k} = 2 \sum_{i=1}^{Nf} \frac{\partial H(\omega_i; \alpha)}{\partial \alpha_j} \frac{\partial H(\omega_i; \alpha)}{\partial \alpha_k}$$

$$+ 2 \sum_{i=1}^{Nf} [H(\omega_i; \alpha) - \widetilde{H}(\omega_i)] \frac{\partial^2 H(\omega_i; \alpha)}{\partial \alpha_j \partial \alpha_k} \qquad (15)$$

Traditional SLP requires only the second term in Eq. (13) to find $\triangle \alpha^{(r)}$ under the following constraints of move limit.

$$-\xi \alpha_j^{(r)} \leqq \triangle \alpha_j^{(r)} \leqq \xi \alpha_j^{(r)} \qquad (j=1\sim N) \qquad (16)$$

where ξ is coefficient that determine the move limit of $\triangle \alpha_j^{(r)}$, and $0.02 \sim 0.2$ is often set as ξ. ξ is to be set smaller in the neighborhood of a optimum point.

In the scheme which we call MSLP(modified SLP), by taking it into consideration that Eq. (13) is a equation of the second degree with respect to $\triangle \alpha^{(r)}$, we try to minimize Eq. (13) by SLP. The problem is that the second-order partial derivatives of transfer function are contained as shown in Eq. (15). In the system under consideration, however, they are easily obtained because the second-order partial derivatives of $\Lambda(\omega)$ and $F(\omega)$ in Eq. (8) and (9) are null matrix and null vector, respectively(see APPENDIX).

The algorithm of MSLP is as follows. We rewrite Eq. (13) for the sake of simplicity of explanation.

$$T \simeq \sum_{j=1}^{N} D_j \beta_j + \frac{1}{2} \sum_{j=1}^{N} \sum_{k=1}^{N} E_{jk} \beta_j \beta_k \qquad (17)$$

where $D_j = (\partial S / \partial \alpha_j)^{(r)}$, $E_{jk} = (\partial^2 S / \partial \alpha_j \partial \alpha_k)^{(r)}$, $\beta_j = \triangle \alpha_j$ and $\beta_k = \triangle \alpha_k$. We omit the

first term $S^{(r)}$ in Eq. (13) because minimization is carried out regardless of the value of $S^{(r)}$. D_j and E_{jk} in Eq. (17) need to be calcurated once in the r-th global iteration. Linearizing Eq. (17) with respect to β by Taylor series expansion,

$$T^{(2)} \simeq T^{(1)} + \sum_{j=1}^{N} (\partial T / \partial \beta_j)^{(1)} \triangle \beta_j^{(1)} \qquad (18)$$

where

$$\partial T / \partial \beta_j = D_j + \sum_{k=1}^{N} E_{jk} \beta_k^{(1)} \qquad (19)$$

Then, $\triangle \beta_j^{(1)}$, $j=1\sim N$, can be determined by using the scheme of LP under the following constraints of move limit.

$$-\zeta \beta_{ju} \leqq \triangle \beta_j^{(1)} \leqq \zeta \beta_{ju} \qquad (j=1\sim N) \qquad (20)$$

where $\beta_{ju} = \xi \alpha_j^{(r)}$ and ζ is coefficient to determine the move limit of $\triangle \beta_j^{(1)}$. We may set 0 to the initial approximation $\beta_j^{(1)}$, $j=1\sim N$.

By the way, by taking it into account that the constraint equation is nothing but Eq. (20), we can determine $\triangle \beta_j^{(1)}$ only from the information about the sign of $\partial T / \partial \beta_j$ as follows.

if $(\partial T / \partial \beta_j)^{(1)} \geqq 0$ then $\triangle \beta_j^{(1)} = -\zeta \beta_{ju}$ (21)

if $(\partial T / \partial \beta_j)^{(1)} < 0$ then $\triangle \beta_j^{(1)} = \zeta \beta_{ju}$ (22)

Thus, local iteration to minimize Eq. (17) can be carried out without using the scheme of LP. After such local iterations, the optimal estimate β_j^* of $\triangle \alpha_j^{(r)}$ is obtained. Then, the new approximation $\alpha_j^{(r+1)}$ in the global iteration is determined by

$$\alpha_j^{(r+1)} = \alpha_j^{(r)} + \beta_j^* \qquad (j=1\sim N) \qquad (23)$$

We adopt the total number of iteration such that 20 as the terminating criterion of local iteration, and Eq. (24) as the terminating criterion of global iteration.

$$S \leqq \varepsilon \qquad (24)$$

where S is the value of objective function and ε is poitive small value such that 10^{-4}.

4 NUMERICAL RESULTS

Since the purpose of this study is to examine the efficiency of MSLP compared with traditional SLP, simulated data for the analytical model is used instead of actually recorded data. The model to be idetified is a linear chain oscillator, as shown in Fig.1, with 5 degree of freedom. The system parameters are shown in Table 1. In the analysis, transfer function between the base and lowest mass(mass 1) of the system is calcurated at 100 frequency points of equal intervals between 0.1Hz $\sim$10.0Hz, which is used as $\widetilde{H}(\omega_i)$ in Eq. (11). Therefore, the transfer function used in this
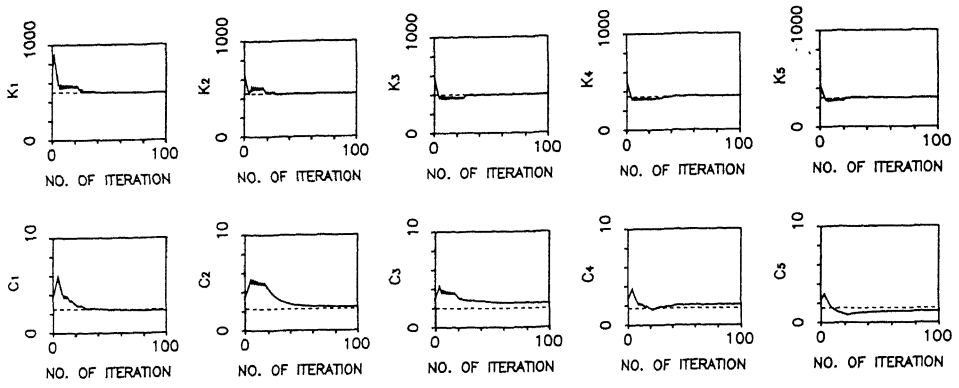
Figure 2   Convergence process of stiffness and damping coefficients by SLP
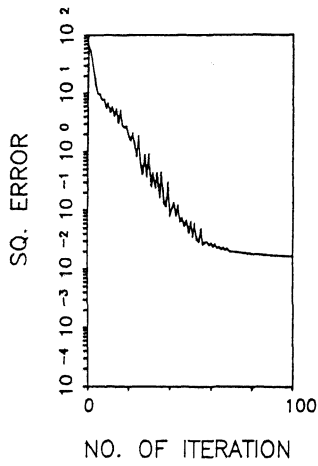
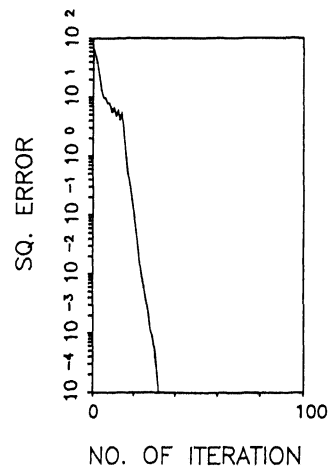

Figure 3   Value of objective
function by SLP

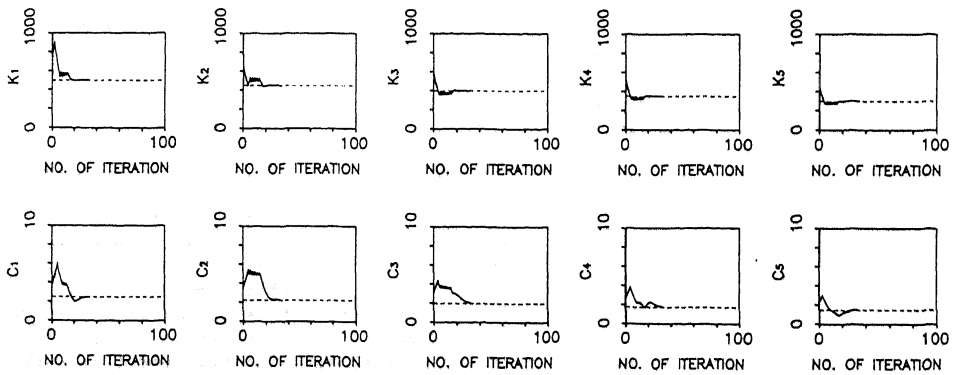Figure 5   Value of objective
function by SLP & MSLP

Figure 4   Convergence process of stiffness and damping coefficients by SLP & MSLP

paper is of noise free. In the following, the stiffness and damping coefficients are identified, assuming known mass. All of the initial approximation of stiffness and damping coefficients are 1.5 muliplied by exact values. We adopt 10^{-4} as ε in Eq. (24).

Table 1 Exact values of parameters

i	m_i [$t \cdot s^2/cm$]	k_i [t/cm]	c_i [$t \cdot s/cm$]
5	0.50	300.0	1.50
4	0.50	350.0	1.75
3	0.50	400.0	2.00
2	0.50	450.0	2.25
1	0.50	500.0	2.50

4.1 Application of SLP

The results of identification by SLP is shown in Fig. 2, Fig. 3 and Table 2. Move limit is gradually smaller when the solution fluctuates in the process of iteration. Fig. 2 is the convergence process of stiffness and damping coefficients. In the figure, horizontal axis is the number of iteration and vertical one is value of each parameter, and broken line corresponds to exact value of each parameter. Stiffness coefficients almost converge to the exact values around 40 iterations. Damping coefficients, however, have not converged by 100 iterations yet. In Table 2, initial values, estimated values and ratios of the former to latter are shown. Damping coefficients contain maximum error of 27% in estimated values.

Table 2 Results of identification by SLP

i	Initial k_i	c_i	Estimated k_i	c_i	Est./Exact k_i/k_i	c_i/c_i
5	450	2.250	300	1.145	0.999	0.763
4	525	2.625	351	2.122	1.003	1.213
3	600	3.000	400	2.545	1.008	1.273
2	675	3.375	451	2.451	1.002	1.089
1	750	3.750	497	2.344	0.994	0.938

Fig. 3 shows the value of objective function in the process of iteration. In the figure, horizontal axis is the number of iteration and vertical one is the sum of square error of transfer functions. Fig. 3 as well as Fig. 2 expose the defect of SLP that convergency is not good in the neighborhood of a optimum point.

4.2 Application of MSLP

The results of identification by MSLP is shown

in Fig. 4, Fig. 5 and Table 3. In order, to save computational time, SLP is used in the early stage of the process, namely, first 15 iterations are carried out by SLP. As to the terminating criterion of local iteration in MSLP, we adopt total number of iteration of 20. Stiffness coefficients almost converge to the exact values about 20 iterations, and another 12 iterations are enough to satisfy the terminating criterion of Eq. (24). Stiffness and Damping coefficients are estimated within the error of 0.1% and 2%, respectively.

Table 3 Results of identification by SLP & MSLP

i	Initial k_i	c_i	Estimated k_i	c_i	Est./Exact k_i/k_i	c_i/c_i
5	450	2.250	300	1.489	1.000	0.993
4	525	2.625	350	1.784	1.000	1.019
3	600	3.000	400	2.034	0.999	1.017
2	675	3.375	451	2.259	1.001	1.004
1	750	3.750	500	2.462	1.000	0.985

Fig. 6 shows the transfer functions calcurated by exact, initial and estimated parameters. Estimated transfer function is in good agreement with exact one.

Thus, it is demonstrated that the convergency of unknown parameters are dramatically improved by using MSLP compared with using only SLP.
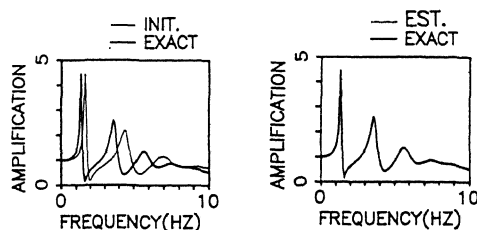

Figure 6 Transfer functions

5 CONCLUSIONS

A simple and efficient optimization scheme based on SLP(Successive Linear Programming) has been developed for identification of stiffness and damping coefficients of a linear chain oscillator. Near a optimum point, the second-order terms in the Taylor series expansion dominate. In this scheme, objective function, which are approximated by up to the second-order terms in the Taylor series expansion, is minimized by SLP in iterations. The features and results in this paper are as follows;
1. Only the information about the sign of the first-order and second-order partial derivative of each parameter is required to perform

identification by SLP and MSLP, in such
problem that does not have any constraints but
move limit of parameters to be identified.
2. The second-order partial derivatives can
easily be derived from the first-order ones in
a linear chain oscillator.
3. The convergency is dramatically improved by
using MSLP.

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APPENDIX

Partial derivatives of transfer function

(1) First-order partial derivatives

Vector of transfer functions between masses
and the base is expressed by

$$H(\omega)=A(\omega)^{-1}F(\omega) \tag{A1}$$

where $H(\omega)=\{X_1(\omega)/X_0(\omega),X_2(\omega)/X_0(\omega),\cdots,$
$X_n(\omega)/X_0(\omega)\}$. In the following, we abbre-
viate (ω) for the sake of simplicity in
expression.
    The first-order partial derivatives of H
with respect to a parameter α_j is

$$\frac{\partial H}{\partial \alpha_j} = A^{-1}\left(\frac{\partial F}{\partial \alpha_j} - \frac{\partial A}{\partial \alpha_j}\right) \tag{A2}$$

where $\partial F/\partial \alpha_j$ and $\partial A/\partial \alpha_j$ are obtained as
follows.

i) $\partial F/\partial \alpha_j$

$\partial F/\partial k_1=\{X_0,0,\cdots,0\}$

$\partial F/\partial k_j=\{0,0,\cdots,0\}$, $j=2,3,\cdots,n$

$\partial F/\partial c_j=i\omega\cdot\partial F/\partial k_j$, $j=1,2,\cdots,n$

$$\tag{A3}$$

ii) $\partial A/\partial \alpha_j$

$$\frac{\partial A}{\partial k_1} = \begin{pmatrix} 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & 0 \\ \vdots & \vdots & & & & \vdots \\ 0 & 0 & \cdot & \cdot & \cdot & 0 \end{pmatrix}$$

$$\frac{\partial A}{\partial k_j} = \begin{pmatrix} 0 & \cdot & \cdot & \cdot & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ \vdots & & & & \vdots & \vdots & \vdots & \vdots & & & & \vdots \\ 0 & \cdot & \cdot & \cdot & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & 0 & 1 & -1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & 0 & -1 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ \vdots & & & & \vdots & \vdots & \vdots & \vdots & & & & \vdots \\ 0 & \cdot & \cdot & \cdot & 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \end{pmatrix} \begin{matrix} 1 \\ \\ j-2 \\ j-1 \\ j \\ j+1 \\ \\ n \end{matrix}$$

$$, j=2,3,\cdots,n$$

$\partial A/\partial c_j=i\omega\cdot\partial A/\partial k_j$, $j=1,2,\cdots,n$

$$\tag{A4}$$

(2) Second-order partial derivatives

The second-order partial derivatives of H
with respect to parameters α_j, α_k is

$$\frac{\partial^2 H}{\partial \alpha_j \partial \alpha_k} = A^{-1}\left(\frac{\partial^2 F}{\partial \alpha_j \partial \alpha_k} - \frac{\partial^2 A}{\partial \alpha_j \partial \alpha_k}H\right.$$
$$\left. - \frac{\partial A}{\partial \alpha_j}\frac{\partial F}{\partial \alpha_k} - \frac{\partial A}{\partial \alpha_k}\frac{\partial F}{\partial \alpha_j}\right)$$
$$, j=1,2,\cdots n$$
$$, k=1,2,\cdots n$$
$$\tag{A5}$$

where α is k or c. The second-order partial
derivatives of A and F are null matrix and
null vector, respectively, taking account of
Eq.(A4). Therefore, Eq.(A5) reduces to

$$\frac{\partial^2 H}{\partial \alpha_j \partial \alpha_k} = -A^{-1}\left(\frac{\partial A}{\partial \alpha_j}\frac{\partial F}{\partial \alpha_k}\right.$$
$$\left. + \frac{\partial A}{\partial \alpha_k}\frac{\partial F}{\partial \alpha_j}\right)$$
$$\tag{A6}$$

Thus, the second-order partial derivatives are
calcurated by first-order partial derivatives
of A and F which are shown in Eq.(A3) and (A4).