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A new approach to analyzing the earthquake response of structural systems with parameter uncertainties

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ABSTRACT: The objective of this paper is to describe a new probabilistic approach for determining the dynamic response of structural systems with uncertain parameters. The approach provides a means of computing the response uncertainty due to uncertainty in model parameters when a structural system is subjected to either deterministic or stochastic excitation. Some applications of the proposed method to the area of earthquake engineering are presented. It is shown that parameter uncertainties may cause very significant changes in the response of structures subjected to earthquake loading.

INTRODUCTION

Uncertainty in model parameters should be accounted for in the description and analysis of real structures. Such uncertainty can arise from assumptions made when modeling geometry, material properties, constitutive laws, and boundary conditions, or from the specification of external loads. The latter uncertainty often arises when the loads result from a physical process, such as an earthquake, so complex that it is best modeled as a stochastic process. The effects of uncertain parameters are often accounted for in design by analyzing systems with various combinations of bounds on the uncertain parameters, or by spectrum broadening (Shinozuka and Jan, 1972; Astill, *et al*, 1972). Alternatively, simple second-order perturbation techniques have been used to obtain approximate analytical estimates of the effects of uncertainty (Liu, *et al*, 1987). All these approaches have limitations on accuracy and the level of information which they provide (Benaroya and Rehak, 1988; Jensen, 1989). These limitations are particularly restrictive when dynamic, especially transient or wave propagation problems must be analyzed.

In this paper a new probabilistic approach is described for studying the dynamic response of linear systems with parameter uncertainties. The parameter uncertainties are modeled in a probabilistic sense through random variables or random fields. In what follows, the proposed method is initially formulated for single-degree-of-freedom systems subjected to deterministic excitation and is then extended to multi-degree-of-freedom and continuous systems. Finally, the method is extended to cases where the excitation is random in time.

FORMULATION

Consider a simple single-degree-of-freedom structural model defined by the equation of motion

$$\ddot{x}(t) + 2\varsigma\dot{x}(t) + \omega^2 x(t) = f(t) \ , \tag{1}$$

where ς is the damping ratio, ω is the natural frequency, $f(t)$ is the external load, and and x is the displacement of the system. The initial conditions are taken to be $x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$.

Let the natural frequency be assumed to be uncertain, while the damping ratio, external load, and initial conditions are assumed to be deterministic. The natural frequency may then be described by means of a random variable which is defined in terms of its mean value and a deviatoric component as:

$$\omega = \bar{\omega} + \lambda\, b \quad , \tag{2}$$

where $\bar{\omega}$ denotes the expected value of the natural frequency, λ is a deterministic coefficient, and b is a random variable with zero mean and unit variance. Therefore, the second-moment representation of the natural frequency is given by $E[\omega] = \bar{\omega}$ and $Var[\omega] = \lambda^2$, where $E[\cdot]$ is the expectation operation and $Var[\cdot]$ represents the variance of the random variable.

The dependent variable $x(t)$ is now an explicit function of the random variable b. It may thus be expanded in a series over b by means of an orthogonal set of polynomials weighted by time dependent functions as

$$x(t,b) = \sum_{j=0}^{NP} x_j(t) H_j(b) \tag{3}$$

where NP is the order of approximation in the random space, $x_j(t)$ is an unknown deterministic function of time, and $\{H_j(b)\}_{j=0}^{\infty}$ is an orthogonal set of polynomials with respect to the mean operation. That is,

$$E[H_i(b)H_j(b)] = \delta_{ij} \tag{4}$$

where δ_{ij} is the Kronecker delta. The precise selection for the set of polynomials, $H_i(b)$, will depend upon the probability density function of the random variable b. For example, Legendre polynomials satisfy the orthogonality condition for a uniform random variable, Hermite polynomials do likewise for a Gaussian random variable, etc. (Jensen, 1989).

A set of differential equations for the deterministic functions of time, $x_j(t)$, may be obtained by means of the weighted residual method together with the use of first- and second-order recurrence relations for the orthogonal set of polynomials. The details of this procedure have been given elsewhere by Jensen and Iwan, (1991).

Once the equations for the unknowns $x_j(t)$ have been solved, an analytical approximation to the solution in the random space is completely defined by equation (3), and the statistical moments of the response can be computed directly. For example, it can be shown that the mean value and the variance of the response are given by

$$E[x(t)] = x_0(t) \quad , \quad Var[x(t)] = \sum_{j=1}^{NP} x_j^2(t) \; . \tag{5}$$

A similar characterization may be given for the velocity and acceleration responses.

In order to illustrate the influence of an uncertain natural frequency on system response, an oscillator with a uniformly distributed natural frequency subjected to an earthquake-like base excitation is considered. For the example presented, the uncertain frequency has a coefficient of variation of 10 per cent with a mean value of 2 Hz. A nominal damping corresponding to 5 per cent of critical based on the mean value of natural frequency is assumed.

Figures 1 and 2 show the mean value and standard deviation of the absolute acceleration response of this simple structure. In this example, the standard deviation of the response is of the same order of magnitude as the peak response. This is typical. The example clearly demonstrates the high degree of variability of the response due to uncertainty in the natural frequency and suggests that great care must be exercised when interpreting the calculated response of a structure if its physical properties are not precisely known.

A sixth-order approximation (NP) in the random space has been used in this example. The need to use such a high order approximation arises due to the high degree of nonlinearity of the response as a function of the uncertain parameters. Validation calculations show that the results from the present method agree quite well with those obtained by simulation techniques. These validation calculations also show the inadequacy of the simple second-order perturbation method (Jensen, 1989). The new formulation can easily be extended to problems with uncertainty in both the natural frequency and damping ratio. The details of this are given elsewhere in Jensen and Iwan (1991).

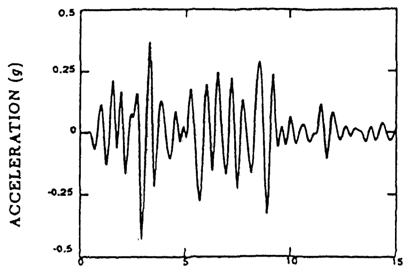

Figure 1: Mean value of the absolute acceleration response of a Single-Degree-of-Freedom structure with uncertain natural frequency subjected to earthquake-like excitation

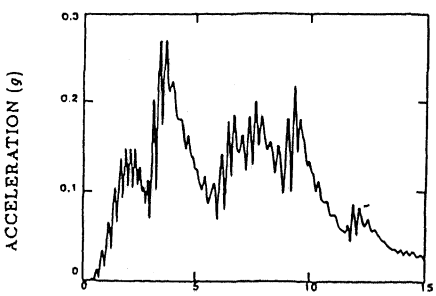

Figure 2: Standard deviation of the absolute acceleration response of a Single-Degree-of-Freedom structure with uncertain natural frequency subjected to earthquake-like excitation

EXTENSION TO MULTI-DEGREE-OF-FREEDOM SYSTEMS

The formulation previously presented can readily be extended to multi-degree-of-freedom systems. Consider a linear structural system with mass, damping and stiffness matrices $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$, and an excitation $\mathbf{f}(t)$. The equation for the response of the system, $\mathbf{x}(t)$, is given by

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \; . \tag{6}$$

The damping and stiffness of the system are assumed to be uncertain. These parameters are taken to be time-invariant and are modeled as random variables. Further, it is assumed that the damping matrix and the stiffness matrix allow the following representations

$$\mathbf{C} = \bar{\mathbf{C}} + \sum_{n=1}^{r} \mathbf{C}_n b_n \quad , \quad \mathbf{K} = \bar{\mathbf{K}} + \sum_{n=1}^{r} \mathbf{K}_n b_n \qquad (7)$$

where $\bar{\mathbf{C}}$, $\bar{\mathbf{K}}$, $\mathbf{C}_n$ and $\mathbf{K}_n$ are deterministic matrices, and $b_n, n = 1, \ldots, r$ are independent zero mean random variables used to represent the uncertainty in damping and stiffness properties. The terms $\bar{\mathbf{C}}$ and $\bar{\mathbf{K}}$ represent the expected values of the damping and stiffness matrices, respectively, and the summations represent their deviatoric component. This representation of the damping and stiffness matrices is similar to the expansion used by Lawrence (1987) for random parameters with known second-order statistics.

In order to solve equation (6), the dependent variable $\mathbf{x}(t)$ is expanded in terms of a series of orthogonal functions which depend upon a random vector $\mathbf{b}$ with components $b_n, n = 1, \ldots r$. Then, the solution takes the form

$$\mathbf{x}(\mathbf{b}, t) = \sum_{0 \le |\mathbf{l}| \le NP} \mathbf{x}_{\ell_1 \ldots \ell_r}(t) \prod_{s=1}^{r} H_{\ell_s}^{b_s}(b_s) \qquad (8)$$

where NP is the order of approximation in the random space, $\mathbf{x}_{\ell_1 \ldots \ell_r}(t)$ is an unknown deterministic function of time, $\mathbf{l}$ is a vector with components $\ell_s, s = 1, \ldots, r$, $|\mathbf{l}|$ stand for the $|\cdot|_1$ norm of $\mathbf{l}$ and $\{H_{\ell}^{b_s}(b_s)\}_{\ell=0}^{\infty}$ is a set of orthogonal polynomials. As above, the selection for the set of polynomials depends on the probability density function of the random variable b_n. A set of differential equations for the coefficients $\mathbf{x}_{\ell_1 \ldots \ell_r}(t)$ can be derived as before by means of the weighted residual method (Jensen, 1989). Once these equations have been solved in time, the response uncertainty and statistics can be computed directly.

To illustrate the application of the method a twenty degree-of-freedom system representing a simple uniform structure is subjected to an impulse base excitation. The structure is assumed to have a nominal fundamental frequency of 2 Hz with nominal Rayleigh damping corresponding to 5 per cent of critical in the first two modes. The stiffness of the structure is assumed to be a one-dimensional homogeneous strongly-correlated Gaussian random field with a 20% coefficient of variation. By means of spectral decomposition of the random field correlation properties, it is easily shown that the stiffness matrix of the system can be written as in equation (7) (Jensen, 1989; Spanos and Ghanem, 1989).

The response variability of the absolute acceleration at the top of the structure is presented in Figure 3. It is noted that the dispersion about the mean value response, as measured by the standard deviation, is substantial being of the order of one-quarter of the peak mean response. The level of response uncertainty shows that the presence of uncertainty in the physical properties of a structure can markedly alter its response characteristics. As in the case of the single-degree-of-freedom-system, validation calculations show that the agreement between the proposed method and simulation is excellent (Jensen, 1989). Similar results are obtained when more complex base excitation, such as an earthquake excitation, is considered. The formulation presented for multi-degree-of-freedom systems can easily be extended to consider continuous systems described by partial differential equations of second and higher order (Jensen, 1989).

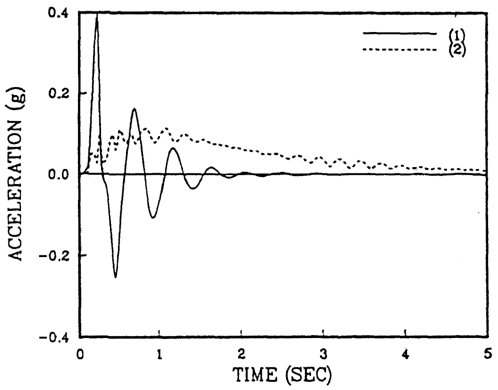

Figure 3: Response of a Twenty DOF structure with uncertain stiffness subjected to an impulsive base excitation. (1) Mean value of the absolute acceleration response at the free end of the structure. (2) Standard deviation of the absolute acceleration response at the free end of the structure.

EXTENSION TO STOCHASTIC EXCITATION

Consider the general class of problems in which the base excitation can be represented as a nonstationary Gaussian white noise process with zero mean. It is well known that the response of a linear system subjected to a Gaussian excitation is also a Gaussian process. If the mathematical model of the system is describable by a multi-degree-of-freedom linear system, then the displacement and velocity vectors of the system are zero-mean jointly Gaussian processes. A Gaussian process is completely defined by its mean vector and covariance matrix, but due to uncertainties in the system properties, this description is itself random. That is, the coefficients of the covariance matrix are random variables. Using a formulation similar to the one used for multi-degree-of freedom systems, a random first-order differential equation for the evolution of the *nonstationary covariance matrix* with time can be derived directly (Jensen, 1989). Then, the covariance equation may be integrated in time and the response variability computed.

As an example of the application to problems with stochastic excitation, the random response of a primary-secondary system to earthquake-like excitation is presented. The primary system is modeled by a five-degree-of-freedom system. The primary structure is assumed to be nominally classically damped with 5% of critical damping in the first two modes, and with a nominal fundamental frequency of 2 Hz. The secondary system, which is attached to the top of the primary structure, is idealized as a single-degree-of-freedom oscillator with 2% of critical damping. The base acceleration is modeled as a filtered, modulated white noise process. The ratio between the natural frequency of the secondary system and the fundamental frequency of the primary system is taken to be 0.75 (a nearly tuned condition), and the secondary system to primary system mass ratio is taken to be 0.01. The primary system stiffness and damping are separately assumed to be uncertain and are modeled as uniform random variables with a coefficient of variation of 30%.

Figure 4 shows the influence of uncertainties in the primary system parameters on the absolute acceleration response of the secondary system.

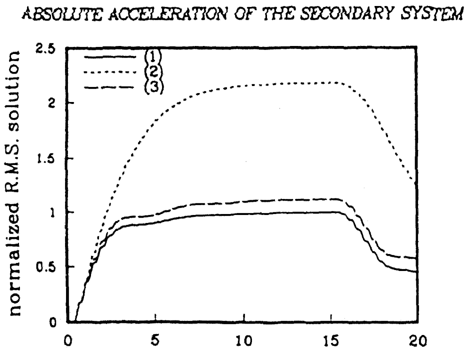

Figure 4: Absolute acceleration response of a SDOF secondary system attached to an uncertain Five DOF primary structure subjected to a stochastic base excitation: (1) Normalized nominal solution. (2) Normalized mean plus one standard deviation value of the solution when the stiffness is uncertain. (3) Normalized mean plus one standard deviation value of the solution when the damping is uncertain.

The duration of the excitation is equal to 15 T, where T is the fundamental period of the primary system. The transient R.M.S. value of the response is normalized by the stationary standard deviation of the response of the nominal system, and the time is normalized by the fundamental period of the primary system. There is very little influence of the uncertainty in the damping on the response of the secondary system for this case. On the contrary, uncertainty in the primary system stiffness has a strong influence on the variability of the response of the secondary system. The mean plus one standard deviation value of the stationary response for this case is more than twice the nominal stationary solution. Thus, the response variability associated with stiffness uncertainty is of the same order of importance as the uncertainty in the input excitation. The high sensitivity of the solution to uncertainty in stiffness in this case is due to the strong influence of tuning on the secondary system response, particularly for small mass ratios, and the likelihood of such tuning when the stiffness is uncertain.

CONCLUSIONS

The proposed method provides a powerful and useful tool for treating uncertainty in models of structural systems excited by earthquake-like excitation. The method can be used to analyze a broad range of complex engineering problems using present computer hardware.

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