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Structural design based on the optimum seismic reliability

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ABSTRACT: An analytical approach to structural design parameters is developed using the principle of optimum seismic reliability. An earthquake-like stationary random excitation being assumed, probabilistic earthquake response is estimated as a solution of a simple simultaneous algebraic equation. Based on this probabilistic earthquake response, seismic structural reliability and its associated derivative with respect to design variables are evaluated, and the optimum parameters are determined. The validity of this approach is demonstrated by examining the perspective and contour lines of the seismic reliability regarded as a function of design parameters for elastic structural systems with three- and five-degree-of-freedom. The application of this approach is also presented to determination of elasto-plastic structural parameters with bilinear hysteretic characteristics.

1 INTRODUCTION

In general the anti-seismic safety of structural systems has been examined on the basis of analysis of structural behavior subjected to actually recorded strong earthquake motions. Structural systems have been checked to ascertain whether the earthquake response of their elements is less than the safety threshold level corresponding to these elements, but optimality has not hitherto been investigated.

The design approach to structural elements developed here is essentially different from previous approaches, which place their emphases on either the minimization of the base shear force or the uniform distribution of the storey drift response. Since one objective of anti-seismic safety design of structural systems lies essentially in offering aseismic safety to the system, a structural design should in turn preserve this objective.

This paper deals with an optimum design of structural systems, i.e. the prediction of the optimum dynamic parameters of the system by selecting probabilistic seismic reliability as an objective function. For this objective, we adopt a stationary earthquake-like non-white random process with a prescribed maximum amplitude, predominant period and spectral shaping factor for the earthquake acceleration functions. A stationary process is used here rather than a non-stationary one, because of its suitability for the parametric surveying method.

Even with a stationary random process, it has so far been by no means easy to estimate the earthquake response of structural systems with multi-degrees-of-freedom (m.d.o.f.), these hitherto having for the real parts of their transfer functions required calculation by the tedious integral estimation method. Here, however, instead of solving differential equation for non-stationary random response, we derive the latter as a solution of a simple first order algebraic simultaneous equation. The analytical procedures developed here are,

1. derivation of the probabilistic second order moment response and its first and second order derivatives with respect to design parameters as a solution of the algebraic simultaneous equation;
2. calculation of the first and second order derivatives of the anti-seismic safety probability for the respective stories;
3. determination of design parameters optimizing probabilistic seismic reliability, that being defined as the aggregate products of the safety probability of each storey, derived using the Newton-Raphson method; and
4. demonstration of the validity of the presented approach through a comparison of its results with those based on another criterion, that of the uniform drift response distribution of the system .

The possible application of the presented approach to elasto-plastic structural systems is also discussed employing bilinear hysteretic systems, which can be replaced with the elastic ones based on the equivalent linearization and the presented approch can be easily applied to.

This approach is partly dependent upon such hypotheses as the stationarity of earthquake-like random excitation, the Poisson probability distribution function (p.d.f) of the anti-seismic probabilities of the respective stories, and the independence of the safety probability, all of which remain to be pursued in the future.

2 ANALYTICAL METHOD

2.1 MOMENT RESPONSE AND ITS DERIVATIVE WITH RESPECT TO DESIGN PARAMETERS

A method was developed by which can be estimated the probabilistic second order moment response of m.d.o.f. systems subjected to non-stationary earthquake-like random excitation by Asano (1985) . Here is briefly summarized the method for the case of stationary response.

A fundamental equation of the motion of structural systems subjected to the earthquake excitation f is

$$\dot{u}_j = \sum_{i=1}^{n} a_{ji} u_i + b_j f \tag{1}$$

in which n is the maximum number of state variables u_j necessary for specifying the motion of the system, j is a variable number, a_{ij} is specified by the stiffness and viscous damping coefficient of the elastic system plus the elastic-limit deformation, the plastic-branch slope and the equivalent linearization coefficients of the elasto-plastic bi-linear system, the predominant period ω_g and the shaping factor h_g of the earthquake excitation, while b_j is the coefficient associated with the excitation intensity. Probabilistic non-stationary second order moment response $m_{ij} = E(u_i u_j)$ is estimated by the differential equations,

$$\dot{m}_{ij} = \sum_{l=1}^{n} (a_{il} m_{lj} + a_{jl} m_{li}) \tag{2}$$

$$m_{nn} = (\sigma_f^2 - 4h_g \omega_g^3 m_{nn-1} - \omega_g^4 m_{n-1n-1})/4h_g^2 \omega_g^2 \tag{3}$$

in which $i = 1 \sim n$, $j = i \sim n-1$,σ_f^2 is the mean square of the excitation, and m_{nn}, appearing on the right hand side of Eq.(2), has to be substituted using Eq.(3) for the numerical calculation. For the stationary case,$\dot{m}_{ij} = 0$ and from Eq.(2) is obtained

$$\sum_{l=1}^{n} (a_{il} m_{lj} + a_{jl} m_{li}) = 0 \tag{4}$$

Substitution of Eq.(3) into Eq.(4) leads to the following simultaneous algebraic equation for $M_J = m_{ij}$,

$$[A]\{M\} = \{B\}\sigma_f^2 \Rightarrow \sum_{J=1}^{N} A_{IJ} M_J = B_I \sigma_f^2 \tag{5}$$

in which the elements of the coefficient matrix [A] are expressed in terms of those of the matrix $[a]$, and the elements of $\{B\}$ are expressed in terms of ω_g and h_g . By selecting the variables $\{X_k\}(k = 1 \sim \mu)$ that specify structural elements as design parameters, first order partial derivatives (sensitivity coefficients) with respect to these variables are derived from Eq.(5) as

$$\left\{\frac{\partial M}{\partial X_k}\right\} = -[A]^{-1} \left[\frac{\partial A}{\partial X_k}\right] \{M\} \tag{6}$$

2.2 DESIGN PARAMETERS BASED ON THE OPTIMUM SEIMIC RELIABILITY

Here seismic reliability R(t) is selected as an objective function in determination of the optimum parameters for structural elements; provided that the p.d.f. of the structural maximum response process is poissonian, it is expressed approximately by

$$R(t) \cong \exp(-\sum_{j=1}^{n} p_j(t)) \tag{7}$$

$$p_j(t) = \frac{t\sqrt{m_{\dot{u}_j \dot{u}_j}}}{\pi \sqrt{m_{u_j u_j}}} \exp\left(-\frac{\bar{u}_j^2}{2m_{u_j u_j}}\right) \tag{8}$$

and where p_j and $\bar{u}_j$ are respectively the probability and the threshold level of safety for a given storey j.

The derivative of R(t) with respect to the variable X_k gained from Eq.(8) is

$$\frac{\partial R(t)}{\partial X_k} = \left(-\sum_{j=1}^{n} \frac{\partial p_j(t)}{\partial X_k}\right) \exp(-\sum_{j=1}^{n} p_j(t)) \tag{9}$$

$$: \frac{\partial p_j(t)}{\partial X_k} = \frac{t}{2\pi} \exp\left(-\frac{\bar{u}_j^2}{2m_{u_j u_j}}\right) \sqrt{\frac{m_{\dot{u}_j \dot{u}_j}}{m_{u_j u_j}}} \times \left[\left(\frac{\bar{u}_j^2}{m_{u_j u_j}} - 1\right) \frac{\partial m_{u_j u_j}}{\partial X_k} / m_{u_j u_j} + \frac{\partial m_{\dot{u}_j \dot{u}_j}}{\partial X_k} / m_{\dot{u}_j \dot{u}_j}\right] \tag{10}$$

and the optimization of R(t) to X_k is tantamount to solving the following algebraic simultaneous equation to X_k

$$\{J_k\} = \left\{\sum_{j=1}^{n} \frac{\partial p_j(t)}{\partial X_k}\right\} R(t) = \{0\} \tag{11}$$

2.3 ITERATIVE METHOD OF EVALUATING DESIGN PARAMETERS

The application of the Newton-Raphson method is effective in the solution of Eq.(11) suggested by Ozaki et al.(1963). This equation is rewritten for the dummy variable number $k = 1, 2, \cdots, \mu$ thus:

$$J_k = \sum_{j=1}^{\mu} \frac{\partial p_{j(t)}}{\partial X_k} R(t) = 0 \qquad (k = 1, 2, \cdots, \mu) \tag{12}$$

The increment amount of the design variable $\{\Delta X_k\}$ with the i-step approximate solution $\{X_k\}$ is given by solving the simultaneous equation,

$$\left[\frac{\partial J}{\partial X}\right] \{\Delta X\} = -\{J\} \tag{13}$$

where $[\partial J/\partial X]$ and $\{J\}$ are calculated using the i-step solution of $\{X\}$. Given that $m_{u_j u_j} = m_{11}$ and $m_{\dot{u}_j \dot{u}_j} =$

m_{22} , the derivatives of J with respect to X are gained by

$$\frac{\partial J_k}{\partial X_{k'}} = \frac{\partial^2 R(t)}{\partial X_k \partial X_{k'}} = \exp(-\sum_{j=1}^{n} p_j(t)) \times \left(\sum_{j=1}^{n} \frac{\partial p_{j(t)}}{\partial X_k} \sum_{j=1}^{n} \frac{\partial p_{j(t)}}{\partial X_{k'}} - \sum_{j=1}^{n} \frac{\partial^2 p_{j(t)}}{\partial X_k \partial X_{k'}}\right) \quad (14)$$

in which

$$\frac{\partial^2 p_j(t)}{\partial X_k \partial X_{k'}} = \frac{t}{2\pi}\sqrt{\frac{m_{22}}{m_{11}}} \exp\left(-\frac{\bar{u}_j^2}{2m_{11}}\right) \times \left[\frac{1}{2}\left\{\left(\frac{\bar{u}_j^2}{m_{11}} - 1\right)\frac{\partial m_{11}}{\partial X_{k'}}/m_{11} + \frac{\partial m_{22}}{\partial X_{k'}}/m_{22}\right\} \times \left\{\left(\frac{\bar{u}_j^2}{m_{11}} - 1\right)\frac{\partial m_{11}}{\partial X_k}/m_{11} + \frac{\partial m_{22}}{\partial X_k}/m_{22}\right\} + \left(1 - \frac{\bar{u}_j^2}{m_{11}}\right)\frac{\partial m_{11}}{\partial X_k}\frac{\partial m_{11}}{\partial X_{k'}}/m_{11}^2 - \frac{\partial m_{22}}{\partial X_k}\frac{\partial m_{22}}{\partial X_{k'}}/m_{22}^2 + \left(\frac{\bar{u}_j^2}{m_{11}} - 1\right)\frac{\partial^2 m_{11}}{\partial X_k \partial X_{k'}}/m_{11} + \frac{\partial^2 m_{22}}{\partial X_k \partial X_{k'}}/m_{22}\right] \quad (15)$$

and $\{\partial^2 m/\partial X_k \partial X_k\}$ is estimated from Eq.(5) as

$$\left\{\frac{\partial^2 m}{\partial X_k \partial X_{k'}}\right\} = -[A]^{-1}\left(\left[\frac{\partial^2 A}{\partial X_k \partial X_{k'}}\right]\{m\} + \left[\frac{\partial A}{\partial X_k}\right]\left\{\frac{\partial m}{\partial X_k}\right\} + \left[\frac{\partial A}{\partial X_{k'}}\right]\left\{\frac{\partial m}{\partial X_k}\right\}\right) \quad (16)$$

3 NUMERICAL EXAMPLES

3.1 ELASTIC STRUCTURAL SYSTEMS

To examine the validity of the presented analytical approach, the optimum distribution of dynamical elastic structural properties is discussed.

Let the given elastic structural system have three-degree-of-freedom, its mass distribution $\{m\}$ be uniform $\{\overline{m}\}$ ($\overline{m}$: a standardized amount of mass) and its stiffness distribution be given as

$$\{k_i\} = \bar{k}\left\{1 - \lambda\left(\frac{i-1}{n-1}\right)^{\nu}\right\} \quad (17)$$

where $\bar{k}$ is a standardized stiffness ,λ and ν are the indicies governing the distribution of $\{k_i\}$, and the damping ratio is assumed to be proportional to the stiffness, assuming five percent of the critical damping as the fundamental mode of vibration. Further, let the level of the envelope function of σ_f , *the shaping factor of the* power spectra h_g and the predominant angular frequency ω_g of the excitation be respectively 50 ~ 100(gal) , 0.3 ~ 0.5 and 3.0 ~ 20.0(rad/sec) , while the standardized mass and stiffness are determined so that the fundamental period of structural systems T_1 is 0.5 ~ 2.0(sec) . The stiffness matrix of the structural system with three-degree-of-freedom is calculated from Eq.(17) as

$$[\bar{k}_{ij}] = \begin{bmatrix} 2-\lambda/2^{\nu} & -(1-\lambda/2^{\nu}) & 0 \\ -(1-\lambda/2^{\nu}) & 2-(1+1/2^{\nu})\lambda & -(1-\lambda) \\ 0 & -(1-\lambda) & 1-\lambda \end{bmatrix} \quad (18)$$

and the elements of the matrix $[a]$ in Eq.(1) with damping proportional to its stiffness are given as

$$[a] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$

$$a_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad a_{23} = \begin{bmatrix} \omega_g^2 & 2h_g\omega_g \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$a_{33} = \begin{bmatrix} 0 & 1 \\ -\omega_g^2 & -2h_g\omega_g \end{bmatrix},$$

$$a_{11} = a_{13} = a_{31} = a_{32} = [0] ,$$

$$a_{21} = \bar{\Omega}^2[\bar{k}_{ij}] \quad : \bar{\Omega}^2 = \bar{k}/\bar{m} ,$$

$$a_{22} = \eta a_{21} \quad :\eta = 2h_1/\omega_1 . \quad (19)$$

Numerical calculations of seismic reliability were made based on Eqs.(5),(7) and (8), using the parameters described immediately above. The results are shown in Figs.(1) and (2), where $T_1 = 1.0sec$, $\sigma_f = 100/3gal$, $\omega_g = 18rad/sec$, $h_g = 0.5$, the safety level in terms of the storey drift angle = 1/200, and the duration of stationary excitation= $10sec$. In Fig.1, seismic reliability is plotted as a function of λ and ν , the latter ranging respectively over 0.1 ~ 0.75 and 0.1 ~ 3.0; and the contour line corresponding to this is plotted in Fig.2.

Figs.(1) and (2) indicate that the seismic reliability is extremely sensitive to λ , but not so sensitive to ν.

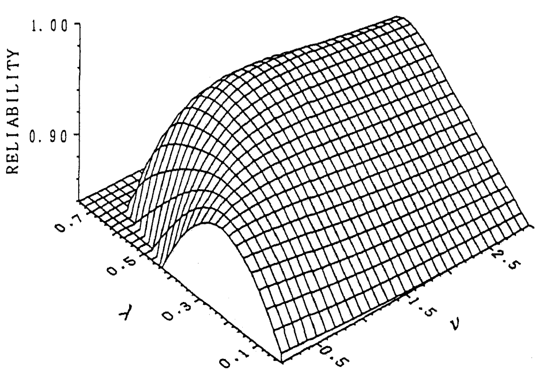

Fig.1 Reliability perspective as a function of λ and ν : 3 d.o.f. elastic system

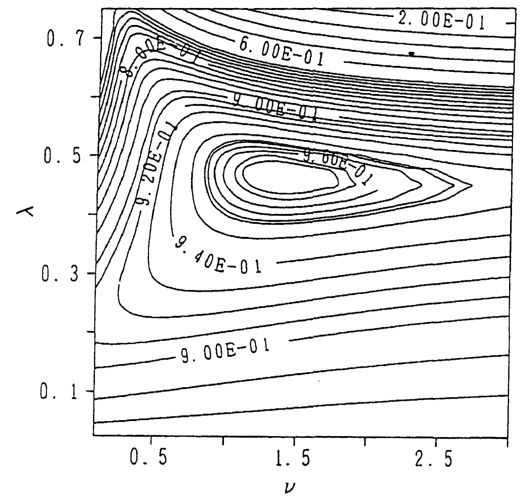

Fig.2 Reliability contour lines as a function of λ and ν : 3 d.o.f. elastic system

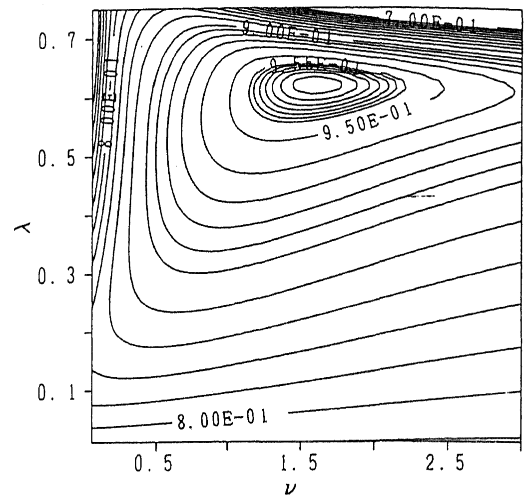

Fig.4 Reliability contour lines as a function of λ and ν : 5 d.o.f. linear system

Parametric survey calculations were also made for the given parameters, the optimum combination of which gives for the maximum seismic reliability of the system the following:

$$\lambda = 0.475, \nu = 1.40 \quad (20)$$

a combination that agrees quite well with that given in the paper by T.Kobori et al.(1970). Here, the optimum parameters based on the presented approach are

$$\lambda = 0.466, \nu = 1.40 \quad (21)$$

In Figs. (3) and (4), the similar perspective and contour lines of the reliability as in Figs. (1) and (2) are plotted for the case of five-degree-of-freedom structural systems, where $T_1 = 1.0sec$, $\sigma_f = 145/3gal$, $\omega_g = 18rad/sec$, $h_g = 0.5$.

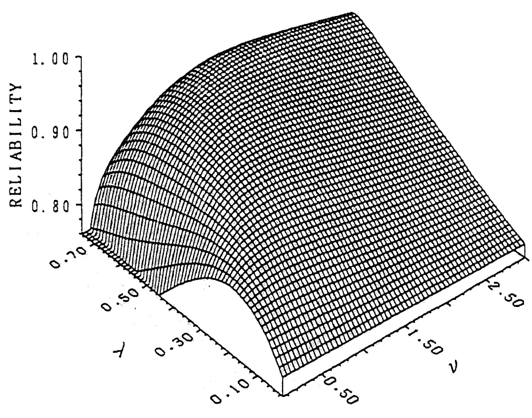

Fig.3 Reliability perspective as a function of λ and ν : 5 d.o.f. elastic system

These figures indicate that the general variation of the perspective and contour lines is similar to tose of three-degree-of-freedom systems, but that the optimum combination of the indices λ and ν slightly differs supposedly dut to the higher order modal response contribution to the total response.

The optimum combination of the indices giving the maximum seismic reliability based on the parametric survey calculation is

$$\lambda = 0.625, \nu = 1.60 \quad (22)$$

while the optimum one based on the presented approach is

$$\lambda = 0.620, \nu = 1.625 \quad (23)$$

The mutual consonance of the indices in Eq.(22) and (23) is also quite satisfactory.

In Figs. (5) and (6), the optimum combination of the indices (λ , ν) for the case of three- and five-degree-of-freedom systems are plotted respectively by choosing the angular frequency ratio $\rho = \omega_g/\omega_1$ as a parameter , where $\omega_1 = 2\pi/T_1$ is the fundamental frequency of the structural system and the used symbols for the ρ are shown upper hight-hand just outside the figures.

These figures indicate that the smaller value of ρ gives the larger and smaller value of λ and ν respectively, while the larger value of ρ gives the smaller and larger value of λ and ν respectively. This means that the softer the soil-ground becomes, the more flexible the structure becomes; i.e. the larger the optimum stiffness tapering becomes. Excitation parameters h_g and σ_f other than ω_g have very little influence on the optimum combination of the indices (λ ,ν), and the associated results with these facts are omitted here due to the space limitaion.

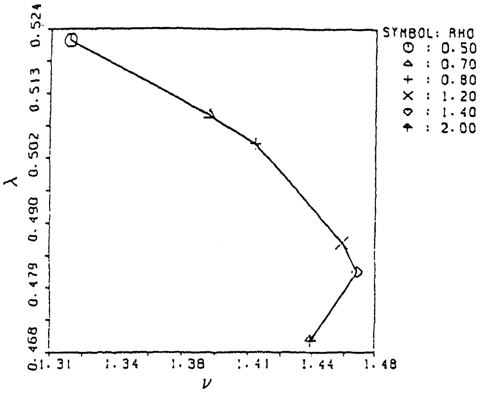

Fig.5 Optimum combinations of indices λ and ν choosing frequency ratio ρ as a parameter

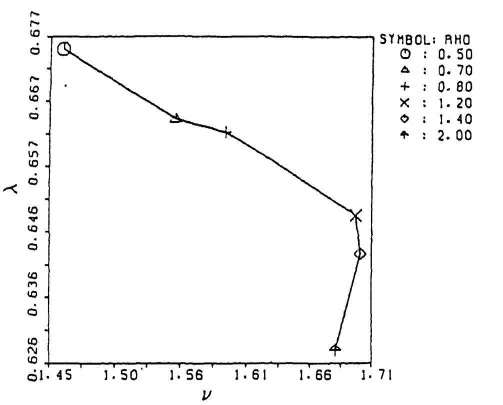

Fig.6 Optimum combinations of indices λ and ν choosing frequency ratio ρ as a parameter

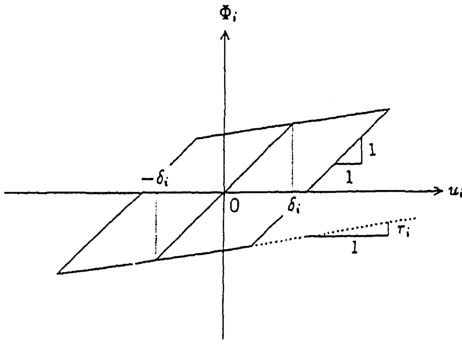

Fig.7 Bilinear hysteretic characteristics

3.2 ELASTO-PLASTIC STRUCTURAL SYSTEMS

Here is discussed the optimum distribution of dynamic elasto-plastic structural properties by taking the bilinear hysteretic characteristics Φ_i shown in Fig. (7) as

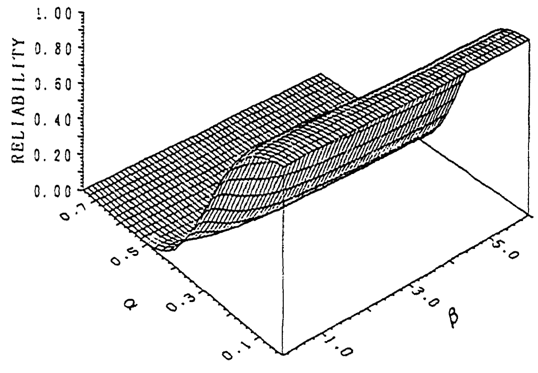

Fig.8 Reliability perspective as a function of α and β : 3 d.o.f. bilinear system

an example where u_i is the i-th interstory displacement.

The parameters considered here are the plastic to elastic stiffness ratio r_i and the elastic-limit deformation δ_i, the distribution of which is given as

$$\{\delta_i\} = \overline{\delta}\left\{1 - \alpha\left(\frac{i-1}{n-1}\right)^{\beta}\right\} \tag{24}$$

where $\overline{\delta}$ is a standardized elastic-limit deformation, and α and β are the similar indices defining the distribution of $\{\delta_i\}$ as in Eq.(24). The elasto-plastic structural system has three-degree-of-freedom, the optimum elastic stiffness distribution just derived above, the uniform mass distribution, the elastic modal critical damping ratio = 0.05 and the elastic fundamental period = $1.0sec$. The excitation parameters $\sigma_f = 280/3gal$, $\omega_g = 18$ rad/sec and h_g = 0.5 are used for numerical calculations, where the structural system with bi-linear hysteretic characteristics were replaced with the stochastic equivalentl linearizatin system developed by Asano et al. (1984).

The elements of the coefficient matrix of [A] corresponding to those in Eq.(5) are expressed for this case in terms of the elastic stiffness and associated damping plus the equivalent ones for bilinear hysteretic characteristics of the respective story. Similar numerical calculations were made on the basis of Eq.(5) ,(7) and (8).

In Figs. (8),(9) and (10),(11), the perspective and corresponding contour lines of the reliability of bi-linear hysteretis structural systems with r_i = 0.1 and 0.5 are plotted respectively as a function of α and β in Eq.(24) utilizing a parametric survey caluculation. These figures indicate that the reliability is highly insensitive to the index β , while very sensitive to the index α regardless of the value r_i. And the reliability attenuates very rapidly with the increse of the index α from zero to 0.5 , which suggests us the optimum elastic-deformation distribution equal to constant,i.e.

$$\{\delta_i\} = \{\overline{\delta}\} \tag{25}$$

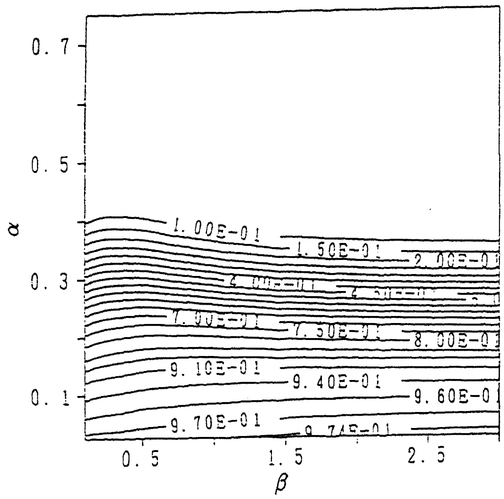

Fig.9 Reliability contour lines as a function of α and β : 3 d.o.f. bilinear system

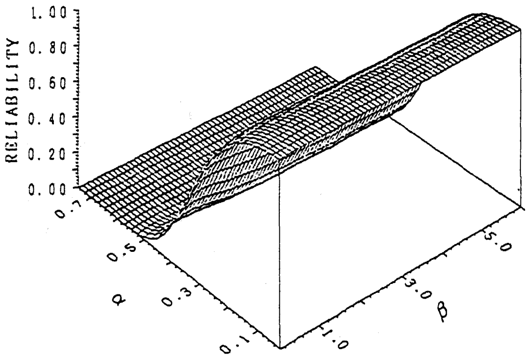

Fig.10 Reliability perspective as a function of α and β : 3 d.o.f. bilinear system

4 CONCLUSIONS

This paper has developed an analytical approach to dynamic design parameters for elastic and elasto-plastic structural systems based on the optimum seismic reliability, and presented illustrative numerical results. The conclusions are summarized as follows:

1. Probabilistic earthquake response was estimated as a solution of simple simultaneous algebraic equations by assuming an earthquake-like random excitation to be a stationary random process with a given excitation level, predominant angular frequency and spectral shaping factor.

2. Based on this probabilistic earthquake response, seismic elastic structural reliability and its associated derivative with respect to design variables were evaluated, and the optimum parameters determined.

3. The validity of the approach was demonstrated by examining the perspective and contour lines of the

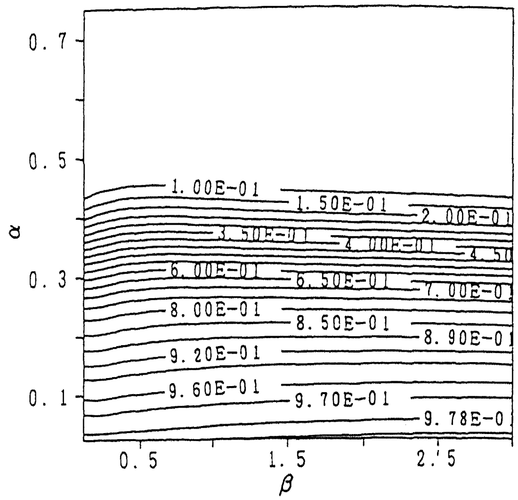

Fig.11 Reliability contour lines as a function of α and β : 3 d.o.f. bilinear system

seismic reliability as a function of design parameters for simple elastic structural systems with three- and five-degree-of-freedom.

4. While the predominant angular frequency has major influence on the optimum elastic parameters, other excitation parameters than this have very little influence.

5. The seismic elasto-plastic structural reliability were evaluated, and the optimum parameters for bilinear hysteretic systems were determined using a parametric survey calculation.

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