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Generalized optimal active control algorithm for nonlinear seismic structures

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ABSTRACT: This paper presents a generalized algorithm for optimal active closed-loop control of nonlinear building structures. Based on the idea that the unknown state variables at the end-point of a time interval should also be minimized, a generalized performance index is selected and the corresponding transversality conditions are derived, so that a time-independent optimal feedback gain matrix is achieved. The state equation of motion is solved in the real time domain by using numerical integration and the concept of unbalanced force correction. Numerical comparisons between the proposed algorithm and the instantaneous algorithm are performed, which indicate that the proposed method is superior in that it is stable while the conventional method is time-dependent.

1 INTRODUCTION

Optimal control can provide protection for building structures from the damaging effects of destructive seismic force or from human discomfort over structural motion induced by strong wind and other types of vibrations. Optimal structural control can be achieved by using passive or active control devices or their combinations. Passive control devices utilize the fact that an energy-dissipating mechanism can be activated by the motion of the structure itself. Active control devices require external energy for their operation. The devices used for active control include: active tendons, active mass dampers, etc.

In recent years, several algorithms for optimal active control of seismic structures have been developed. Among them, instantaneous optimal active closed-loop control algorithm has been studied by J.N. Yang et al (1987), T.T. Soong et al (1987), and F.Y. Cheng et al (1986, 1987, 1988). Application of instantaneous algorithm to nonlinear structures has also been made by J.N. Yang et al (1988). Current studies by F.Y. Cheng (1991) indicate that in the instantaneous optimal closed-loop control algorithm is time-dependent on the incremental time intervals used in the response analysis. Using different time intervals yields various control forces and structural responses of a given structure subjected to the same earthquake. Thus, control effectiveness cannot be ensured for a structure subjected to different earthquakes during its lifetime.

In this paper a generalized optimal active closed-loop control algorithm for seismic nonlinear structures is proposed. Based on the idea that the unknown state variables at the end-point of a time interval should also be minimized, a generalized performance index is selected and the corresponding transversality conditions are derived, so that a time-independent optimal feedback gain matrix is achieved. The state equation of motion is solved in the real time domain by using numerical integration and the concept of unbalanced force correction.

2 FORMULATIONS

2.1 Motion equation for active tendon control

The motion equation for a one-dimensional N-story nonlinear structure equipped with active tendons at some floors, as shown in Fig.1, can be expressed as

$$[M]\{\ddot{x}(t)\}+\{F_D(t)\}+\{F_K(t)\} = [\gamma]\{u(t)\}+\{\delta\}\ddot{X}_g(t) \quad (1)$$

where [M] is N × N mass matrix; {x(t)} of N × 1 and {u(t)} of r × 1 are relative displacement and control force vectors, respectively, where r is the number of active controllers; and [γ] of N × r and {δ} of N × 1 are location matrix for control force and coefficient vector for the earthquake ground

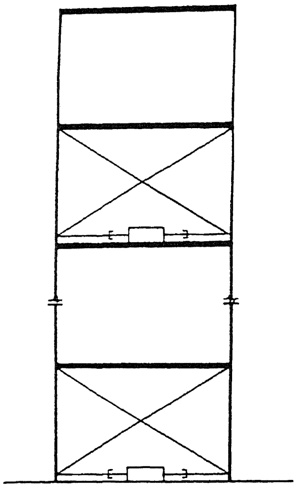

Figure 1. Building structure equipped with active tendon

acceleration, $\ddot{X}_g(t)$; $\{F_D(t)\}$ and $\{F_K(t)\}$ of $N \times 1$ are damping and restoring force vectors, respectively, which can be approximated by the following expression

$$\begin{aligned}\{F_D(t)\}&=\{F_D(t-\Delta t)\}+[C(t-\Delta t)](\{\dot{x}(t)\}-\{\dot{x}(t-\Delta t)\})\\ \{F_K(t)\}&=\{F_K(t-\Delta t)\}+[K(t-\Delta t)](\{x(t)\}-\{x(t-\Delta t)\})\end{aligned} \quad (2)$$

where $[C(t-\Delta t)]$ and $[K(t-\Delta t)]$ of $N \times N$ are damping and stiffness matrices at time $t-\Delta t$, the coefficients of which are defined by

$$\left.\begin{aligned}c_{ij}(t-\Delta t)&=\frac{F_{Di}(t-\Delta t)-F_{Di}(t-2\Delta t)}{\dot{x}_j(t-\Delta t)-\dot{x}_j(t-2\Delta t)}\\ k_{ij}(t-\Delta t)&=\frac{F_{Ki}(t-\Delta t)-F_{Ki}(t-2\Delta t)}{x_j(t-\Delta t)-x_j(t-2\Delta t)}\end{aligned}\right\} \quad (3)$$

Substitution of Eq.(2) into Eq.(1) yields

$$\begin{aligned}&[M]\{\ddot{x}(t)\}+[C(t-\Delta t)]\{\dot{x}(t)\}+[K(t-\Delta t)]\{x(t)\}\\ &=\{F(t-\Delta t)\}+[\gamma]\{u(t)\}+\{\delta\}\ddot{X}_g(t)\end{aligned} \quad (4)$$

where

$$\begin{aligned}\{F(t-\Delta t)\}=&[C(t-\Delta t)]\{\dot{x}(t-\Delta t)\}+[K(t-\Delta t)]\{x(t-\Delta t)\}\\ &-\{F_D(t-\Delta t)\}-\{F_K(t-\Delta t)\}\end{aligned} \quad (5)$$

By defining the state-vector

$$\{z(t)\}=\begin{Bmatrix}\{x(t)\}\\ \{\dot{x}(t)\}\end{Bmatrix} \quad (6)$$

Eqs. (1) and (2) can be combined as

$$\begin{aligned}\begin{Bmatrix}\{\dot{x}(t)\}\\ \{\ddot{x}(t)\}\end{Bmatrix}=&\left[\begin{array}{c|c}[0] & [I]\\ \hline -[M]^{-1}[K] & -[M]^{-1}[C]\end{array}\right]\begin{Bmatrix}\{x(t)\}\\ \{\dot{x}(t)\}\end{Bmatrix}\\ &+\begin{bmatrix}[0]\\ \hline [M]^{-1}[\gamma]\end{bmatrix}\{u(t)\}+\begin{Bmatrix}\{0\}\\ \hline [M]^{-1}\{\gamma\}\end{Bmatrix}\ddot{X}_g(t)\\ &+\begin{Bmatrix}[0]\\ \hline [M]^{-1}\{F\}\end{Bmatrix}\end{aligned} \quad (7)$$

in which $[K]=[K(t-\Delta t)]$, $[C]=[C(t-\Delta t)]$ and $\{F\}=\{F(t-\Delta t)\}$.

In compact form, Eq. (3) becomes

$$\{\dot{z}(t)\}=[A]\{z(t)\}+[B]\{u(t)\}+\{C\}\ddot{X}_g(t)+\{F\} \quad (8)$$

2.2 Generalized performance index

Suppose that the performance index is to be minimized in the time interval $[t_{i-1}, t_i]$. Since the values of the state vector at the right end-point t_i, $\{z(t_i)\}$, are unknown, which implies that the problem involved is a free end-point boundary value problem, $\{z(t_i)\}$ should be minimized, i.e., a function of $\{z(t_i)\}$ should be included in the expression of the performance index. Therefore, a new performance index J_i, called generalized performance index, is proposed as follows

$$\begin{aligned}J_i=&g(\{z(t_i)\})\\ &+\frac{1}{2}\int_{t_{i-1}}^{t_i}(\{z(t)\}^T[Q]\{z(t)\}\\ &+\{u(t)\}^T[R]\{u(t)\})dt\\ =&g(\{z(t_i)\})+\int_{t_{i-1}}^{t_i}\bar{f}(t)\,dt\end{aligned} \quad (9)$$

where $[Q]$ is a $2N \times 2N$ positive semidefinite matrix; $[R]$ is an $r \times r$ positive definite matrix; and $g(\{z(t_i)\})$ could be chosen in the form of

$$g(\{z(t_i)\})=\{z(t_i)\}^T[S]\{z(t_i)\} \quad (10)$$

in which $[S]$ is a $2N \times 2N$ positive semi-definite matrix.

2.3 Transversality conditions

Since the problem involved is a free end-point boundary value problem, in order to minimize the generalized performance index, J_i, not only Euler Equations, but also the transversality conditions should be met at the end-point ti.

Suppose that the end conditions relating the end-point variables are given by

$$\left.\begin{aligned}&t_{i-1}=t_o+\sum_{k=1}^{i-1}\Delta t_k\\&\{z(t_{i-1})\}=\{z_{i-1}\}\\&t_i=t_o+\sum_{k=1}^{i}\Delta t_k\end{aligned}\right\} \tag{11}$$

where Δt is the time increment.

Eq. (11) can also be written in the following form

$$\{\Omega\}=\begin{Bmatrix}\Omega_1\\\{\Omega_2\}\\\Omega_3\end{Bmatrix}=\begin{Bmatrix}(t_{i-1}-t_o)-\sum_{k=1}^{i-1}\Delta t\\\{z(t_{i-1})\}-\{z_{i-1}\}\\(t_i-t_o)-\sum_{k=1}^{i}\Delta t\end{Bmatrix}=\{0\} \tag{12}$$

By introducing multipliers $\{\mu\}$ and $\{\lambda\}$ and forming the following augmented functions

$$G=g+\{\mu\}^T\{\Omega\} \tag{13}$$

$$\begin{aligned}F=&\bar{f}+\{\lambda(t)\}^T([A]\{z(t)\}+[B]\{u(t)\}\\&+\{C\}\ddot{X}_g(t)+\{\bar{F}\}-\{\dot{z}(t)\})\\=&\bar{f}+\{\lambda(t)\}^T\{f(t)\}\end{aligned} \tag{14}$$

the transversality condition can be expressed as

$$dG-\left(\left\{\frac{\partial F}{\partial\{\dot{z}(t)\}}\right\}^T\{\dot{z}(t)\}-F\right)dt\Bigg|_{t_{i-1}}^{t_i}+\left\{\frac{\partial F}{\partial\{\dot{z}(t)\}}\right\}^T\cdot d\{z(t)\}\Bigg|_{t_{i-1}}^{t_i}=0 \tag{15}$$

Substitution of Eqs.(13) and (14) into Eq.(15) yields

$$[S]\{z(t_i)\}-\{\lambda(t_i)\}=\{0\} \tag{16}$$

2.4 Determination of feedback gain matrix

By applying Euler equation in Eq.(9), the following characteristic equations can be obtained

$$[Q]\{z(t)\}+[A]^T\{\lambda(t)\}+\{\dot{\lambda}(t)\}=\{0\} \tag{17}$$

$$[R]\{u(t)\}+[B]^T\{\lambda(t)\}=\{0\} \tag{18}$$

Eq. (18) can be rewritten as

$$\{u(t)\}=-[R]^{-1}[B]^T\{\lambda(t)\} \tag{19}$$

For a closed-loop control system, the relation between the state vector $\{z(t)\}$ and the control force vector can be given by

$$\{u(t)\}=[G]\{z(t)\} \tag{20}$$

where $[G]$ is called feedback gain matrix.

By combining Eqs.(19), (20) and transversality condition Eq.(16) the expression of the feedback gain matrix at each end-point t_i is obtained as follows

$$[G(t_i)]=-[R]^{-1}[B]^T[S] \tag{21}$$

It is noted that the feedback gain matrix $[G(t_i)]$ is actually a constant matrix, it is neither a function of time t_i nor a function of time increment Δt. Therefore, during the computation process, Δt can be arbitrarily changed within the range of precision.

It is also noted that the weighting matrix $[Q]$ makes no contribution to the feedback gain matrix. Therefore, for simplicity, $[Q]$ can be chosen as $[0]$.

Furthermore, it can be seen that if $[S]$ is chosen to be the algebraic Ricatti matrix $[P]$, i.e., $[S]=[P]$, the feedback gain matrix can be written as

$$[G]=-[R]^{-1}[B]^T[P] \tag{22}$$

which is the same as the gain matrix of the Ricatti closed-loop control algorithm. Therefore, the Ricatti closed-loop control algorithm is obviously included in this generalized algorithm.

2.5 Solution technique

By substituting Eqs.(20) and (21) into Eq.(8) and employing Wilson-θ method, the solution of the state equation of motion can be derived as

$$\begin{aligned}\{z(t)\}=&([I]+[A_2][R]^{-1}[B]^T[S])^{-1}\\&\cdot(\{D(t-\Delta t)\}+\{A_1\}\ddot{X}_g(t))\end{aligned} \tag{23}$$

where

$$\begin{aligned}\{D(t-\Delta t)\}=&[A_3]\{z(t-\Delta t)\}+[A_4](\{F_D(t-\Delta t)\}\\&+\{F_S(t-\Delta t)\})+[A_5]\{u(t-\Delta t)\}\\&+\{A_6\}\ddot{X}_g(t-\Delta t)\end{aligned} \tag{24}$$

in which matrices $[A_i]$ $(i=1,2,\ldots,6)$ are functions of $t-\Delta t$.

3 NUMERICAL ILLUSTRATION

3.1 Structural models

A single DOF structural model is used to illustrate the fundamental behavior of the control algorithms. Structural properties of the model are: floor mass M = 345.6 tons; translational stiffness K = 3.404×10^5 kN/m; and linear viscous damping coefficient C = 734.3 kN sec/m; the undamped frequency and damping ratio are 5HZ and 0.034, respectively; the yielding deformation for each of

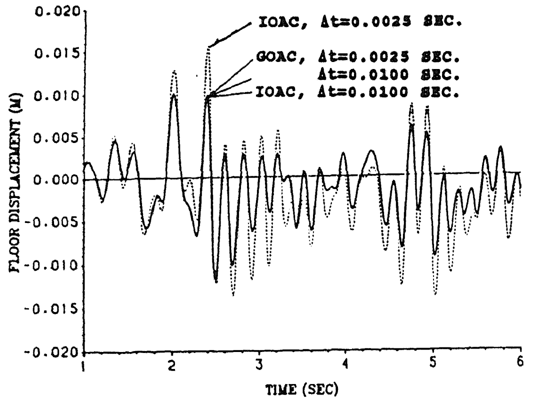

Figure 2.(a) Influence of time increment on floor displacements

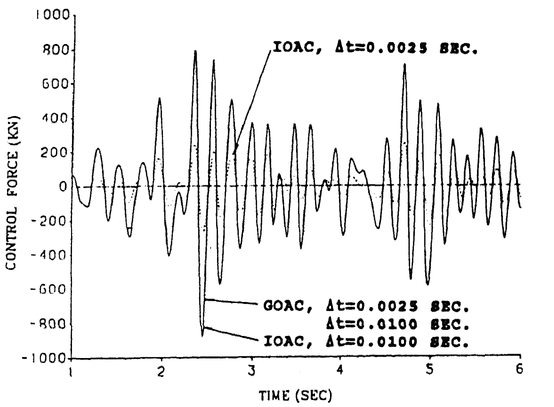

Figure 2.(b) Influence of time increment on control force

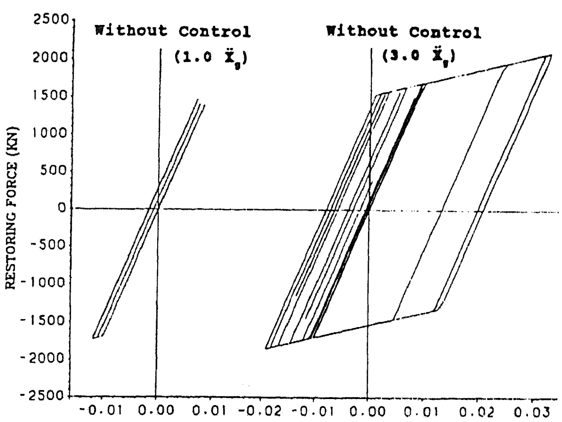

Figure 3. Hysteresis loop for uncontrolled structure

the columns is 1.0cm. N-S component of El-Centro earthquake of May 18, 1940 is used as ground acceleration input. The structure is equipped with active tendon. Both instantaneous (IOAC) and generalized optimal active control (GOAC) of nonlinear closed-loop algorithms are employed in the study. In the IOAC algorithm, the ratio between the elements of the diagonal [Q] matrix, $q_1 = q_2$, and the element of [R] matrix, r_1, is chosen to be 2×10^8; in the GOAC algorithm, the ratio between the elements of the diagonal [S] matrix, $s_1 = s_2$, and the element of [R] matrix, r_1, is chosen to be 1×10^6.

3.2 Comparison of response by IOAC with GOAC

Figures 2(a) and (b) show the influence of time-increment Δt on the floor displacement and control force of the structure subjected to the earthquake excitation with magnitude increased by a factor of 2.0. It can be seen that for IOAC algorithm the influence of Δt is remarkable. When two different time increments of 0.0025 and 0.01 sec. are used, the responses of displacements and control forces associated with the two time increments are significantly different. However, Δt does not influence the response for the GOAC algorithm when the two different time increments are employed. Apparently, IOAC is time-dependent and sensitive to time increment while GOAC is independent of varying time increments. In structural analysis, the time increments of the selected earthquake records could be different. Therefore, the response and control force of the structure should both be free from the variable Δt.

3.3 Active control effectiveness on nonlinear structures

Figs.3(a) and (b) show the columns' hysteresis loops of the uncontrolled structure subjected to 1.0 and 3.0 times of the earthquake acceleration, respectively; while Figs.4(a) and (b) show those of the controlled structure. These figures indicate that the active control effectiveness is very significant: for the first case (1.0 $\ddot{X}_g$), the floor relative displacement is reduced from 1.19 cm to 0.59 cm; for the second case (3.0 $\ddot{X}_g$), the displacement is reduced from 3.27 cm to 2.16 cm.

3.4 Earthquake response of the actively controlled structure

Fig.5 gives the relative floor displacements of the uncontrolled and controlled structure under 3.0 times of the earthquake acceleration. By comparison it can be seen that using active control devices the inelastic response of structures can be significantly reduced.

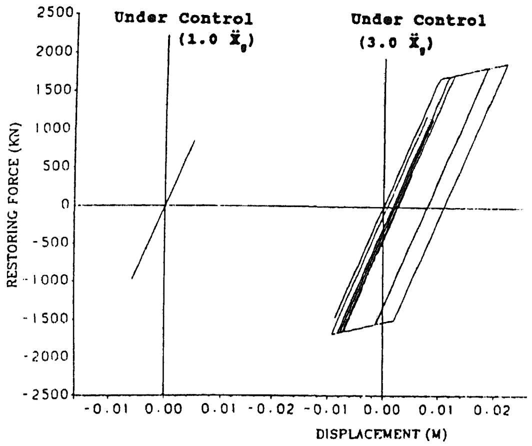

Figure 4. Hysteresis loop for the actively controlled structure

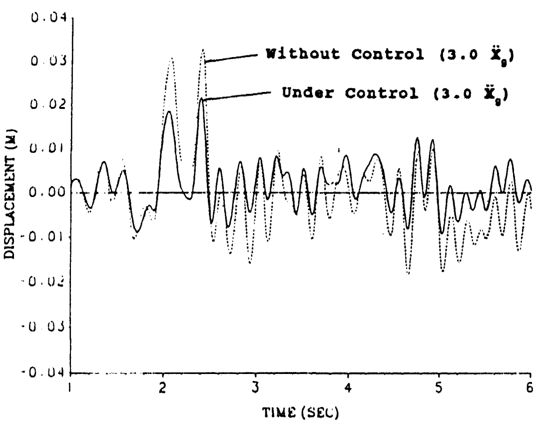

Figure 5. Comparison of the displacements between uncontrolled and controlled structure

Figs.6 (a) and (b) show the displacements and control forces of the actively controlled structure subjected to 1.0 and 3.0 times of the earthquake acceleration, respectively.

4 CONCLUSIONS

This paper presents a generalized optimal active control algorithm for nonlinear seismic structures. By introducing a generalized performance index and employing the transversality conditions, a generalized feedback gain matrix is derived. Unlike the instantaneous algorithm, this generalized feedback gain matrix is not dependent on incremental time intervals. Therefore, if the generalized algorithm is employed, the control force and structural response will be identical when different time increments are used.

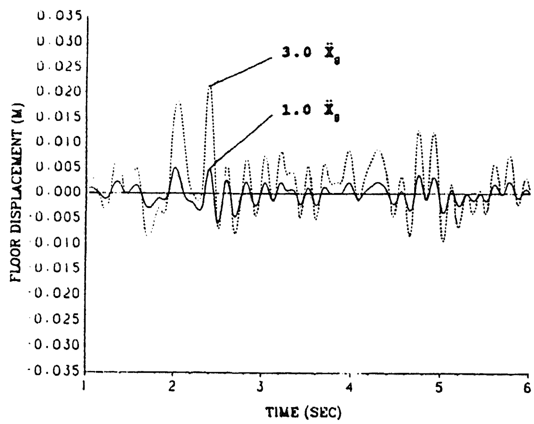

Figure 6.(a) Displacements under different earthquake ground motion

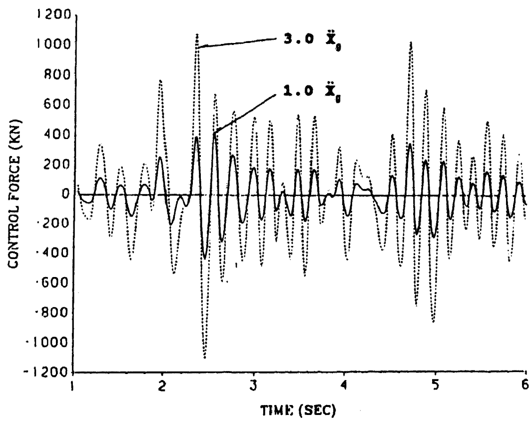

Figure 6.(b) Control force under different earthquake ground motion

Furthermore, during the computation process, the time increment can be arbitrarily changed within a range of precision not leading to discontinuous results.

It is also found that this generalized approach can match the algebraic Ricatti matrix method if the weighting matrix [S] is properly selected. The numerical examples show the effectiveness of the active control on nonlinear structures. By using active devices, the inelastic deformation of structures can be significantly reduced.

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