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Structural safety assessment of building structures under earthquake hazard by the stochastic equivalent linearization method

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ABSTRACT: An efficient and fairly accurate methodology for the evaluation of the probability of failure of nonlinear hysteretic degrading reinforced concrete building structures under earthquake hazard, in which the stochastic equivalent linearization method and the differential hysteretic models play a very important role, is presented. The structural modelling is based on an association of a shear-building with hysteretic columns with a shear-wall connected by hinged bars with infinite axial stiffness. The theoretical development is followed by a small numerical application in order to emphasize the practical interest of the method.

1 INTRODUCTION

The design and safety checking of structures to withstand earthquake actions is a problem of utmost difficulty in civil engineering due to the uncertainties in the evaluation of the seismic hazard, the complexity of the earthquake vibrations, the dynamic nature of the structural response and the need to exploit the energy dissipating capacity of the structures to ensure, under acceptable economic conditions, their survival under strong ground motions. Moreover, the structural stochasticity and the possibility of consideration of different levels of sophistication of structural modelling are aspects that still increase the complexity of the problem.

It is thus understandable that very sophisticated methods of dynamic analysis have been developed (Cunha, 1990; Roberts and Spanos, 1990; Casciati and Faravelli, 1991) to enable the designers to predict the seismic structural behaviour and that the use of those methods has been increasingly recognized in the earthquake resistant regulations, namely the Eurocode 8, the future unified European seismic code.

It is however fundamental to associate those methods of structural analysis with appropriate design and safety checking techniques, what can be done at different levels, according to the sophistication of the algorithms used and to the complexity of the structural models (Duarte, 1991).

The safety assessment of nonlinear hysteretic degrading reinforced concrete building structures can be performed by idealizing the material nonlinearity as a variation of the Takeda model (Campos-Costa, 1990), using a digital simulation technique and a step-by-step integration scheme to evaluate the nonlinear structural response and introducing the concept of vulnerability function (Duarte, 1991).

This paper presents an alternative methodology, fairly accurate and rather less time consuming, for the evaluation of the probability of failure of this kind of structures under seismic actions in which the stochastic equivalent linearization method (Cunha, 1991) and the differential hysteretic models (Wen, 1989) play a very important role.

2 STRUCTURAL DESIGN AND SAFETY CHECKING. THE CONCEPT OF VULNERABILITY FUNCTION

The development of very powerful methods to analyse the behaviour of nonlinear hysteretic degrading structures calls for the improvement of criteria of design and safety checking of structures under severe earthquake actions, what implies an appropriate stochastic modelling of the seismic excitations, the establishment of relations between the main characteristics of the excitation and suitable indicators of the severity of the earthquake effects in structures and a convenient definition of a set of structural requirements (no collapse or serviceability requirements).

Although this safety assessment can be performed at different levels opening a large spectrum of design procedures, as it has been recently emphasized by Duarte (1991), the evaluation of the probability of failure as an indicator of structural performance is commonly recommended.

One way of evaluating that probability of failure is based on the introduction of the concept of vulnerability function ($c_i = V_i(\alpha)$) which relates some intensity measure of the excitation (α) with a convenient response control variable (c_i). These quantities are given by functionals that establish the mapping of the time histories either of the ground motion acceleration ($\ddot{u}_g(t)$) or of the structural response quantities ($r_i(t)$) into scalar values.

Although there is a large number of possibilities for selecting such descriptive functionals, the intensity measure of the earthquake action is currently assumed as the peak ground acceleration, whereas the response control variables can be the average values of the absolute maximum responses, $E[\max|r_i(t)|]$, in some time interval T.

The structural response time histories $r_i(t)$ must be selected so that the corresponding control variables c_i constitute appropriate indicators of the structural damage. Therefore the variables c_i are often assumed as maximum deformations (e.g. maximum interstorey drifts (Algan, 1982)), taking into account the existing relationship between damage and ductility ratio. It is worth mentioning however that a better option can be the consideration of a damage index that, beyond evaluating the ductility demand, also quantifies the influence of low cycle fatigue, e.g. the index presented by Park, Ang and Wen (1987):

$$D_i = \frac{|r_i|_{max}}{r_u} + \frac{\beta \int dW_d}{F_{y_i} r_u} \qquad (1)$$

where: D_i - damage index ($D_i \geq 1$ means collapse); $|r_i|_{max}$ - maximum deformation; r_u - ultimate deformation under monotonic load; β - non-negative constant; F_{y_i} - yield strength; dW_d - incremental dissipated hysteretic energy.

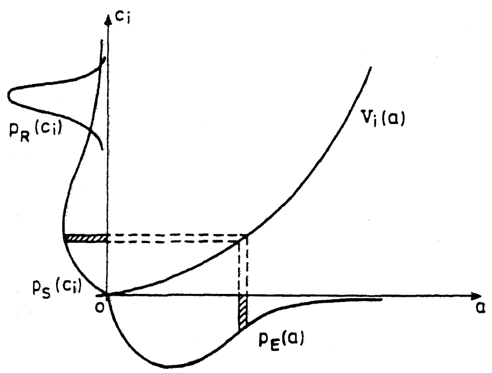

Figure 1. Probability of failure evaluation based on the vulnerability function.

The knowledge of the vulnerability function $c_i = V_i(a)$ permits the evaluation of the probability density function $p_S(c_i)$ of the random variables c_i that measure the severity of the seismic effects, provided that the probability density function of the seismic intensity measure $p_E(a)$ can be defined based on an appropriate probabilistic model of the earthquake action (Oliveira, 1979).

The probability of failure P_f can be then quantified based on the simultaneous knowledge of the probabilistic density functions $p_R(c_i)$ of the structural resistant properties expressed in terms of the control variables c_i, as

$$P_f = \int_0^{\infty} p_S[V_i(a)] \int_0^{V_i(a)} p_R(c_i) dc_i dV \qquad (2)$$

3 GROUND MOTION STOCHASTIC MODELLING

An earthquake induced strong ground motion can be usually modelled as a sample of finite duration of a zero mean stationary filtered white noise whose frequency content is described by a Kanai-Tajimi spectrum of the form

$$S_{\ddot{u}_g}(w) = S_0 \frac{w_f^4 + 4\xi_f^2 w_f^2 w^2}{(w_f^2 - w^2) + 4\xi_f^2 w_f^2 w^2} \qquad (3)$$

where S_0 is the intensity scale factor of the PSD function and w_f and ξ_f are shape parameters (filter coefficients) dependent on the epicentral distance, the earthquake magnitude and the ground layer rigidity.

Based on the Fourier amplitude spectra for the strong motion phase and treating a large number of strong motion records, Moayyad and Mohraz (1982) obtained the power spectra for soft, intermediate and hard grounds. Sues, Wen and Ang (1983) evaluated the appropriate Kanai-Tajimi parameters using a least squares technique and introduced correction factors (scale factors) F_f in the evaluation of the variance given by the area of that spectrum, in order to eliminate the meaningless contribution of the high frequencies. Thus, that variance can be evaluated as (Chu, 1985)

$$\sigma_{\ddot{u}_g}^2 = F_f S_0 \frac{w_f \pi}{2\xi_f}(1 + 4\xi_f^2) \qquad (4)$$

where F_f assumes the values: 0.81 (soft), 0.83 (intermediate) and 0.79 (hard soils).

Considering the peak ground acceleration $E[\max|\ddot{u}_g|]$ as the descriptive functional corresponding to the intensity measure of the earthquake action (a), one can write according to Vanmarke and Lai (1980) that

$$a = E[\max|\ddot{u}_g|] = p_{\ddot{u}_g} \sigma_{\ddot{u}_g} \qquad (5)$$

with the peak factor

$$p_{\ddot{u}_g} = \begin{cases} \sqrt{2\ln(2T_d/T_f)} & , \quad T_d \geq 1.36T_f \\ \sqrt{2} & , \quad T_d < 1.36T_f \end{cases} \qquad (6)$$

where T_d is the duration of the strong motion phase of the ground excitation and T_f is the dominant period of the ground motion. Using mean values of T_d, Sues, Wen and Ang (1983) concluded that the peak factor is almost insensitive to the duration and predominant period of the ground motion and suggested the following approximate values: $p_{\ddot{u}_g} = 3$ (soft) and $p_{\ddot{u}_g} = 2.9$ (intermediate and hard soils).

On the other hand, seismicity studies can also lead to the knowledge of a probability density function of the peak ground acceleration $p_E(a)$ (Oliveira, 1979; Campos-Costa, 1992).

The non-stationary nature of the seismic excitation can still be considered using appropriate deterministic modulating functions that take into account the time variability of the frequency content and/or of the earthquake intensity (Cunha, 1990).

4 MATERIAL NONLINEARITIES IDEALIZATION AND STRUCTURAL MODELLING

Two types of models may be used to idealize material nonlinearities (Duarte, 1991). The first type assumes a concentration of nonlinearities in some parts of the structure (e.g. a plastic hinge), whose behaviour is governed by a defined relationship between a small number of kinematic variables and the corresponding internal forces. This can be done, for reinforced concrete structures, using the Takeda model or its interesting variation used in LNEC (Campos-Costa, 1990). The second type considers a discretization of some parts of the structure in many small elements governed by known stress-strain relationships. This is the case of the filament model used by Vaz (1990), based on force-deformation loops defined for the filaments (e.g. Giuffré-Pinto model for steel and Kent-Park model for concrete).

When the stochastic equivalent linearization method is employed to analyse the random structural response, it is extremely convenient however to use differential hysteretic models (Wen, 1988; Roberts and Spanos, 1990) what can easily permit to overcome the strong restrictions imposed by the well known Krylov-Bogoliubov assumption (Caughey, 1960).

Moreover, such kind of models still have the remarkable virtue of permitting to consider stiffness and strength degradation (Baber and Wen, 1980), pinching effects (Noori, 1984) and bidirectional bending (Park *et al.*, 1986), although some criticism can also arise from their incapability of total agreement with the classic plasticity theory, namely in terms of possible violations of the complementary rule or of the Drucker postulate (Casciati and Faravelli, 1991).

On the other hand, the structural modelling of hysteretic degrading structures may also be performed at different levels of complexity when methods of stochastic structural dynamics are applied.

The shear-building is a very common model specially suitable for the study of plane frames whose beams have a rather high stiffness and strenght, being the nonlinear behaviour essentially restricted to the columns (Sues, Wen and Ang, 1983). More sophisticated plane and three dimensional models with linear bars connected by hysteretic degrading plastic hinges have also been used (Casciati and Faravelli, 1991).

5 STOCHASTIC RESPONSE ANALYSIS OF NONLINEAR SYSTEMS UNDER EARTHQUAKE EXCITATIONS

Consider, as an example, the case of a plane building structure that can be modelled as the association of a shear-building with hysteretic columns with a shear-wall connected by hinged bars with infinite axial stiffness (Fig. 2), submitted to an earthquake ground acceleration idealized by a zero mean gaussian stochastic process $\ddot{u}_g(t)$.

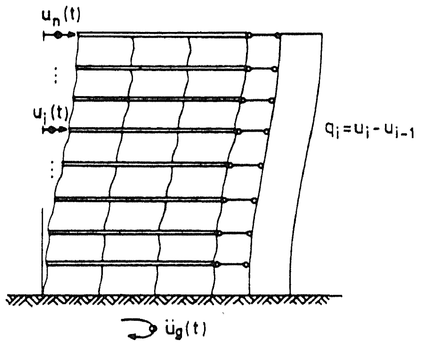

Figure 2. Structural model.

The motion of the i-th floor may be described by the equation

$$m_i\left[\sum_{j=1}^{i}\ddot{q}_j(t) + \ddot{u}_g(t)\right] + c_i\dot{q}_i(t) - c_{i+1}\dot{q}_{i+1}(t) + g_i(t) - g_{i+1}(t) = 0 \qquad (7)$$

where m_i is the mass of the floor, c_i is an appropriate damping constant and the restoring force g_i represents the sum of the contributions of the i-th column level (g_i^{SB}) and of the corresponding portion of the shear-wall (g_i^{SW}).

Using the Bouc-Wen model, g_i^{SB} and g_i^{SW} can be both expressed in the general form

$$g_i^* = \alpha_i^* k_i^* q_i + (1-\alpha_i^*) k_i^* z_i^* \qquad (8)$$

where the corresponding endocrin variables z_i^{SB} and z_i^{SW} are governed by nonlinear differential equations of the type

$$\dot{z}_i^* = A_i^* \dot{q}_i - \beta_i^* |\dot{q}_i| |z_i^*|^{n_i^*-1} z_i^* - \gamma_i^* \dot{q}_i |z_i^*|^{n_i^*} \qquad (9)$$

which can be linearized in the form

$$\dot{z}_i^* = c_{e_i}^* \dot{q}_i + k_{e_i}^* z_i^* \qquad (10)$$

where $c_{e_i}^*$ and $k_{e_i}^*$ are linearization coefficients.

The coloured excitation $\ddot{u}_g(t)$ may be assumed as the absolute response acceleration $\ddot{u}_f^t$ of a linear filter with unit mass, natural frequency w_f and damping factor ξ_f, whose base is subjected to an acceleration idealized by a white noise $\ddot{u}_g^0(t)$ with a spectral density $S_0(t)$, being its motion described in terms of the relative displacement $u_f(t) = u_f^t(t) - u_g^0(t)$ by

$$\ddot{u}_f(t) + 2\xi_f w_f \dot{u}_f(t) + w_f^2 u_f(t) = -\ddot{u}_g^0(t) \qquad (11)$$

Taking into account that equations (7), (8) and (10) can be written in the following matricial forms

$$\underline{M}\ddot{\underline{q}} + \underline{C}\dot{\underline{q}} + (\underline{K}^{SB} + \underline{K}^{SW})\underline{q} + \underline{G}^{SB}\underline{z}^{SB} + \underline{G}^{SW}\underline{z}^{SW} = -\underline{M}_d \underline{1} \ddot{u}_g \qquad (12)$$

$$\dot{\underline{z}}^{SB} = \underline{C}_e^{SB}\dot{\underline{q}} + \underline{K}_e^{SB}\underline{z}^{SB} \qquad \dot{\underline{z}}^{SW} = \underline{C}_e^{SW}\dot{\underline{q}} + \underline{K}_e^{SW}\underline{z}^{SW} \qquad (13)$$

and introducing the state vector $\underline{y}^T = [\underline{q}^T, \dot{\underline{q}}^T, \underline{z}^{SB^T}, \underline{z}^{SW^T}, u_f, \dot{u}_f]$, the hysteretic behaviour of the structure, as well as the motion of the system and the filter may be expressed by matricial relations of the type (Cunha, 1990)

$$\underline{D}\dot{\underline{y}}(t) + \underline{E}\underline{y}(t) = \underline{f}(t) \quad \text{or} \quad \dot{\underline{y}} = \underline{A}_e \underline{y}(t) + \underline{x}(t) \qquad (14)$$

where the matrices $\underline{E}$ and $\underline{A}_e$ depend on the values of the linearization coefficients and $\underline{f}(t)$ or $\underline{x}(t)$ depend on the white noise $\ddot{u}_g^0(t)$.

The evaluation of the covariance matrix $\underline{\mu}_y$ that completely characterizes the probabilistic distribution of the response, assumed as gaussian, can be made solving the first order differential system

$$\dot{\underline{\mu}}_y = \underline{A}_e \underline{\mu}_y + \underline{\mu}_y \underline{A}_e^T + 2\pi \underline{S}_0 \qquad (15)$$

where $\underline{S}_0$ depends on the spectral density of the white noise $\ddot{u}_g^0(t)$.

If the earthquake excitation is idealized as a sample of finite duration of a stationary process, the left hand side of equation (15) vanishes, leading to the well known Liapunov equation, whose solution may be efficiently reached by using the numerical algorithm presented by Bartels and Stewart (1972).

Hence, an iterative procedure may be followed updating the linearization coefficients c_{e_i} and k_{e_i} based on the Atalik conditions (Baber and Wen, 1980) and, indirectly, the state matrix $\underline{A}_e$ until convergence is achieved.

6 MAXIMUM RESPONSE STATISTICS

Although no exact solution for the probabilistic distribution of the absolute maximum responses has been discovered yet, several approximate solutions have been developed, namely the asymptotic approximation of Yang and Liu (1981), which is based on the simulation results obtained by Shinozuka and Yang (1971), showing that the distribution of the nonstationary global extreme in a time interval $[t, t+T]$ may be approximately discribed by a Weibull distribution.

Thus, assuming that the extremes in that interval are statistically independent and that their total number N is large, the mean global extreme value is given by (Chu, 1985)

$$E\left[\max_T |r_i|\right] = (D + \gamma D^{1-\theta})\sigma_{r_i} \qquad (16)$$

where $\gamma = 0,577216$ is the Euler constant, σ_{r_i} is the standard deviation of r_i and

$$D = (\theta \ln N)^{1/\theta} = \left[\theta \ln \int_t^{t+T} 2 f_0^+(t)\,dt\right]^{1/\theta} \qquad (17)$$

being $f_0^+(t)$ the time varying zero upcrossing rate.

If the stochastic process $r_i(t)$ is stationary, the Weibull distribution of the global extreme values reduces to a Rayleigh distribution with $\theta = 2.0$ and equation (16) leads to the well known expression of Davenport.

7 APPLICATION

To illustrate the application of the method previously described for the evaluation of the structural probability of failure using the stochastic equivalent linearization method, a simple example of a 2 storey shear-building with hysteretic columns already studied before by Chang (1985) and Cunha (1990), under different kinds of seismic excitations, was considered.

The masses of each floor are $m_1 = 36310\,Kg$, $m_2 = 18155\,Kg$ and the parameters of the Bouc-Wen model are $k_1 = 1218.6\,KN/m$, $k_2 = 513.7\,KN/m$, $\alpha_1 = \alpha_2 = 0.04$, $A_1 = A_2 = 1.0$, $n_1 = n_2 = 1$, $\beta_1 = \gamma_1 = 11.647/m$ and $\beta_2 = \gamma_2 = 6.562/m$. The damping matrix $\underline{C} = \alpha_c \underline{M}_d + \beta_c \underline{K}_L$ was evaluated considering $\underline{K}_L$ as the initial stiffness matrix, $\alpha_c = 0.263/s$ and $\beta_c = 0.00852s$.

The earthquake excitation was idealized as a sample of finite duration ($T = 10s$) of a stationary filtered white noise characterized by a Kanai-Tajimi spectrum with $\omega_f = 15.56\,rad/s$ and $\xi_f = 0.64$.

The vulnerability function corresponding to the first level of columns was plotted (Fig. 3) considering an incremental variation of the spectral density S_0 and evaluating for each value the average maximum displacement of the first floor using an appropriate computer program developed in Porto University (Cunha, 1990) based on the stochastic equivalent linearization technique. The relation between S_0 and the descriptive functional corresponding to the intensity measure of the earthquake action, $a = E[\max|\ddot{u}_g|]$, was established by equations (4-6) with $F_f = 1$. The ductility demand, ratio between the average maximum displacement and the yielding displacement (ultimate restoring force of the hysteretic component divided by its initial stiffness)

$$q_{y_1} = \frac{(1-\alpha_1)k_1 z_{1\max}}{(1-\alpha_1)k_1 A_1} = \frac{1}{A_1}\left(\frac{A_1}{\beta_1 + \gamma_1}\right)^{1/n_1} \quad (18)$$

was considered as the descriptive functional that measures the severity of the seismic effects, c_1.

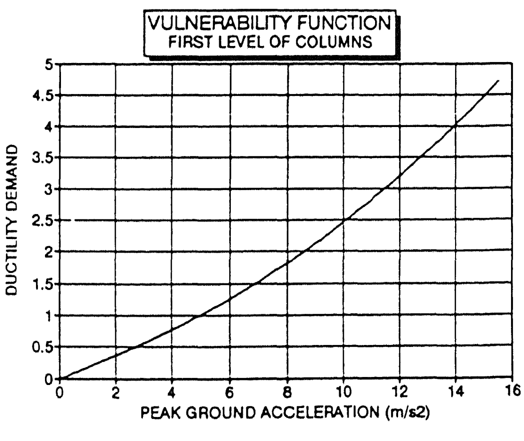

Figure 3. Vulnerability function of the first level of columns.

The probability density function of the seismic intensity measure $p_E(a)$ may be often assumed as a Gumbel function of type I

$$p_E(a) = \nu \ \exp[y - \exp(y)] \quad (19)$$

with $y = -\nu(a-u)$, where ν and u must be chosen according to seismic hazard studies like those developed by Campos-Costa (1992) for several regions of Portugal. Supposing $\nu = 10.24 \times 10^{-3}$ and $u = 57.72$ it was possible to obtain the probability density function $p_S(c_1)$ of the response control variable c_1, plotted in Fig. 4. Considering the ultimate ductility characterized, in this case, by a gaussian distribution (also plotted in Fig. 4) with a mean value $m = 3.0$ and a standard deviation $\sigma = 0.2$, the application of equation (2) leads to a failure probability $P_f = 1.6 \times 10^{-5}$.

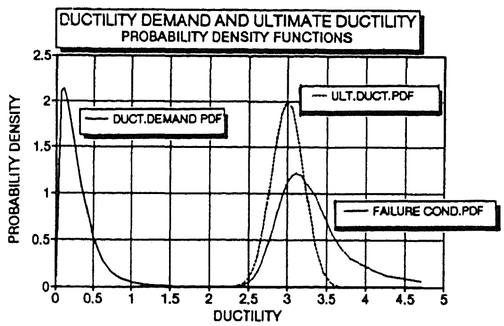

Figure 4. Probability density functions of the ductility demand, $p_S(c_1)$, and of the ultimate ductility, $p_R(c_1)$.

8 CONCLUSIONS

The application of digital simulation techniques for the estimation of vulnerability functions and evaluation of failure probabilities of nonlinear hysteretic degrading reinforced concrete building structures under earthquake hazard presents the remarkable disadvantage of being a too much heavy and time consuming procedure, as the reduction of the statistical uncertainties to an acceptable level can imply the generation and treatment of a large number of realizations.

This inconvenient can be overcome using the fairly accurate and rather efficient methodology presented in this paper, based on the use of the stochastic equivalent linearization technique and suitable differential hysteretic models.

Furthermore, more sophisticated structural models may still be considered and suitable values of behaviour coefficients used as aposteriori correction factors in simplified linear analysis may be easily estimated.

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