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Double spline approximation in nonlinear dynamic response analysis

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ABSTRACT: Double polynomial spline approximation is used for the nonlinear dynamic response analysis of structures with smooth nonlinear hysteretic model. This method is versatile and considerably simpler compared with other methods proposed in the literature. Furthermore, the accuracy of the method is verified against Monte-Carlo simulation for the response levels. The corresponding programs have been compiled for the suggested methods in this paper.

1 INTRODUCTION

Most current structural design codes require the design of structures to resist a certain level of earthquake forces. These designs are either based on a pseudo-static analysis using a response spectrum or a dynamic response analysis using an accelerogram (Clough et al. (1975)). Dynamic response analysis is a complex task due to the structural nonlinear characteristics. In stochastic response analysis of structures, a smooth nonlinear hysteretic system has been presented. The corresponding equivalent linearization method for this system under random excitation has been proposed. In this paper, we introduce this system to analyze the structural nonlinear dynamic response. The accuracy of the approximation is verified against Monte-Carlo simulation. A computer code is developed by the authors specifically for this investigation.

2 SMOOTH HYSTERETIC NONLINEAR MODEL

A smooth hysteretic nonlinear model has been proposed in the literature (Baber and Wen (1979)). Without loss of generality, we concentrate our attention on the response analysis of a single degree of freedom system shown in Fig. 1 having the following equation of motion

$$M\ddot{u}+C\dot{u}+\alpha Ku+(1-\alpha)Kz=F(t) \tag{1}$$

in which u is the displacement relative to the ground motion, and M, C, and K are structural mass, damping, and stiffness, respectively. In (1), F(t) = -M × $\ddot{a}(t)$, with $\ddot{a}(t)$ being an earthquake acceleration record. Furthermore, z is the hysteretic component of the restoring force in the system. It can be constructed by requiring u and z to satisfy the following differential equation:

$$\frac{dz}{du}=A\pm(\gamma\pm\beta)z^{n} \tag{2}$$

in which n, γ, β, and A are constants. (2) can be integrated in close form to obtain a hysteretic relationship between z and $\dot{u}$. The scale and general shape of the hysteretic loop are governed by A, γ, and β, while the smoothness of the force-displacement curve is controlled by n. Therefore, by adjusting the values of these constants, a variety of restoring forces, such as these exhibiting hardening or softening, and those having narrow or wide-band systems, can be constructed. As n increases, the transition region shrinks, and the limiting case of n = ∞ corresponds to a true elastoplastic system.

For convenience in analysis, we set $\xi = \pm(\gamma\pm\beta)$. For different values of z and $\dot{u}$, ξ has different values as

$$for\ z\geq 0,\ \dot{u}\geq 0;\quad \xi=-(\gamma-\beta) \tag{3}$$

$$for\ z\geq 0, \dot{u}<0;\quad \xi=(\gamma+\beta) \tag{4}$$

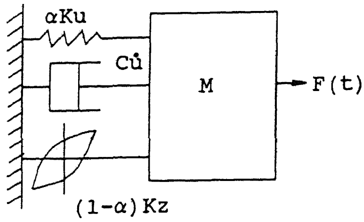

Fig.1 A single degree of freedom system with smooth nonlinear hysteretic system

$$for\ z<0,\dot{u}<0;\quad \xi=(\gamma-\beta) \tag{5}$$

$$for\ z<0,\dot{u}\geq 0;\quad \xi=-(\gamma+\beta) \tag{6}$$

3 DOUBLE SPLINE FUNCTIONS FOR SMOOTH NONLINEAR HYSTERETIC MODEL

Assume the structural displacement u in (1) to be represented by the following third order polynomial function D_3 (cheney (1980))

$$D_3=a_0+\frac{a_1\tau}{1!}+\frac{a_2\tau^2}{2!}+\frac{a_3\tau^3}{3!}=\{1,\frac{\tau}{1!},\frac{\tau^2}{2!},\frac{\tau^3}{3!}\}\{A\} \tag{7}$$

in which

$$\{A\}=\{a_0,a_1,a_2,a_3\}^T \tag{8}$$

The nonlinear hysteretic variable z in (2) is assumed to be represented by the following second order polynomial function H_2 of u

$$H_2=b_0+\frac{b_1u_i}{1!}+\frac{b_2u_i^2}{2!}=\{1,\frac{u_i}{1!},\frac{u_i^2}{2!}\}\{B\} \tag{9}$$

in which

$$\{B\}=\{b_0,b_1,b_2\}^T \tag{10}$$

For different structural and hysteretic characteristics, we generally choose n=1, or n=2 as discussed in the following subsections.

3.1 *Case 1: n=1*

In order to determine the coefficient vectors {A} and {B}, we assume θ=1. The five initial conditions at time t can be given as follows:

$$D_3(0)=a_0=u(t) \tag{11}$$

$$\dot{D}_3(0)=a_1=\dot{u}(t) \tag{12}$$

$$\ddot{D}_3(0)=a_2=\ddot{u}(t) \tag{13}$$

$$b_0+b_1u(t)+\frac{b_2u(t)^2}{2}=z(t) \tag{14}$$

$$\dot{z}(t)=b_1\dot{u}(t)+b_2u(t)\dot{u}(t) \tag{15}$$

Two dynamic equilibrium conditions at time t+Δt can be established as

$$M\ddot{u}(t+\Delta t)+C\dot{u}(t+\Delta t)+Ku(t+\Delta t)+(1-\alpha)Kz(t+\Delta t)=F(t+\Delta t) \tag{16}$$

$$\frac{dz(t+\Delta t)}{du}=A+\xi z(t+\Delta t) \tag{17}$$

in which

$$z(t+\Delta t)=b_0+b_1u(t+\Delta t)+\frac{b_2}{2}u(t+\Delta t)^2=H_2[u(t+\Delta t)] \tag{18}$$

(11) to (18) can be combined to solve for the unknown coefficients. In the following, the two cases, for which $\dot{u}(t)=0$ and $\dot{u}(t)\neq 0$, are discussed separately.

1. For $\dot{u}(t)=0$

$$d_1=M\ddot{u}+C\ddot{u}\Delta t+\alpha Ku+\alpha\frac{K}{2}\ddot{u}\Delta t^2+(1-\alpha)K[z+\ddot{u}\frac{\Delta t^2}{2}(A+\xi)z]-F(t+\Delta t) \tag{19}$$

$$d_2=M\Delta t+\frac{C}{2}\Delta t^2+\alpha\frac{K}{6}\Delta t^3+(1-\alpha)\frac{K}{6}\Delta t^3[A+\xi z] \tag{20}$$

$$d_3=\frac{(1-\alpha)K\ddot{u}(t)}{8}\Delta t^4 \qquad (21)$$

$$d_4=\frac{\xi}{2}\ddot{u}(t)\Delta t^2[A+\xi z] \qquad (22)$$

$$d_5=\frac{\xi}{6}\Delta t^3(A+\xi z) \qquad (23)$$

$$d_6=\ddot{u}^2\frac{\Delta t^4}{8}-\ddot{u}\frac{\Delta^2}{2} \qquad (24)$$

Then

$$a_3=\frac{d_3d_4-d_1d_6}{d_2d_6-d_3d_5} \qquad (25)$$

$$b_0=z(t)-Au(t)-\xi z(t)u(t)+\frac{1}{2}u(t)^2b_2 \qquad (26)$$

$$b_1=A+\xi z(t)-b_2u(t) \qquad (27)$$

$$b_2=\frac{d_1d_5-d_2d_4}{d_2d_6-d_3d_5} \qquad (28)$$

2. For $\dot{u}(t)\neq 0$

$$d_1=M\ddot{u}+C\dot{u}+C\ddot{u}^2\Delta t+\alpha K(u+\dot{u}\Delta t+\ddot{u}\frac{\Delta t^2}{2})+(1-\alpha)K(z+\dot{z}\Delta t+\ddot{u}\frac{\Delta t^2}{2\dot{u}})-F(t+\Delta t) \qquad (29)$$

$$d_2=M\Delta t+\frac{C}{2}\Delta t^2+\alpha\frac{K}{6}\Delta t^3+(1-\alpha)K\dot{z}\frac{\Delta t^3}{6\dot{u}} \qquad (30)$$

$$d_3=(1-\alpha)K(\frac{\ddot{u}\Delta t^4}{8}+\frac{\dot{u}^2\Delta t^2}{2}+\frac{\dot{u}\ddot{u}\Delta t^3}{2}) \qquad (31)$$

$$d_4=A+\xi(z+\dot{z}\Delta t+\frac{\dot{z}\ddot{u}\Delta t^2}{2\dot{u}})-\frac{\dot{z}}{\dot{u}} \qquad (32)$$

$$d_5=\frac{\xi\Delta t^3z}{6\dot{u}} \qquad (33)$$

$$d_6=\xi(\frac{\dot{u}^2\Delta t^2}{2}+\frac{\ddot{u}^2\Delta t^4}{8}+\frac{\dot{u}\ddot{u}\Delta t^3}{2})-\dot{u}\Delta t-\frac{\ddot{u}\Delta t^2}{2} \qquad (34)$$

Then

$$a_3=\frac{d_3d_4-d_1d_6}{d_2d_6-d_3d_5} \qquad (35)$$

$$b_0=z(t)-\frac{\dot{z}u}{\dot{u}}+\frac{u(t)^2b_2}{2} \qquad (36)$$

$$b_1=\frac{\dot{z}}{\dot{u}}-ub_2 \qquad (37)$$

$$b_2=\frac{d_1d_5-d_2d_4}{d_2d_6-d_3d_5} \qquad (38)$$

The structural response for the next time step is obtained as

$$u(t+\Delta t)=D_4(\Delta t) \qquad (39)$$

$$\dot{u}(t+\Delta t)=\dot{D}_4(\Delta t) \qquad (40)$$

$$\ddot{u}_4(t+\Delta t)=\ddot{D}_4(\Delta T) \qquad (41)$$

$$z(t+\Delta t)=b_0+b_1u(t+\Delta t)+b_2u(t+\Delta t)^2 \qquad (42)$$

3.2 *Case 2: n=2*

In order to determine the coefficient vectors {A} and {B} for n=2, the same procedure as in Case 1 is followed, and mostly similar equations are obtained. For n=2, (17) is replaced by

$$\frac{dz(t+\Delta t)}{du}=A+\xi z(t+\Delta t)^2 \tag{43}$$

1. For $\dot{u}(t)=0$

$$d_4=\frac{\xi}{2}[\ddot{u}(t)\Delta t^2(A+\xi z)+\frac{1}{4}\ddot{u}^2\Delta t^4] \tag{44}$$

$$d_5=\frac{\xi}{6}[\Delta t^3(A+\xi z)+\frac{\ddot{u}^2\Delta t^4}{8}] \tag{45}$$

$$d_6=\ddot{u}^2\frac{\Delta t^4}{8}-\ddot{u}\frac{\Delta^2}{2}+\xi(\frac{\ddot{u}^2\Delta t^4}{8}-\frac{\ddot{u}\Delta^2}{2}) \tag{46}$$

2. For $\dot{u}(t)\neq 0$

$$d_4=\frac{\xi}{2}[\ddot{u}(t)\Delta t^2(A+\xi z)+\frac{1}{4}(4\ddot{u}^2\Delta t^2 +\ddot{u}^2\Delta t^4+4\dot{u}\ddot{u}\Delta t^3)(z+\dot{z}\Delta t)] \tag{47}$$

$$d_5=\frac{\xi}{6}[\Delta t^3(A+\xi z)+\frac{\dot{z}\ddot{u}\Delta t^2}{\dot{u}}] \tag{48}$$

$$d_6=\xi(\ddot{u}^2\frac{\Delta t^4}{8}-\ddot{u}\frac{\Delta t^2}{2}+z^2 +\dot{z}^2\Delta t^2+2z\dot{z}\Delta t+\frac{\dot{z}^2\ddot{u}\Delta t^3}{\dot{u}}) \tag{49}$$

4 NUMERICAL STABILITY AND ACCURACY

The stability of the proposed method has been systematically investigated. It is found that this method is unconditionally stable. The accuracy of this method is verified against Monte-Carlo simulation for the response levels (Yener, Shen, and Gong (1991)). In order to illustrate the validity of the proposed method, a computer code is developed by the authors specifically for this investigation.

5 CONCLUSIONS

The primary purpose of the method presented in this paper is to increase the numerical accuracy of the analysis of earthquake structural resistant response. In this paper, we introduce a smooth hysteretic model for the nonlinear analysis of structural dynamic response by representing the displacement by a third order polynomial function and the hysteretic variable by second order polynomial function. It is suggested that, in the future, a higher order polynomial is used in representing the response of smooth nonlinear system.

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