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# Loading path dependency

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ABSTRACT: A systems' ultimate strength represented by the collapse load is influenced by the loading path. In other words, the collapse load of a system may decrease in particular loading path. However, quantitative observation has not yet been done. In this paper we discuss loading path dependency using a simple loading path model. It is demonstrated that not only material behavior but also several factors such as geometry of the system and load location may cause loading path dependency of ultimate strength of structures.

## 1 INTRODUCTION

Loading path is defined as the possible path from a certain point up to a particular point in the load space. There is an infinite number of possible loading paths. Since it is impossible to investigate all of these, we propose a simple loading path model. Based on this model loading path dependency is discussed for using several structural systems. A yield loci is represented by a plot of the components of the ultimate load Pu of a system for any given load locations. With the aid of the yield locus, we investigate loading path dependency.

## 2 Analysis

### 2.1 Material behavior

Three types of material behavior are considerd here: brittle, semi-brittle and perfectly ductile. The general model of material behavior is shown in Fig.1, where three independent parameters are used to reprensent the material behavior. Those parameters are ( see Fig. 3.1):

1. Yield stress ratio, $\alpha = \sigma_y{}^C / \sigma_y{}^T$;
2. Softening factor for tension, $\eta^T$ and
3. Softening factor for compression $\eta^C$

The softening factors $\eta^T$ and $\eta^C$ take values beteween 0 and 1. For instance, when the material is perfectly brittle in compression $\eta^C$ is zero. An element which is made of this elastic until its initial failure. After it fails in compression or tension, the force in the element drops to a constant post-yield level which is determined by the softening factor $\eta$. Then the force remains at this constant post-yield level.

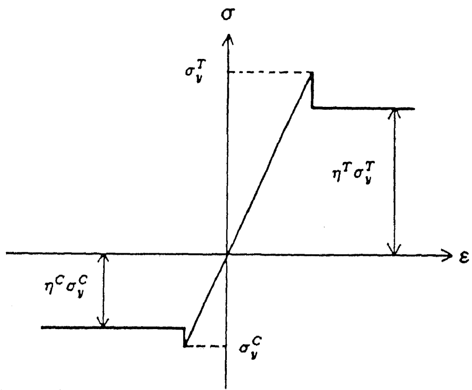


Fig.1 Material behavior

### 2.2 Geometry of systems

A three-bar truss shown in Fig.2 and five-bar truss shown in Fig.3 are respectively analyzed for an example of symmetric geometry and asymmetryc geometry. Each truss is subjected to an external load P which varies from 0° to 360°. All members are of equal areas. The pertinent information is provided in each figure.

### 2.3 Member replacement method

The response of truss structure with semi-brittle elements is obtained using a simplified method called the "member replacement" method. In this method, after an element of

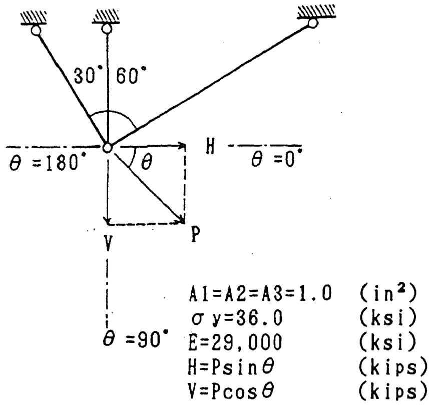


Fig.2 Three-bar truss

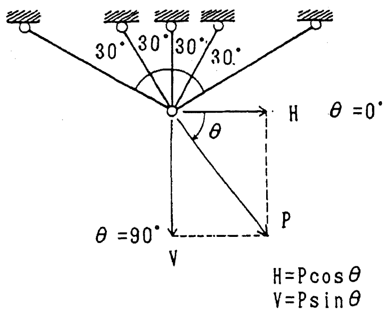


Fig.3 Five-bar truss

a system fails, the element is removed from the system and the effect is accounted for by applying a set of forces, which is equal to the post-yield force at the nodes at the ends of the elements. The forces are maintained at the same level for the subsequent step of the analysis. Fig.4 shows yield locus of the three-bar truss shown in Fig.2 with respect to the yield stress ratio of 1.0,

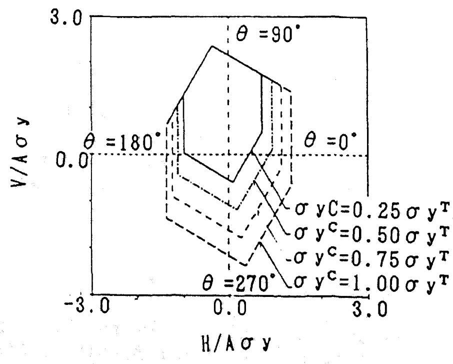


Fig.4 3-bar truss yield loci(collapse) with respect to different stress ratios

0.75, 0.5, and 0.25, respectively. The distances between the origin and yield loci represent the collapse load Pc. Fig.5 shows the yield locus with respect to collapse for softening factors $\eta$=0.8 and $\alpha$=0.5.

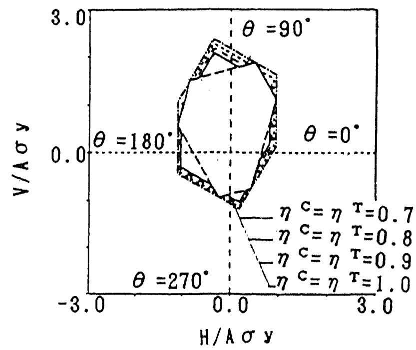


Fig.5 Yield locus(3-bar truss) Differernt material behavior

## 3 Loading path model

### 3.1 Loading path model

In yield locus, the distance between the origin and that point represents the collapse load of the system, Pcr, for the corresponding load combination. This critical load Pcr can be named " the collapse load associated with proportional loading". We limit ourselves in the 2-D load space, namely to the components Px, Py(i.e., Pcrcos$\theta$ Pcrsin$\theta$). The loading path model shown Fig. 6 consists of two distinct load vectors:

1. Loading path vector 1 :from the origin to Pl
2. Loading path vector 2 :from Pl to Pcr

wherePl is the intermediate point defined as

Plx=(tPcrcos$\theta$ , 0)

where t denotes the loading path parameter which varies from zero to unity. It can be noted that when t is zero, the loading path is the proportional path between the origin and Pcr and when t is one, the loading path is an extreme one whose first vector goes to the point M( Pcrcos$\theta$ ,0 ). The second path parallel to the vertical axis from M to Pcr goes to Pcr. We can conclude that if the collapse load corresponding to loading path parameter t is invarriant, the loading path does not affect the ultimate strength of systems. Otherwise, something causes loading path dependency.

### 3.2 Examples

Four examples are analyzed to clarify the

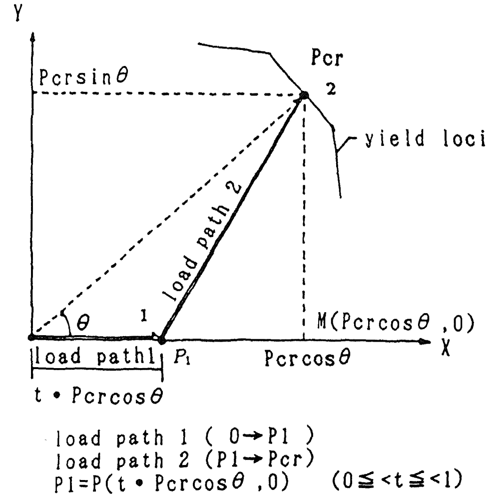


Fig.6 Load path model

effects of loading path dependency factors discussed previouly. The yield locus of each example is shown in Fig. 7 to Fig. 10. The conditions of each system are as follows:

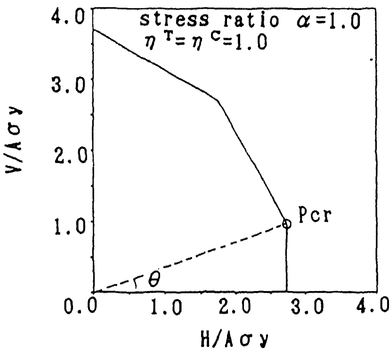


Fig.7 Load path dependency: example 1

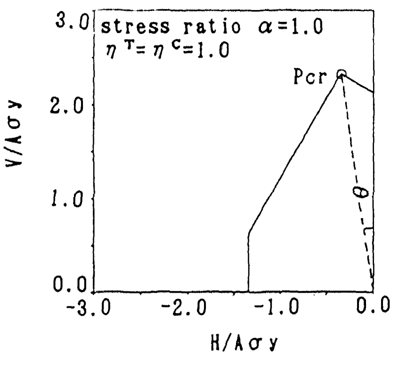


Fig.8 Load path dependency: example 2

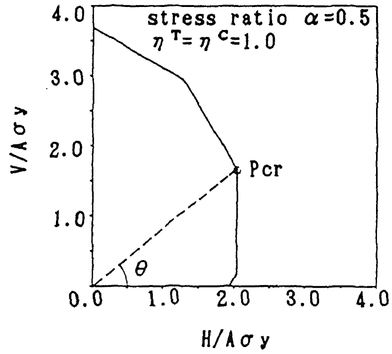


Fig.9 Load path dependency: example 3

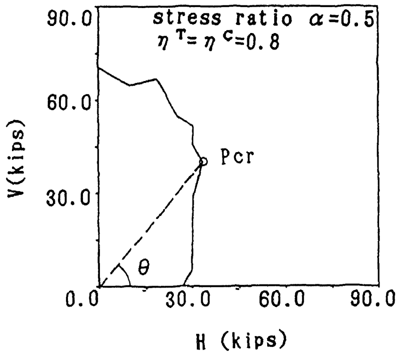


Fig.10 Load path dependency: example 4

1. $\eta^T=\eta^C=1.0$, $\alpha=1.0$, symmetric geometry (5-bar truss, Fig. 7)
2. $\eta^T=\eta^C=1.0$, $\alpha=0.5$, symmetric geometry (5-bar truss, Fig. 8)
3. $\eta^T=\eta^C=1.0$, $\alpha=1.0$, asymmetric geometry (3-bar truss, Fig. 9)
4. $\eta^T=\eta^C=0.8$, $\alpha=0.5$, asymmetric geometry (3-bar truss, Fig.10)

The results of loading path dependency with respect to the parameter t of each case are shown in Fig.11 to Fig.14. In these figures, the vertical axes represent the ratio of maximum load obtained by the loading path model in Fig.6. to the proportional loading from the origin to the particular point Pcr on the collapse locus. We can see three patterns of influence on the ultimate strength.

1.There is no effect of the loading path (Fig.11).

2.The ultimate strength suddenly drops to a constant level at a certain point t and the system collapses (Fig.12 and Fig.13).

3. The ultimate strength suddenly drops to a certain value and gradually decreases to a certain value according to the increase in t and again the strength suddenly drops to a lower constant value. After that the system collapses (Fig.14).

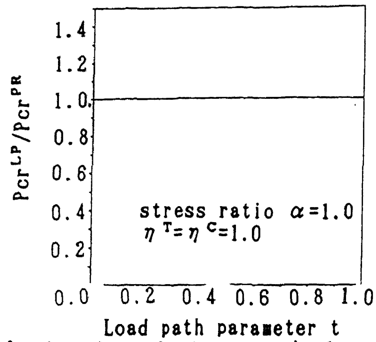


Fig.11 Load path analysis :example 1

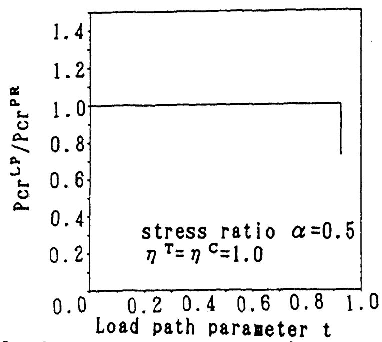


Fig.12 Load path analysis :example 2

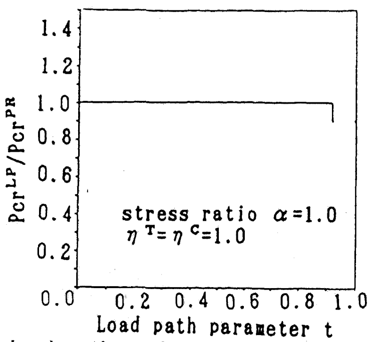


Fig.13 Load path analysis :example 3

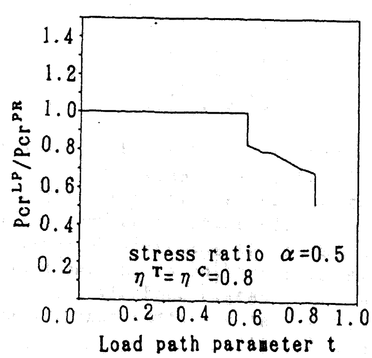


Fig.14 Load path analysis :example 4

## 4 Conclusions

Some examples were investigated to evaluate the effects of factors for loading path dependency. The following observations are noted:

1. It is observed that the loading path dependency of a structural system is influenced by material properties, stress ratio, load location and the system geometry.
2. The loading path model discussed is an useful tool to visualize the effects of loading path dependency. The gradual decrease in the ultimate strength which corresponds to the increase in loading path parameter t indicates the existence of concavity in the collapse locus of the system.

## REFERENCES

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