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Drift design of tall buildings

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ABSTRACT : Using the unit load method and the Lagrange multiplier λ, a structure improvement method for minimizing the lateral drift of tall buildings caused by earthquakes or strong winds is proposed in this paper.

1. INTRODUCTION

Drift problem as the horizontal displacement of tall buildings is one of the most serious issues in tall building design, relating to the dynamic characteristics of the building during earthquakes and strong winds.

Drift shall be caused by the accumulated deformation of each member, such as a column, beam, brace and shear wall. Therefore, when we want to control the quantity of displacement by changing its design, we can not figure out which member of the structure should be changed, only from one result of computer calculation.

This paper aims at proposing a new method on drift control in structural design. This method can be regarded as a kind of design sensitively analysis which is newly used for structural analysis programs. Finally, I apply the method to a tall building structure.

2. THE CHANGE OF DISPLACEMENT CAUSED BY CHANGING SECTION OF MEMBERS

2.1 Unit Load Method for Calculating the Displacement of Structures

When we perform structural analysis with computers, we generally use Matrix method which defines the displacement of each node as the unknown. In this case, the displacements of a structure are printed out automatically and we can check them easily.

On the other hand, the displacement of a structure shall be caused by the accumulation of each member's deformation as mentioned before, The computer output can not indicate which member's deformation has a large share in its displacement.

I would like to mention Unit Load Method for calculating displacements derived from the principles of Virtual work.

According to References [2], the process of this method are as follows.

"The procedure for calculation a displacement by means of the unit-load method using Eq.1 may be summarized as follow : (1) determine the stress resultants N_L, M_L, and V_L in the structure caused by the actual loads : (2) place a unit load on the structure corresponding to the displacement D that is to be found : (3) determine the stress resultants N_U, M_U, and V_U caused by the unit load : (4) form the terms shown in Eq.1 and integrate each term for the entire structure : and (5) sum the results to obtain the displacement D."

$$D = \int \frac{N_U N_L}{EA} dx + \int \frac{M_U M_L}{EI} dx + \int \frac{\alpha V_U V_L}{GA} dx \quad \text{Eq. 1}$$

Additionally, Prof. S.P.Timoshenko described the validity efficiency of this method as follows.

"The unit-load method can be used not only for beams, trusses, and other simple kinds of structures, but also for very complicated structures having many members. Furthermore, the unit-load method is suitable for finding all types of displacements, including the deflection of a point in the structure, the rotation of the axis of a member, the relative displacement between two points, and others. Theoretically, it may be used for either statically determinate or indeterminate structures, although for practical purposes the method is limited to determinate structures because its use requires that the stress resultants be known throughout the structure."

For these reasons, this method has been used only for hand calculation exercises at universities. In practical design, computers usually calculate structural deformation. There are very few engineers who take the trouble to calculate it by themselves using this method.

Today, Stress resultants of statically indeterminate structures can be obtained by computers. Unit load method has the advantage that it can analyze the source of displacement of certain point of a structure.

Therefore, this method can be practically used not only for statically determinate structures, but also for any type of structures.

2.2 Application to Statically Determinate Structures

As the simplest model, series springs are shown in Fig. 1.

Spring a, b are assumed to have the same spring constant k_0 ($= EA_0 / l_0$) : It is apparent that the axial forces of spring a, b can be represented by P : $N_{La} = N_{Lb} = P$. When a unit load is applied to point 3, the axial forces are expressed as, $N_{Ua} = N_{Ub} = 1$.

From Eq.1 of Unit load method, the displacement of point 3, defined as D_3, can be written as

$$D_3 = \int_{l_0} \frac{1\,P}{EA_0} dx + \int_{l_0} \frac{1\,P}{EA_0} dx$$

$$= \frac{Pl_0}{EA_0} + \frac{Pl_0}{EA_0} = \frac{P}{k_0} + \frac{P}{k_0} \qquad \text{Eq. 2}$$

By setting $d_a = P / k_0$, $d_b = P / k_0$, this equation becomes

$$D_3 = d_a + d_b \qquad \text{Eq. 3}$$

We can think of d_a, d_b as constituents of D_3.

Then, I consider a modified case in which spring constants of spring a, b are multiplied by α_a, α_b respectively.

The displacement of point 3, defined as D_3', can be similarly obtained from Eq.2.

$$D_3' = \frac{P}{\alpha_a k_0} + \frac{P}{\alpha_b k_0} = \frac{d_a}{\alpha_a} + \frac{d_b}{\alpha_b} \qquad \text{Eq. 4}$$

This model is a statically determinate structure. Even if spring constants are changed, the stress resultants of them shall stay unchanged. Therefore the result of this calculation is a right solution.

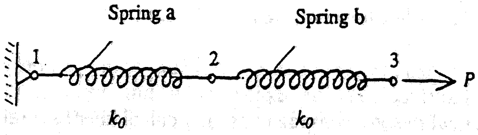

Fig. 1 Simple Statically Determinate Structure

2.3 Application to Statically Indeterminate Structures

As the simplest model of a statically indeterminate structure, two parallel springs are shown in Fig.2.

D_2 can be expressed as $P/2k_0$ from basic technical knowledge. I figure out D_2 by means of the same method of statically determinate structures.

The axial forces of spring a, b caused by P can be represented by $P/2$, $N_{La} = N_{Lb} = P/2$. When a unit load is applied to point 2, the axial forces are expressed as, $N_{Ua} = N_{Ub} = 1/2$. D_2 can be obtained with these values using Eq.1

$$D_2 = \int_{l_0} \frac{1/2 \; P/2}{EA_0} dx + \int_{l_0} \frac{1/2 \; P/2}{EA_0} dx = \frac{P\,l_0}{4EA_0} + \frac{P\,l_0}{4EA_0} \qquad \text{Eq. 5}$$

By putting, $k_0 = EA_0 / l_0$

$$D_2 = \frac{P}{4k_0} + \frac{P}{4k_0} = \frac{P}{2k_0} \qquad \text{Eq. 6}$$

By setting $d_a = P / 4k_0$, $d_b = P / 4k_0$, this equation becomes

$$D_2 = d_a + d_b \qquad \text{Eq. 7}$$

We can think of d_a, d_b as constituents of D_2.

According to Eq.4, assuming that the quantity of each spring's deformation changes in proportion to the change of each spring constant respectively, I propose a new simplified calculation method. A new equation can be obtained from Eq.7, by dividing d_a by α_a and dividing d_b by α_b and adding both of them:

$$D_2' = \frac{d_a}{\alpha_a} + \frac{d_b}{\alpha_b} \qquad \text{Eq. 8}$$

I propose quite a daring assumption for structural deformation as follows.

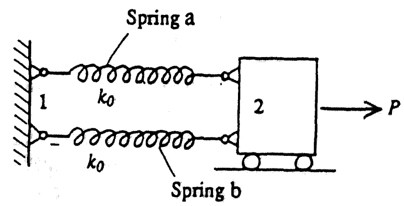

Fig. 2 Simple Statically Indeterminate Structure

"The quantity of each member's deformation which causes the displacement of certain point of a structure changes in proportion to the change of its member's

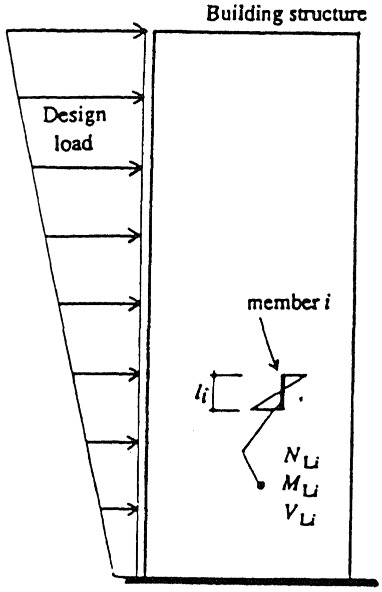

Fig. 3.a Stress Resultants
Caused by Design Load

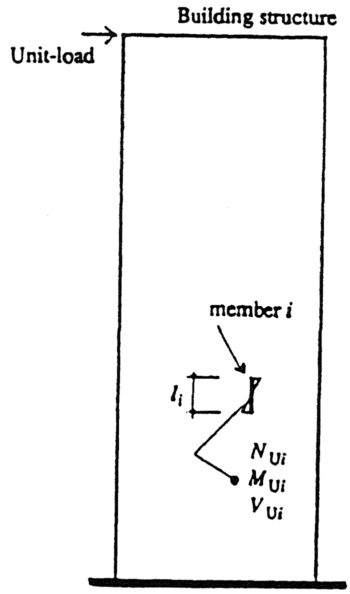

Fig. 3.b Stress Resultants
Caused by Unit Load

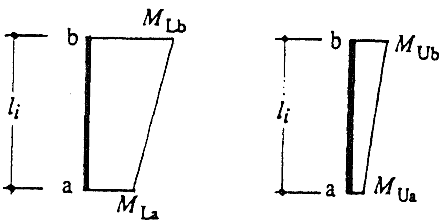

Fig. 4 Moment Distribution

stiffness, and this quantity is not affected by the change of any other member's stiffness."

This assumption is perfectly right for statically determinate structures. If there are no big changes in stress distribution, this assumption is almost right for statically indeterminate structures.

3. CONSTITUENTS OF DRIFT OF THE TOP OF A BUILDING CAUSED BY LATERAL FORCE

3.1 A New Calculation Method

Fig.3.a indicates the stress resultants of a building caused by design lateral load. Fig.3.b indicates the stress resultants of the same building when Unit load is applied to the top of the building. In these figures, i means the sequence number of each member and l_i, w_i mean the length and weight of member i.

By using the stress resultants of Fig.3.a, and Fig.3.b, and integrating Eq.1, the lateral displacement of the top of this building can be expressed as

$$D_{top} = \sum_{i=1}^{m} d_i \qquad \text{Eq. 9}$$

where

$$d_i = \frac{N_{Ui} N_{Li}}{EA_i} l_i + \int_{l_i} \frac{M_{Ui} M_{Li}}{E I_i} dx + \frac{\alpha V_{Ui} V_{Li}}{GA_i} l_i \qquad \text{Eq. 10}$$

The second term of Eq.10 can be easily obtained from Fig.4 as follows.

$$\int_l \frac{M_U M_L}{EI} dx = \frac{l}{6EI} (M_{Ua}(2M_{La}+M_{Lb})+M_{Ub}(M_{La}+2M_{Lb})) \qquad \text{Eq. 11}$$

Total weight of the structure can be given as

$$W_{total} = \sum_{i=1}^{m} w_i \qquad \text{Eq. 12}$$

3.2 Practical application to big buildings

When the structure is small, we can easily check and decompose each member's deformation to examine the source of the displacement of the top of a building. In practical design, generally a building has more than ten thousand members. Besides, the decomposition of each member's deformation shall significantly increase the number of data.

Consequently, The number shall become so great that we can hardly grasp all of them. Spread Sheet Programs for personal computers such as *Excel* and *Wingz*, will be quite useful to analyze these data. For example, they can sum up the deformations of columns, beams, braces and shear walls, separately.

4. THE WAY TO MINIMIZE THE DISPLACEMENT WITHOUT CHANGING TOTAL WEIGHT

The displacement of the top of a building can be expressed as

$$D_{top} = \sum_{i=1}^{m} d_i \qquad \text{Eq. 13}$$

where d_i = the deformation of member i

Total weight of steel can be expressed as

$$W_{total} = \sum_{i=1}^{m} w_i \qquad \text{Eq. 14}$$

where w_i = the weight of member i

On the assumption that section of members can be changed only by controlling the thickness of the steel plate without changing its type of figure, the Area and Moment Inertia and Weight of member i, A_i, I_i, w_i, shall be changed in proportion to the coefficient α_i which controls the thickness of the plate.

According to 2.3, the deformation of changed member i can be written as d_i / α_i

Therefore, the displacement of the top of the changed building is expressed as

$$D'_{top} = \sum_{i=1}^{m} \frac{d_i}{\alpha_i} \qquad \text{Eq. 15}$$

From the assumption, total weight of steel is constant. Therefore this is given by

$$W_{total} = W'_{total} = \sum_{i=1}^{m} \alpha_i w_i \qquad \text{Eq. 16}$$

The problem to be solved here is finding the minimum of Eq.15 under the subsidiary condition Eq.16.

By applying Lagrange multiplier λ, Eq.15 can be rewritten as

$$D'_{top} = \sum_{i=1}^{m} \frac{d_i}{\alpha_i} + \lambda \left(\sum_{i=1}^{m} (\alpha_i w_i) - W_{total} \right) \qquad \text{Eq. 17}$$

By partially differentiating Eq.17 by α_i (i = 1,2,.... m) and λ, and then setting these equations to be equal zero, we can find the values of α_i (i = 1,2,.... m) and λ which minimize D'_{top}. These equations become

$$\frac{\partial D'_{top}}{\partial \alpha_i} = -\frac{d_i}{\alpha_i^2} + \lambda w_i = 0 \quad (i = 1, 2...m) \qquad \text{Eq. 18}$$

$$\frac{\partial D'_{top}}{\partial \lambda} = \sum_{i=1}^{m} (\alpha_i w_i) - W_{total} = 0 \qquad \text{Eq. 19}$$

From Eq.18, α_i can be written as

$$\alpha_i = \sqrt{\frac{d_i}{\lambda w_i}} \quad (i = 1, 2... m) \qquad \text{Eq. 20}$$

By substituting Eq.20 into Eq.19, consequently we obtain the weight of changed member i as follows

$$w'_i = \alpha_i w_i = \frac{\sqrt{d_i w_i}}{\sum_{j=1}^{m} \sqrt{d_j w_j}} W_{total} \quad (i = 1, 2... m)$$

Eq. 21

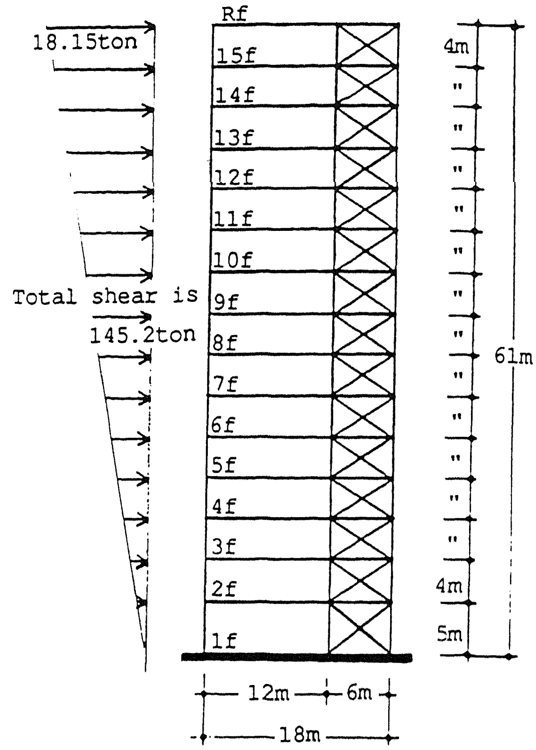

Fig. 5 15 Story Tall Building

Table 1 Sections of Columns, Girders and Braces

Columns		Girders		Braces	
12f-15f	Box-450x450x9	2f-Rf	H-600x200x11x17	11f-15f	H-150x150x7x10
8f-11f	Box-450x450x12			6f-10f	H-175x175x7.5x11
4f-7f	Box-450x450x16			1f-5f	H-200x200x8x12
1f-3f	Box-450x450x19				

Table 2 Calculation Procedure

group	*wi* (tons)	*di* (cm)	$\sqrt{d_i w_i}$	*w'i* (tons)	αi	*d'i* (cm)
1	3.16	.42	1.149	1.72	.545	.77
2	3.35	.56	1.371	2.05	.613	.91
3	2.56	.64	1.285	1.93	.751	.86
4	1.95	.76	1.216	1.82	.935	.81
5	6.32	12.90	9.030	13.54	2.141	6.02
6	6.70	6.71	6.706	10.05	1.500	4.47
7	5.13	1.84	3.068	4.60	.897	2.05
8	3.90	.45	1.334	2.00	.512	.89
9	18.99	11.60	14.844	22.25	1.172	9.90
10	9.50	.17	1.265	1.90	.200	.85
11	3.66	2.00	2.705	4.06	1.109	1.80
12	2.90	1.33	1.964	2.94	1.016	1.31
13	2.27	.46	1.020	1.53	.673	.68
total	70.39	39.84	46.957	70.39		31.32

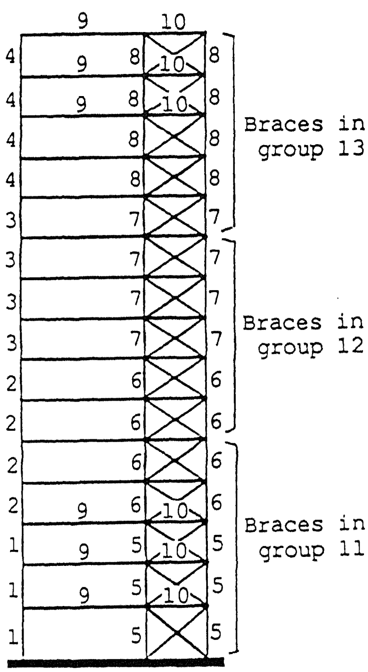

Fig. 6 Classification to 13 groups

where the solutions of this problem α_i (i = 1,2,.... m), λ can be easily obtained.

In conclusion, the weight of member i whose section was changed to minimize the displacement of the top, can be obtained by allocating total weight of steel in proportion to the square root of the product of d_i and w_i. This simple formula could be understood intuitively and helpful to structural designers.

5. APPLICATION TO TALL BUILDING

For example, this method is applied to a steel structure as schematized in Figure 1. Although the structure is fifteen-story high and is not actually a high-rise building, this example is suitable for proving the usefulness of the theory. The sections of columns and girders and braces in original design are determined by simplified calculation method based on Japanese seismic design standard.

The shapes of these sections are listed on Table 1. When the distributed horizontal force as shown in Figure 5 is subjected to the structure, the horizontal displacement of the top is figured out to be 39.8 cm. As mentioned in section 3.2, this structure has so many members that 105 members are classified into 13 groups as shown in Fig.6 and the calculation for optimum design is performed. In section 4, *di* and *wi* represent the deformation and weight of member i respectively. In this example, di and wi can be defined as the sum of the deformations and weights of all members which belong to group i.

The calculation process is shown in Table 2. di and wi are the calculation results of original structure. αi is the coefficient to change the section of group i members.

The case that αi is more than 1.0, indicates the structural deformation shall be decreased by changing the section of group i members for larger one. The case that αi is less than 1.0, indicates the structural deformation shall be increased a little even if smaller sectional members are replaced in group i. Accordingly, using smaller sectional members for reducing weight could be efficient.

$d'i$ indicates the sum of deformations of all changed members which belong to group i. The summation of $d'i$ (i=1~13) represents the estimated horizontal displacement at the top of the structure (=31.3 cm).
At the top of changed building, the horizontal displacement caused by the previous force, becomes 29.9 cm. The calculation result is smaller than the estimated value by 1.4 cm, and consequently the method is proved to be quite efficient. The reason why the calculation result is not equal to the estimated value, is that this structure is a statically indeterminate one and the stress distribution varies by changing sections.

The original and changed structure have the same total weight (70.4 ton). The displacement at the top can be reduced to be 75 % of original design by this method.

6. CONCLUSION

Design is the product of human creativity. Even if Optimum Design Method is completed by computers, I think very few structural engineers will use computer design directly without any consideration. This paper considers mainly structural deformation and proposes the new method to improve structural design.

In Computer age, technical intuitions are more difficult to be obtained as compared with Hand-calculation age. Therefore, some conservative engineers are afraid that frequent use of computers might be ruining the quality of engineers. Although I do not dare to oppose it, actually computer has been making great progress day by day and I think the important thing is how to use computers more efficiently. We should try to find out new methods which enhance engineers' technique and creativity by using computers. I believe that providing the process of computer calculation could be helpful, and I try to perform it in this paper.

This paper has developed a method on the assumption that the stiffness of each member is proportional to its weight. Next time, I would like to proceed with the method considering the change of each member's stiffness corresponding to the change of its depth and width. Furthermore, I will try to study deformation analysis which includes plastic deformation caused by large external loads.

7. ACKNOWLEDGEMENT

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