Vibration tests and earthquake analysis of two actual axisymmetric structures

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ABSTRACT: Two actual axisymmetic structures, namely, a natural draft hyperbolic cooling tower and a thermal power house chimney, have been analyzed for their self-weight and earthquake forces. Two types of analysis, i.e., 3D axisymmetric finite element analysis and 2D beam type analysis, have been carried out and the results from the two types of analysis have been compared. Theoretical response due to scaled down El Centro ground motion has been evaluated to its two mutually orthogonal horizontal components along with the vertical component. The contribution of the vertical component of the motion and that of higher modes on the total response of the structures has been investigated. To verify the method of analysis and the assumptions involved therein, the natural periods of vibration of the two structures have been measured by carrying out wind excited vibration tests on the structures at the site.

1 INTRODUCTION

To verify the method of analysis and the assumptions involved therein, it is desirable to have dynamic measurements of the actual structures. The measurement that could be made on a prototype without causing any damage to it would be to find its natural period of vibration which for a structure depends upon its mass and stiffness distribution. Axisymmetric structures in the true sense are exceptions rather than a rule. Certain assumptions are, therefore, made in their representation for theoretical evaluation of periods of vibration. The measurement and verification of assumptions in the theoretical evaluation of time period is essential to prove the usefulness of the analytical method.

Two typical actual structures, i.e., a natural draft cooling tower and a chimney, have been chosen for study. The cooling tower is supported on columns, and the chimney has a flue duct opening near its base. Both the structures have been approximated as axisymmetric systems. The cooling tower is representative of rather a squatty structure while chimney of a slender structure.

Theoretical response to three componental earthquake ground motion of El Centro type having spectral intensity of 100 cm has been evaluated for both the structures.

2 BASIC THEORY

2.1 Axisymmetric analysis

Utilizing the semi-analytical finite element process (Zienkiewicz, 1977) and considering external loads symmetric about $\theta=0$ plane, the loads as well as displacements are expanded into a Fourier series. Thus, the radial, vertical and tangential components of displacement are expressed as

$$\begin{split} \mathbf{u}(\mathbf{r},\mathbf{z},\theta) &= \Sigma(\mathbf{u}_{\mathbf{n}}^{\mathbf{S}}(\mathbf{r},\mathbf{z}) \cos n\theta + \mathbf{u}_{\mathbf{n}}^{\mathbf{A}}(\mathbf{r},\mathbf{z}) \sin n\theta) \\ \mathbf{w}(\mathbf{r},\mathbf{z},\theta) &= \Sigma(\mathbf{w}_{\mathbf{n}}^{\mathbf{S}}(\mathbf{r},\mathbf{z}) \cos n\theta + \mathbf{w}_{\mathbf{n}}^{\mathbf{A}}(\mathbf{r},\mathbf{z}) \sin n\theta) \\ \mathbf{v}(\mathbf{r},\mathbf{z},\theta) &= \Sigma(\mathbf{v}_{\mathbf{n}}^{\mathbf{S}}(\mathbf{r},\mathbf{z}) \sin n\theta - \mathbf{v}_{\mathbf{n}}^{\mathbf{A}}(\mathbf{r},\mathbf{z}) \cos n\theta) \end{split}$$

where, u, w and v are the radial, vertical and tangential displacements, respectively, and n is the harmonic number. The summations extend over as many terms as are necessary for proper representation of loading. The superscripts 's' and 'a' refer to symmetric and antisymmetric components of displacements, respectively. The stress in an element'e' is computed from these generalized displacements by the relation

$$\underline{\sigma}^{e} = \underline{D}^{e} \underline{B}^{e} \underline{\delta}^{e}$$

where σ = stress vector, D = elasticity matrix; B = strain displacement matrix; and δ = vector of generalized displacements.

For the analysis of response due to earthquake excitation including the dead load of the structure, only the first two terms in the Fourier series survive and, therefore, only two separate 2-D analyses are required. The n=1 term would give response due to horizontal component of ground motion, and the term n=0 would give due to vertical component of ground motion and the super-structure.

2.2 Beam analysis

It is a 2-noded element with translational (u) and rotational (θ) degrees of freedom per node. Deformations due to bending and shear and the effect of rotatory inertia have been included. The stiffness matrix of each element \underline{K} will be a 4×4 matrix:

$$\underline{K} = \begin{matrix}
EI \\
-3(1+\phi)
\end{matrix}$$

$$\begin{bmatrix}
12 & 6L & -12 & 6L \\
6L & (4+\phi)L^2 & -6L & (2-\phi)L^2 \\
-12 & -6L & 12 & -6L \\
6L & (2-\phi)L^2 & -6L & (4+\phi)L^2
\end{bmatrix}$$

in which L = length of beam element; G = moudulus of rigidity; E = modulus of elasticity; I = moment of inertia, ϕ = (12 EI/G As L¹); and As = effective area in shear equal to area times shape factor.

The elements of the diagonal mass matrix are m and I, in which m and I are the mass and the mass moment of inertia of the element lumped at the node, respectively.

3 IDEALIZATION OF THE STRUCTURES

Figs. 1 and 2 show the axisymmetric finite element as well as beam idealization of cooling tower and chimney. The cooling tower is a 128 m high reinforced cement concrete (RCC) structure with its diameter at top, throat and bottom as 58.1m, 53.1m and 107.05m, respectively. The tower is supported on 85cm diameter columns. In the finite element idealization, these columns have been represented by an equivalent cylindrical shell having the same moment of inertia as that of columns about the axis through centre line of tower. The chimney is a 150 m high RCC structure having a flue duct of 5.5 m width and 8.7 m height near the base. The flue duct has been modelled in axisymmetic analysis as an equivalent cylinderical shell having the same EI value as that of the actual section. In the beam analysis, actual moment of inertia has been considered at the flue duct section.

For the purpose of finite element discretization, 6-noded paralinear element

has been used with quadratic displacement variation in the direction of height and linear displacement variation in the direction of thickness as the thickness of concrete shell is relatively much lesser as compared to the height of the tower.

The properties assumed for concrete are modulus of elasticity = $30 \times 10^6 \text{ kN/m}^2$, Poisson's ratio = 0.20 and unit weight = 24 kN/m. The structures are assumed to be fixed at the base. In the beam analysis, the shape factor for shear deformation has been taken as 1/2.

Free vibration analysis has been done for finding mode periods and associated mode shapes under horizontal and vertical vibrations due to El Centro type ground motion. The stiffness matrix of the system has been calculated by using numerical integration at 3 x 2 Gauss integration points. Mass matrix has been assumed as diagonal. Damping has been taken as 5% of critical in all modes of vibration. To investigate the effect of higher modes, single mode analysis has also been done.

4 RESULTS AND DISCUSSION

4.1 Time periods

i) Cooling tower

Fig. 3(a) shows the typical portion of wind excited vibration record of the cooling tower which gives the fundamental mode period as 0.31 sec. The analytically calculated first three periods and the associated mode participation factors of the tower obtained from axisymmetric and beam analyses are summarized in Table 1. The fundamental mode periods from axisymmetric and beam analyses are found to be 0.32 sec and 0 35 sec. respectively. A comparison of time periods shows that the axisymmetric analysis gives an excellent comparison with the experimentally observed fundamental period of the tower. The beam analysis, however, predicts about 9% higher value of period than the axisymmetric analysis. .

ii) Chimney

The experimentally measured fundamental period of Chimney from the vibration record shown in Fig.3(b) is 4.0 sec. The fundamental periods as calculated from axisymmetric and beam analyses are found to be 3.55 sec and 3.65 sec, respectively, as listed in Table 2. It is also observed that the periods from the two types of analysis are very close to each other.

Table 1 - Periods (T) and mode participation factors (MPF) of Cooling Tower.

Mode	Axisymm Analy.		Beam Analy.	
No.	T	MPF	T	MPF
	(a)	Horizontal	Vibratio	n
1	0.320	1.52	0.350	1.50
2	0.150	0.75	0.140	1.77
3	0.104	0.92	0.082	0.51
	(p)	Vertical V	ibration	
1	0.134	1.36	0. 129	1.43
2	0.092	0.36	0.051	0.72
3	0.086	0.18	0.033	0.56

Table 2. Periods (T) and mode participation factors (MPF) of Chimney

Mode	Axisym	m. analy.	Beam Analys.					
No.	T	MPF	T	MPF				
(a) Horizontal Vibration								
1 2 3 4 5	3.552 0.863 0.360 0.202 0.130 0.096	1.38 0.92 0.67 0.52	3.650 0.868 0.361 0.202 0.134 0.097	1.35 0.91 0.66 0.50				
(b) Vertical vibration								
1 2 3 4 5 6	0.238 0.098 0.060 0.042 0.032 0.027	0.71 0.41 0.28 0.22	0.239 0.098 0.060 0.043 0.033 0.027	0.71 0.41 0.28 0.21				

NOTE: The experimental fundamental mode periods of cooling tower and chimney were observed to be 0.31 sec and 4.0 sec, respectively.

4.2 Stresses

i) Cooling tower

The distribution of maximum displacement and various stresss, i.e. bending, shear, axial and principal stresses, over the height of the cooling tower has been plotted in Fig. 4. It is observed from the figurethat the maximum stresses occur near El 25m and below this level there is a reduction in stresses

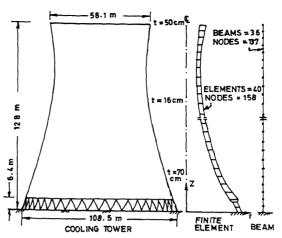


FIG. 1 _MATHEMATICAL IDEALIZATION OF COOLING TOWER

because of an abrupt increase in the cross sectional area. The figure also shows the comparison of maximum displacements and stresses obtained from axisymmetric and beam analyses due to El Centro type ground motion. It is seen that the axial stresses due to self-weight, bending and shear stresses due to two orthogonally horizontal components of motion, and the maximum and minimium principal stresses obtained from the two types of analysis show very good comparison, the maximum difference being less than 10%.

ii) Chimney

The distribution of maximum displacement and various stresses over the height of chimney is presented in Fig.5. An excellent agreement is observed between the displacement and stress pattern in both types of analysis. The maximum stresses occur between El 80 m and 100m. The stresses are significant above the flue duct portion at El 15m. The stress pattern shows that the stresses are not uniformly distributed over the height which is indicative of the fact that the structural material has not been properly utilised.

4.3 Effect of vertical component of earthquake

In both the structures, the contribution of vertical component of earthquake is too small and can be ignored. The hoop stress are also insignificant.

4.4 Effect of higher modes

In case of cooling tower, the higher mode periods are sufficiently small and, therefore, one- mode analysis was done to investigate the effect of higher modes on the

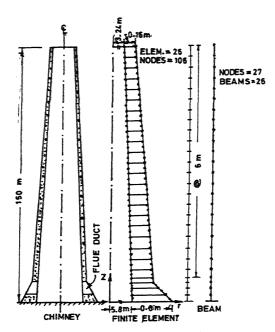


FIG. 2 _MATHEMATICAL IDEALIZATION OF CHIMNEY

(a) COOLING TOWER

MMMMMM

Ty = 0.31 sec

(P) CHINNEA

FIG. 3 _TYPICAL WIND-EXCITED VIBRATION RECORDS

total reponse of tower. The contribution of higher modes is found to be insignificant and one-mode analysis is, therefore, adequate for design purpose. In case of chimney, higher modes may influence the response as it is a flexible structure. Therefore, 6-mode analysis was also carried in addition to 3-mode analysis to ascertain the effect of higher modes. It is found that higher modes beyond three do not contribute to the response of chimney from design point of view.

5 CONCLUSIONS

(1) The experimentally observed fundamental periods for cooling tower as well as chimney exhibited an excellent agreement with the ones obtained theoretically using axisymmetric analysis. The chimney is a very

flexible structure as compared to the cooling tower.

- (2) The two methods of analysis, i.e., the axisymmetric and beam, gave very good comparison of periods and mode participation factors for chimney. However, for cooling tower the periods from the two methods of analysis were agreeable within 10%.
- (3) The two types of analysis gave good comparison of stresses also.
- (4) The stresses due to vertical component of earthquake for both the structures were found to be negligibly small. The response, therefore, can be calculated considering only the horizontal components of earthquake.
- (5) The effect of higher modes on the total response of the cooling tower was found to be insignificant and for all practial purposes one- mode analysis is adequate. But for chimney, first three modes must be considered.

REFERENCE

Zienkiewicz, O.C. 1977. The Finite Element Method, 3rd Edition, McGraw-Hill.

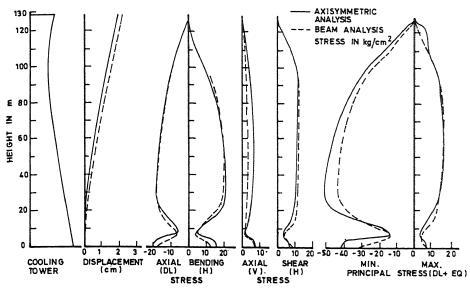


FIG. 4 _ MAXIMUM STRUCTURAL RESPONSE OF COOLING TOWER TO EL CENTRO
TYPE MOTION DL= DEAD LOAD , H=HORIZONTAL EQ., V=VERTICAL EQ.

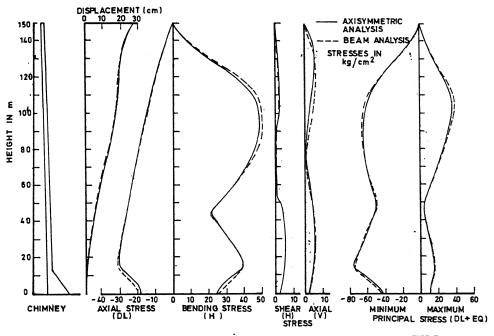


FIG. 5 - MAXIMUM STRUCTURAL RESPONSE OF CHIMNEY TO EL CENTRO TYPE GROUND MOTION