Deformation capacity of reinforced concrete beams used high strength concrete and reinforcement

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ABSTRACT: On structural design for high rise reinforced concrete buildings, ultimate strength and behavior of beams after flexural yielding is realized should be obvious. This paper presents an experimental study of the beams, and those of flexural yielding precedes shear and bond failure. The most effective factor in the beam is the shear reinforcement ratio for deformation capacity of the beams, and the most effective factor in the slab is the tensile strength of the bars to which the longitudinal reinforcement of the beams are parallel in slab for ultimate strength of beams with slab.

1 INTRODUCTION

Recently, on structural design for high rise earthquake resistant reinforced concrete buildings, it is usual that beam yielding should govern hinge mechanism of the buildings. Consequently, if an end of a beam is designed to be hinge, the beam must be ductile, and must not be stronger than the ends of columns which are the same joint, even considering effects of floor slab. It is hence important to estimate exactly ultimate strength, secant stiffness at yielding point, and deformation capacity after flexural yielding is realized. of beams. This paper presents an experimental research of the beams used high strength concrete and reinforcement. In this research, we have three objectives. The first objective is to make clear the required ratio and detail of shear reinforcement to prevent bond failure of beams after the flexural yielding is realized. The second one is to investigate the required ratio and detail of hinged part of beams in order that it might have good deformation capacity. The third one is to investigate the effects of slab on flexural performance of the beams.

2 METHODS OF EXPERIMENTS

2.1 Specimens

All specimens are estimated 1/3 scale. Primary factors of the specimens are shown in Table 1., and the relations of some factors during each specimens are shown in Figure 1. and Figure 2. The specimens for the first objective are from No.1 to No.7. Variational factors are ratio and detail of shear reinforcements. It

<table>
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<th>No</th>
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<th>shear reinforcement (%)</th>
<th>Pw (MPa)</th>
<th>σwy (MPa)</th>
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<td>863</td>
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<td>967</td>
<td>71.5</td>
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</table>

Table 1. Primary factors of specimens

factors of slabs (both sides)

section bars transverse bars  σwy σy (MPa) (MPa)

all specimens

section

0.20m

0.27m

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can be observed that effects of the ratio, spacing, and the detail (using sub-ties) for failing types of the beams after flexural yielding is realized. The specimens for the second objective are from No.8 to No.12. The part of the center of them is made, so that it might not be failed on bond. It can be observed that the effects of the ratio and the detail of shear reinforcements for deformation capacity of the beams. Now, No.12 is made to be different from No.9 at tensile reinforcement ratio, but the same at the ultimate strength. The specimens for the third objective are from No.13 to No.18. It can be observed that the effects of the difference, of strength of the bars in the slab to which longitudinal reinforcements in the beams are parallel (We will call them slab bars), compressive strength of slab concrete, spacing of the bars in the slab which longitudinal reinforcements in the beams meet at right angle (We will call them transverse bars in slab), and the slab itself for ultimate strength, initial stiffness, secant stiffness at yielding point, and flexural crack moment. And effective width of the slab by clear span: is for the factors can be observed also.

Design strength of concrete is 68.6MPa of all the specimens, but the part of the slab at No.15 is 29.4MPa.

2.2 Loading method

Loading method of the specimens from No.1 to No.12 is a statical cyclic loading with double curvature, and which from No.13 to No.18 is a statical cyclic cantilever type loading. Each methods are shown in Figure 3. and Figure 4. respectively. Loading path of all the specimens is shown in Figure 5.
3 RESULTS

Matter of convenience, the first and the second objectives are put together. Now, limited angle and yielding angle are defined as deformation angle, when strength degrades 80% against the maximum strength, and when tensile reinforcements yield respectively. And we consider that ultimate strength of beams is little different from yielding strength of the beams.

3.1 Effects of strength margins for shear force and sub-ties

The relations of strength margins for shear force; Qy/Qmu and the yielding and the limited angles are shown in Figure 6. The notations indicate angles and failing types of the specimens after the flexural yielding are realized. And the lateral axis, that is, the Qy/Qmu means ratio of shear strength;Qy against shear force when the beam indicates the ultimate strength;Qmu. The Qy in the case of considering yielding hinge and keeping good deformation capacity more than 1/50 is obtained with the formula (1), and the Qmu is obtained with (2).

\[ Q_y = b \cdot j \cdot p \cdot w \cdot \sigma_{yw} \cdot \cot \phi + \tan \theta \cdot (1 - \beta) \cdot b \cdot d \cdot \nu \cdot \sigma_{yw} / 2 \]  

\[ \beta = \frac{(1 + \cot \phi) \cdot p \cdot w \cdot \sigma_{yw}}{\nu \cdot \sigma_{yw}} \]  

\[ \tan \theta = \sqrt{\left(\frac{h_y}{d} + 1\right) - \frac{h_y}{d}} \]  

\[ \cot \phi = \min(\cot \phi_1, \cot \phi_2, \cot \phi_3) \]  

\[ \cot \phi_1 = 1.0 \]  

\[ \cot \phi_2 = \frac{1}{d \cdot \tan \theta} \]  

\[ \cot \phi_3 = \frac{\nu \cdot \sigma_{yw}}{p \cdot w \cdot \sigma_{yw} - 1.0} \]  

\[ \nu \cdot \sigma_{yw} = 160 \cdot \sigma_{yw} \]  

\[ \nu = 0.25 \cdot \nu \]  

IF \( \sigma_{yw} > 25 \cdot \sigma_{yw} \) THEN \( \sigma_{yw} = 25 \cdot \sigma_{yw} \)  

IF \( p \cdot w \cdot \sigma_{yw} > \nu \cdot \sigma_{yw} / 2 \) THEN \( p \cdot w \cdot \sigma_{yw} = \nu \cdot \sigma_{yw} / 2 \)

Obviously, the specimens which are low Qy/Qmu are not ductile, and the Qy/Qmu separates every failing type clearly. The specimens which are high Qy/Qmu; more than 2.5, have good deformation capacity, and are failed on compressive shear. The next good ones are failed on bond, and the worst ones; less than 1.0, are failed on tensile shear. The yielding angles are little different each other. Comparing with No.3 and No.6, the specimen arranged sub-ties has a better deformation capacity than the one which is not arranged sub-ties and the Qy/Qmu is the same as the specimen. This result shows that sub-ties make beams ductile.

3.2 Effects of strength margins for bond stress

The relations of strength margins for bond stress; \( \tau_{yw} / \tau \) and the limited angles are shown in Figure 7. And the lateral axis, that is, the \( \tau_{yw} / \tau \) means ratio of bond strength; \( \tau_{yw} \) against design bond stress; \( \tau \). The relations of the Qy/Qmu and the \( \tau_{yw} / \tau \) are shown in Figure 8. The design bond stress in the case of considering yield hinges both ends of the beam is obtained with the formula (3), and the bond strength in the case of those is obtained with (4), but the case for top reinforcement of the beams should be 0.8 times.

\[ \tau = d_s \cdot \Delta \sigma / (4 \cdot (h_s - d)) \]  

IF \( b_s < b_c \) (corner split type) THEN

\[ \tau = 60 \cdot p \cdot w \cdot b \cdot (N_s + 2) / (N_s \cdot \Sigma d_s) + 0.4 \]  

IF \( b_s < b_c \) (whole split type) THEN

\[ \tau = (70 \cdot A_{in} - b_s) / (A_{in} \cdot d_s) \]  

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Figure 6. Deformation capacity and \( Q_y / Q_{mu} \)
Figure 7. shows that the falling types are separated clearly not only with $Q_y/Q_{mu}$ but also with $\tau_{bu}/\tau_r$. And it seems that the $\tau_{bu}/\tau_r$ needs more than 0.5 to prevent bond failure, and as a result, the beams keep good deformation capacity. Referring Figure 8., the falling types are separated more clearly than Figure 7. Each specimens which are the same falling types are collected with the $Q_y/Q_{mu}$ and the $\tau_{w}/\tau_r$, and if a beam is made to be high $Q_y/Q_{mu}$, high $\tau_{w}/\tau_r$, and good deformation capacity will be consequently attained. The specimen of No.11 is the only exception. It has good deformation capacity, but the $Q_y/Q_{mu}$ of this is low. It is considered that the $Q_y$ of No.11 is calculated less than the others because the strength of shear reinforcement is the lowest in all the specimens, but it has enough shear reinforcement ratio to be ductile. Consequently, shear reinforcement ratio governs deformation capacity of the beam.

3.3 Effects of slab factors

The factors which are influenced by the slab are as follows: the initial stiffness $Ke$, the secant stiffness at yielding point $\alpha_y$, the flexural crack, the ultimate strength $Mc$, $Mu$. The experimental results of the specimens of the third objective are shown in Figure 9. It shows that lines drawing the cracking points and the yielding ones. All of them do not indicate the ultimate point till the deformation angle 1/20. Accordingly, the ultimate deformation angles of them are estimated as 1/20. The effective width of the slab by clear span $ws/ho$ for the four factors, as mentioned above, are shown in Figure 10.,11. And the lateral and the longitudinal axis, that is, the $ws/ho$ means the ratio of $ws$ against ho in the each factors. $ws$ is an assumed length of the slab in calculating of factors, so that it is coincided with experimental value. The ho is clear span of the specimens. The $Mu$ is obtained with the formula (5), the $Mc$, the $Ke$, and the $\alpha_y$ are obtained with the formula (6), (7), (8), respectively.

$$W_y = 0.9 \left( \Sigma (a_n \cdot \sigma + a_s \cdot \sigma_s) \right) \cdot d \quad \quad (5)$$

$$W_y = 180 \cdot (\sigma + \sigma_s)^{0.6} \cdot Z_s \quad \quad (6)$$

$$Z_s = \frac{I_s}{X}$$

$$X = ((l - b) \cdot t \cdot b \cdot D^2 \cdot (a_n + a_s) \cdot (t - 2) \cdot (n - 1)) / ((l - b) \cdot t \cdot b \cdot D + (2 \cdot a_n + a_s) \cdot (n - 1))$$

Figure 7. $\tau_{bu}/\tau_r$ and limited angle

Figure 8. $Q_y/Q_{mu}$ and $\tau_{bu}/\tau_r$

Figure 9. Skeltons (No.13 - No.18)
\[ I = \frac{((l-s)b - t - (s^2/3) + (2X - t)^2 + bD(D^2/3) + (2X - D)^2))}{4a_r - ((b-d)x^2 + (D - x)^2)} - (h - 1) \]

\[ K_r = \frac{1/(\sigma_r + \sigma_s)}{\sigma_s} \]

\[ \delta = \frac{h_r^2}{(3E - 1)} \]

\[ \delta_r = \frac{\kappa_1}{} \]

\[ G_r = \frac{E}{2(1 - \nu)} \]

\[ \alpha_r = \frac{bD(2a_1 + a_2)}{(h - 1)} \]

\[ h_r = \frac{h + bD}{2} \]

\[ K_r = \frac{\alpha_r \cdot K_s}{\alpha_s} \]

\[ \alpha_s = \frac{(0.043 + 0.655 - P) + 0.043 - a/D}{(a/b)} \]

\[ P_r = \frac{a + a_s}{(l - b + bD)} \]

For reference, the exactness of the calculated values with these formulas against the experimental values for the specimen of which section is rectangular must be certified. Accordingly, we show the exactness which are in the case of No.17 with Table 2. This indicates that the calculated values of the factors are nearly coincided with experimental value, except the \( \alpha_r \).

Obviously, comparing with No.13 and No.17, the yield strength of No.13 is made higher than that of No.17 with the slab, but yield deformation is not made longer. The \( ws/h_o \) of the specimen of No.18, that is short clear span, is shorter than the other specimens in all the factors. That of No.14, that is used high strength slab bars, is longer than the other specimens, except the \( \alpha_s \). On considering the \( \alpha_r \), it seems that it is more suitable to observe the tangent stiffness during the cracking point and the yielding point (We will call it second stiffness), than considering itself. The yielding point of No.14 indicates higher than the others, but the second stiffness of it is not different from the others with slab. And the yielding deformation of the others: No.13, No.15, No.16, are not different from No.17. Consequently, the \( \alpha_r \) of beams with slab should be estimated from yielding point. The yielding strength is obtained with the formula (5), and the yielding deformation is obtained as the same value as the beam of which section is rectangular. If high strength slab bars are used in a beam, the yielding point should be obtained at the intersection of next two lines. One is the lateral line which indicates the yielding strength obtained with (5), and the other is the line which indicates second stiffness of the beams in which usual strength slab bars are used. The \( ws/h_o \) will be generally considered about 0.25, 0.15, 0.2, all of slab, for the \( \alpha_r \), the \( Ke \), the \( Mc \), the \( Mu \), respectively.

<table>
<thead>
<tr>
<th>Ke(N/m)</th>
<th>Ky(N/m)</th>
<th>Mc(N*m)</th>
<th>My(N*m)</th>
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Figure 10. \( ws/h_o \) for strength

Figure 11. \( ws/h_o \) for stiffnesses
4. CONCLUSIONS

1) Behaviors of beams after flexural yielding is realized are governed with mainly shear reinforcement ratio. It is shown in Figure 12, which failing types of the specimens are added on Figure 1. Obviously, they are separated with the shear reinforcement ratio clearly. If it is less than 0.4%, members will be failed on tensile shear, and if it is near 0.4%, members will be failed on bond, and if it is greater than 0.4%, members will have good deformation capacity.

2) It will be able to get simultaneously, high Qy/Qru, high \( r_{\omega}/r_{\eta} \), and good deformation capacity for beams.

3) The most effective factor for beams in slab is tensile strength of the slab bars.

4) Effects of slab make ultimate strength of beam higher, but do not make yielding deformation higher, and accordingly, it is important to estimate to consider the effects of floor slab for performance of beams.

![Figure 12. Failing types of specimens](image)

### Symbols

- \( b \): width
- \( t \): thickness of slab
- \( D \): depth
- \( d \): effective depth
- \( h \): clear span
- \( G \): shear modulus
- \( s \): spacing of shear reinforcement
- \( p \): shear reinforcement ratio
- \( h \): effective clear span of specimen
- \( w \): effective width of slab
- \( \sigma_{\omega} \): strength of shear reinforcement
- \( \sigma_{\eta} \): compressive strength of concrete
- \( \sigma_{\omega} \): compressive strength of concrete at slab
- \( E \): Young's modulus of concrete
- \( n \): Young's modulus ratio of concrete to steel
- \( a \): area of tensile reinforcement
- \( a_{\omega}\): area of reinforcement in slab
- \( \alpha \): tensile strength of reinforcement
- \( \alpha_{\omega}\): tensile strength of slab reinforcement
- \( \chi \): shape factor of shear deformation
- \( P \): tensile reinforcement ratio
- \( K \): initial stiffness
- \( A \): equivalent area
- \( \delta \): shear deformation
- \( \delta_{\omega}\): flexural deformation
- \( M \): ultimate moment
- \( M_{\omega}\): flexural crack moment
- \( Q \): shear strength
- \( M_{\omega}\): flexural yield moment
- \( I_{\omega}\): equivalent inertia
- \( g \): gravity
- \( K_{\omega}\): secant stiffness at yielding point
- \( Z \): equivalent section modulus
- \( X \): distance from face to centroid
- \( \alpha \): ratio of stiffness at yielding to initial point
- \( \tau \): bond stress due to flexure
- \( \tau_{\omega}\): bond strength
- \( \tau_{\omega}\): bond strength of concrete
- \( \tau_{\omega}\): bond strength of stirrup
- \( j \): distance between top and bottom bars
- \( A \): area of shear reinforcement covering corner steel
- \( d \): diameter of longitudinal reinforcement (diameter of corner steel)
- \( d \): depth of corner concrete from the center of corner steel
- \( \Delta \sigma \): stress difference of longitudinal reinforcement at the two ends of a member in the yield mechanism ensuring design
- \( N \): number of flexural steels directly hooked by sub-ties
- \( N \): number of flexural steels not hooked by sub-ties
- \( N \): total number of flexural steels

### Reference


### Acknowledgment

This paper is responsible to Dr. Otani who is a chief of Structural Division, High Strength Steel and Concrete Project. The right of this paper is reserved by this author.