

Ductility of reinforced concrete columns with various reinforcing arrangements

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ABSTRACT: This paper reports on the evaluation method of the inelastic deformation capacity of reinforced concrete square columns confined by various reinforcement, subjected to shear force, flexural moment and high axial force. The concrete model was derived, taking into account the effects of the details of the sections. Lateral load-deflection relations were calculated using the concrete models with some modifications, taking into account the confinement provided by the elastic region of the specimen and the rigid base stub adjacent to the plastic hinge region. The simplified method to calculate the deformation capacity of reinforced concrete columns was proposed and compared with experiments with various reinforcing arrangements and found to be useful to evaluate the deformation capacity of the members.

1 INTRODUCTION

Recent earthquake resistant design concept of structures places explicit emphases on the inelastic deformation capacity in addition to the previously accepted resisting capacity. This paper reports on the evaluation method of the inelastic deformation capacity of reinforced concrete square columns confined by various reinforcement, subjected to shear force, flexural moment and high axial force.

The concrete model was derived from test results with axial load only, taking into account the effects of the details of the sections such as concrete strength, diameter and spacing of hoops, hoop strength, hoop shape (hoop with hook or spiral) and interior ties.

The moment-curvature relations calculated using this concrete model were roughly comparable with the test results subjected to the constant moment distribution. However, the lateral load-displacement relations calculated with this model generally underestimated the test results subjected to the bending moment and the shear force. This is because the confinement was provided by the elastic region of the specimen and the rigid base stub adjacent to the plastic hinge region in addition to the confinement by the hoop reinforcement. This confinement was derived from the test results in this study.

On the other hand, it is not convenient to calculate the whole load-displacement relations to obtain the deformation capacity of the specimen. The simplified method to

calculate the deformation capacity of reinforced concrete columns was proposed and compared with experiments with various reinforcing arrangements.

2 STRESS-STRAIN RELATIONSHIP FOR CONCRETE

The stress-strain relationship of the concrete was used to calculate the moment-curvature relations of column sections. The confinement of the concrete, subjected to axial load only, is provided mainly by the hoop reinforcement. However, that of the column, subjected to axial, shear and moment loads, is provided by elastic region of the specimen and the rigid base stub adjacent to the critical section in addition to the hoop reinforcement.

The concrete model confined by the square hoop reinforcement only was developed, taking into account the effects of the concrete strength, the diameter and the spacing of hoops, the hoop strength, the hoop shape (hoop with hook or spiral) and the presence of interior ties (see Kato, 1991). The maximum axial stress and the strain of the confined concrete and the slope of the falling branch, are given using the confining stress by the hoop reinforcement. The confining stress by the hoop reinforcement is given, taking account of the details of reinforcement.

The confining stress provided by the hoop reinforcement and the elastic region is given by adding the confining stress by the elastic region to that by the hoop reinforcement.

The maximum stress σ_{cp} , the strain ϵ_{cp} and the slope of the falling branch E_{up} of the concrete confined by the hoop reinforcement and the elastic region are as follows:

$$\sigma_{cp} = \sigma_c + 4.1 \cdot (\sigma_{tp} + \sigma_{tb1}) \quad (1)$$

$$\epsilon_{cp} = \epsilon_c + 0.00153 \cdot (\sigma_{tp} + \sigma_{tb1}) \quad (2)$$

$$E_{up} = \frac{E_u}{1 + 12.8 \cdot (\sigma_{tp} + \sigma_{tb2})} \quad (3)$$

where σ_c , ϵ_c and E_u are the strength, the axial strain and the slope of the falling branch of the plain concrete and σ_{tp} is the confining stress provided by the square hoop reinforcement in Mpa. σ_{tb1} and σ_{tb2} are confining stresses provided by the elastic region of the column and the base stub of the specimen in Mpa, which are discussed in Section 3. The confining stress σ_{tp} is

$$\sigma_{tp} = \frac{\epsilon_c}{(2.8 + 0.00357 \cdot eK_{cf}) / K_{cf} - 0.00153} \quad (4)$$

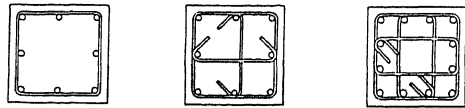
where

$$\sigma_t = \frac{a_w \cdot \sigma_y}{D_c \cdot S} \quad (5)$$

$$K_{cf} = \frac{a_w \cdot E_s}{D_c \cdot S} \quad (6)$$

$$eK_{cf} = K_{cf} \cdot \frac{\sqrt{S/D_c}}{N_{div}} \quad (7)$$

where a_w , σ_y , E_s (in Mpa), D_c and S are the area, the yielding stress, the modulus of elasticity of the hoop reinforcement, the core depth and the hoop spacing. N_{div} is the number of core regions divided by ties as follows.



$N_{div} = 1$

$N_{div} = 2$

$N_{div} = 3$

Note that σ_{tp} must be equal to σ_t in case that σ_{tp} given by equation (4) is less than 0 or greater than σ_t . On the other hand, σ_{tp} must be equal to 0 in case that the concrete model is used for the cover concrete not confined by the hoop reinforcement.

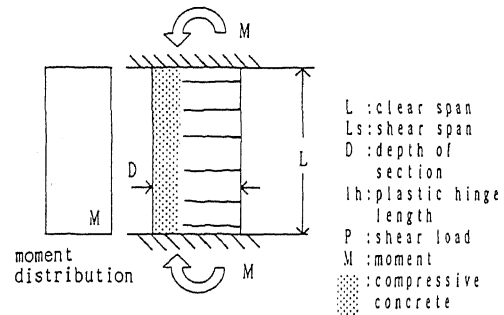
3 CONFINEMENT OF CONCRETE IN HINGE REGION PROVIDED BY ELASTIC REGION

The confining stress of the concrete in the

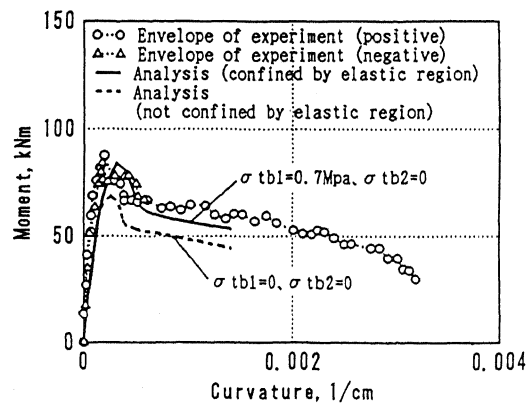
plastic hinge region provided by the undamaged elastic region of the column and the rigid base stub adjacent to the hinge region, was derived from the experimental results. Namely, observed load-deflection relations of the column specimens subjected to axial load were compared with theoretical load-deflection relations using the confined concrete model, varying the confining stress provided by the elastic region and the stub.

Theoretical moment-curvature curves for column sections with flexure and axial load were derived on the basis that plane sections before bending remain plane after bending. The deformation of the column was derived on the assumption that the curvature of the plastic hinge region is constant (see Fig. 2(a)). The elastic shear deformation was also taken in account in this study.

The calculated moment-curvature relations are compared with the test result in Fig. 1(b). The test was conducted by Hiraishi et



(a) Crack pattern and compressive concrete of plastic hinge region

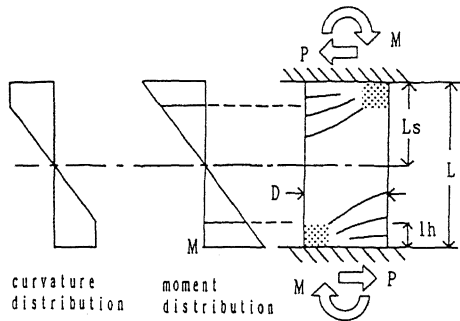


(b) Comparison of moment-curvature relations between experiment and analysis

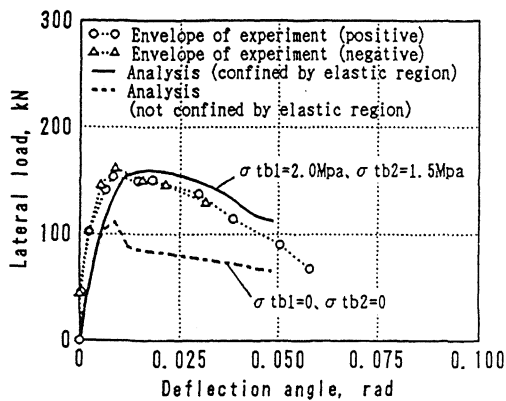
Figure 1. Behaviors of column subjected to constant moment distribution

al. at Building Research Institute in Japan (1990). The specimen with 250mm square section, 500mm height and transverse reinforcement ratio of 0.64% (40mm spacing) was subjected to constant axial load of 991 kN and constant moment distribution along the column. Two 13mm diameter deformed bars were used as the tensile longitudinal reinforcement. The strength of concrete, longitudinal reinforcement and transverse reinforcement were 33.7 Mpa, 658 Mpa and 996 Mpa, respectively. Figure 1(a) shows the idealized crack pattern and the compressive concrete of the plastic hinge region.

The calculated moment-curvature relation with the concrete model confined by hoop reinforcement only is plotted in a dashed line in Fig. 1(b). The calculated moment is somewhat lower than that of the experiment but the shape of the envelope curve is similar to that of the experiment, which shows that the confinement provided by the elastic region is not so significant in this



(a) Crack pattern and compressive concrete of plastic hinge region



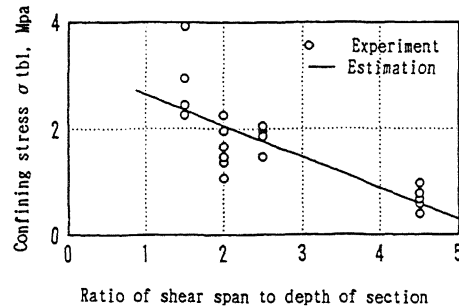
(b) Comparison of lateral load-deflection relations between experiment and analysis

Figure 2. Behaviors of column subjected to shear and moment

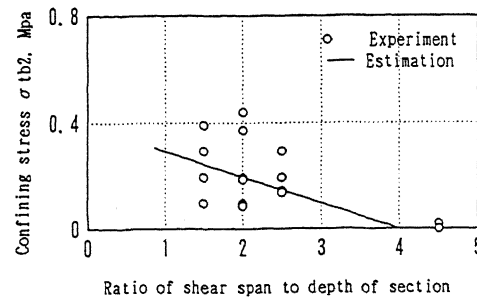
case. A better agreement is shown in the case with the concrete confined by the elastic region of 0.7 Mpa, which was arbitrarily taken to match the observed curve.

The calculated lateral load-deflection angle relations are compared with the test result in Fig. 2(b). The deflection angle, the top displacement divided by overall height, was used to describe the deflection of the column specimen in this paper. The specimen, with the same section and reinforcement as the specimen shown in Fig. 1, was subjected to constant axial load of 991 kN, shear and moment. The height of the specimen was 1250mm, the shear span ratio of which was 2.5. Figure 2(a) shows the idealized crack pattern and the compressive concrete of the plastic hinge region. Note that the area of the compressive concrete in the plastic hinge region is very small compared with that of Fig. 1(a), which means that the confinement provided by the undamaged elastic region of the specimen and the rigid base stub adjacent to the plastic hinge region is significant.

The calculated lateral load-deflection relation with the concrete model confined by



(a) Confining stress at maximum stress point of concrete



(b) Confining stress after maximum stress point of concrete

Figure 3. Estimation of confinement provided by elastic region of column and base stub

hoop reinforcement only is plotted in a dashed line in Fig. 2(b). The calculated lateral load is much lower than that of the experiment and the shape of the envelope curve is quite different from that of the experiment, which shows that the confinement provided by the elastic region is not negligible in this case. A good agreement is shown in the case with the concrete model confined by the elastic region of 2.0 and 1.5 Mpa, which were arbitrarily taken to match the observed curve.

Figure 3(a) shows the effects of the shear span ratio, the ratio of the shear span to the depth of the section, on the confining stress provided by the elastic region at the maximum point of the stress-strain curve. The confining stresses were arbitrarily taken to match the observed curves of 25 specimens including 2 specimens shown in Fig 1 and 2. The imaginary shear span ratios of the columns subjected to constant moment distribution, were assumed to be 4.5, taking the area of the plastic hinge region into account. Figure 3(b) shows the effects of the shear span ratio on the confining stress provided by the elastic region after the maximum point of the stress-strain curve. The solid lines in Fig 3 were obtained by the method of least squares and expressed by

$$\sigma_{tb1} = 3.2 - 0.6 \cdot (L_s/D) \quad (\geq 0) \quad (8)$$

$$\sigma_{tb2} = 0.4 - 0.1 \cdot (L_s/D) \quad (\geq 0) \quad (9)$$

where L_s/D is the shear span ratio. Note that the unit of the stress is Mpa.

4 DEFORMATION CAPACITY OF COLUMNS

The method to calculate the deformation capacity of reinforced concrete columns is

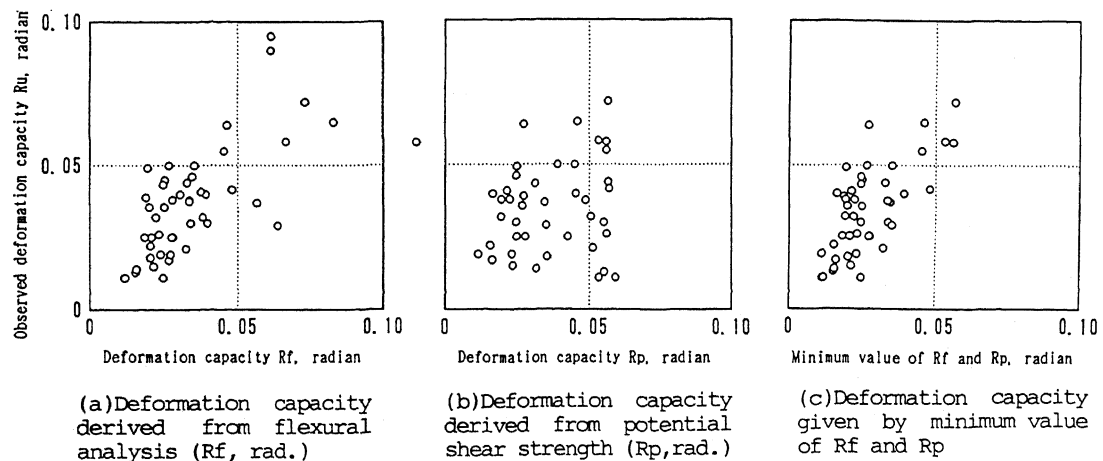


Figure 5. Comparison of deformation capacity between experiment and models

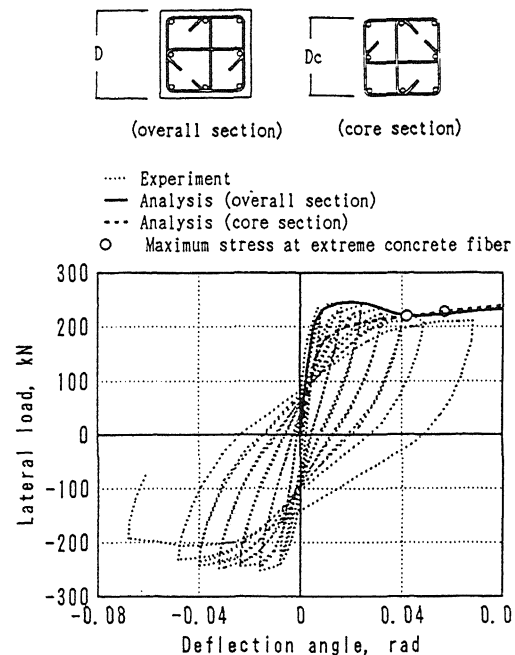


Figure 4. Analytical comparison of load - deflection curves between column with overall section and that with core section only

described in this section and the results are compared with the observed deformation capacity during the experiments.

4.1 Calculated deformation capacity derived from flexural analysis

The calculated lateral load-deflection relations with different two models are

compared with the test result in Fig. 4. The test was conducted by Kato et al. at Niigata University in Japan (1990). The specimen with 250mm square section, 750mm height and transverse reinforcement ratio of 1.3% (30mm spacing) was subjected to constant axial load of 746 kN, shear and moment. Three 13mm diameter deformed bars were used as the tensile longitudinal reinforcement. The strength of concrete, longitudinal reinforcement and transverse reinforcement were 29.1 Mpa, 362 Mpa and 390 Mpa, respectively.

The calculation with the overall section is plotted in a solid line and that with the core section only is plotted in a dashed line in Fig. 4. A good agreement was shown between the experiment and the calculation with the overall section. On the other hand, calculated lateral load with the core section was lower than that of experiment in a small displacement range. However, the calculation with the core section gave a good agreement with another two curves after the deflection angle of 0.04 radian. The deflection angles when the extreme concrete fibers of core sections reached the maximum stress point in calculations, plotted in circles on the calculated lines, were approximately comparable with the deflection angle when the observed lateral load decreased to 80% of the maximum strength. This means that this deflection angle, the extreme concrete fibers of the core section reached the maximum stress point, represents the deformation capacity of the column.

The curvature of the core section without cover concrete, when the extreme concrete fiber reached the maximum stress point, can be simplified to the equation (10), neglecting the effect of the longitudinal reinforcement.

$$\phi f = \frac{S1 \cdot Bc}{N} \quad (10)$$

where Bc, N and S1 are the core width, the axial load and the hysteresis area of the core concrete from the origin point through the maximum stress point on the stress-strain plane. The deformation capacity Rf, derived from the flexural analysis written as equation (11), was obtained with equation (10) on the assumption that the curvature of the plastic hinge region was constant.

$$Rf = \frac{n}{n+1} \cdot \sigma_{cp} \cdot \epsilon_{cp} \cdot \frac{Bc \cdot 2 \cdot Ls + 2 \cdot lh - lh^2 / Ls}{6} \quad (11)$$

$$(n = E_{cp} \cdot \epsilon_{cp} / \sigma_{cp})$$

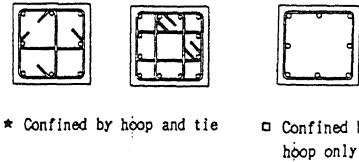
where Ecp is the modulus of the elasticity of the confined concrete. The hinge length lh, written as equation (12), was proposed by Yoshioka et al. (1979).

$$lh = 0.5(Ls/D)d \quad (12)$$

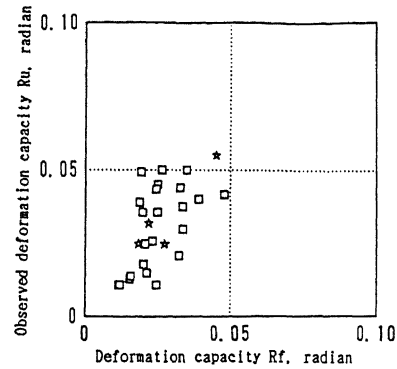
where d is the effective depth, the distance between the extreme fiber of compressive concrete and the tensile steel.

4.2 Calculated deformation capacity derived from potential shear strength

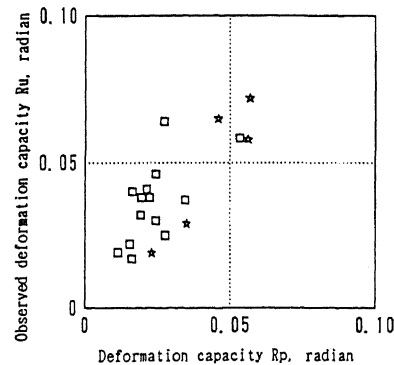
The deformation capacity Rp, derived from potential shear strength, was proposed in the design guidelines for earthquake



* Confined by hoop and tie □ Confined by hoop only



(a) Specimens with deformation capacity derived from flexural analysis (Rf < Rp type)



(b) Specimens with deformation capacity derived from potential shear strength (Rf > Rp type)

Figure 6. Comparison between experiment and models varying reinforcing arrangements

resistant reinforced concrete buildings based on ultimate strength concept by Architectural Institute of Japan in 1990.

4.3 Result

The deformation capacity observed during the test was defined as the deformation, at which the restoring force of the column decreased to 80% of the maximum lateral load.

The deformation capacity R_f given by equation (11) and R_p derived from the potential shear strength were compared with the test results of 55 specimens in Figure 5(a) and (b), respectively. These specimens were tested in Japan and the range of the main variables were $f'_c = 19$ to 30Mpa, $L_s/D = 1.0$ to 3.0, $N/bDf'_c = 0$ to 0.91. These two figures show the wide scattering. On the other hand, Figure 5(c) of the relationship between the minimum values of R_f and R_p and the test results shows a good agreement.

Figure 6(a) compares the deformation capacity R_f with the test results of the specimens with various reinforcing arrangements, the deformation capacity R_f of which were less than R_p . The deformation capacity R_f roughly predicted the observed deformation capacity. A good agreement was shown in Figure 6(b) with the deformation capacity R_p also.

5 CONCLUSION

The moment-curvature relations calculated using the concrete model proposed were roughly comparable with the test results subjected to the constant moment distribution. However, the lateral load-displacement relations calculated with this model underestimate the test results subjected to the moment and the shear load. This is because the confinement was provided by the elastic region of the specimen and the rigid base stub adjacent to the plastic hinge region in addition to the confinement by the hoop reinforcement. This confinement was derived from the test results as equations (8)(9).

The simplified method to calculate the deformation capacity of reinforced concrete columns, derived from the flexural analysis, was proposed as equation (11) and this deformation capacity R_f was compared with experiments with various reinforcing arrangements. The minimum value of R_f and R_p , the deformation capacity derived from the potential shear strength, was found to be useful to evaluate the deformation capacity of the members.

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