

## Axial stress ratio and drift allowed in R/C columns

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**ABSTRACT :** The maximum available drift of columns in the seismic design which is determined by considering both the stable limit and "the overlap of the inelastic strain in concrete" is discussed in this paper. It is shown that the maximum available drift is depending on the axial stress ratio and the extent of concrete confinement.

### 1 INTRODUCTION

The authors pointed out the followings in Ref.1 and Ref.2 that :

1) For columns subjected to monotonic lateral loads or columns with low axial stress ratio subjected to cyclic lateral loads, a significant degradation in the resisting moment occurred at the deformation of the stable limit which had been proposed by the authors as the deformation capacity of columns after flexural yielding. The columns also lost the axial load carrying capacity there.

2) However, for columns with high axial stress ratio subjected to cyclic lateral loads, a significant moment deterioration and a increase of centroidal compressive strain due to the cyclic loading appeared in the large deformation region. These phenomena were comprehensively explained by the cyclic behavior of concrete. Namely at the neighborhood of the centroid of the critical section, the strain in concrete was compressive in both the positive and negative loading, and once the concrete experienced large inelastic compressive strain in the positive direction, the compressive stress of the concrete in the negative direction was considerably reduced due to the unloading because the compressive strain was smaller than that in the positive loading. Therefore in order to balance against the axial load, the compressive strain at the entire section increased. This increase of the compressive strain resulted in the moment deterioration successively. In this paper this reason for the cyclic moment deterioration and axial shortening of columns is refereed as "the overlap of inelastic strain in concrete". The column eventually lost axial load carrying capacity at the point which was named as quasi-stable limit by the authors.

To determine the available maximum drift of columns in the seismic design, both the stable limit and the cyclic moment deterioration should be taken into consideration, because it may be expected in the usual seismic design that there is little degradation in the resisting moment and little axial shortening until the available maximum drift.

In this paper, first, the proposed methods for estimating the strains and the curvature at the stable limit are examined by test results. Secondly it is shown that "the overlap of the inelastic strain in concrete" has a remarkable influence on the seismic behavior of columns. Finally the available maximum drift angle based on the stable limit and "the overlap of the inelastic strain in concrete" is proposed and shown in equations and figures, assuming that the stress-strain characteristics of core concrete is represented by the simplified bi-linear model.

### 2 EVALUATION METHODS FOR STABLE LIMIT

#### 2.1 The strain in the extreme compressive concrete fiber

The strain in the extreme compressive concrete fiber at the stable limit,  $\epsilon_{c,sl}$ , is given as the strain of the intersecting point of the stress-strain curve of concrete and the line which represents the compressive stress level in concrete (Fig.1)(Ref.3).

The strain in the extreme compressive concrete fiber at the quasi-stable limit,  $\epsilon_{c,sl'}$ , is also obtained from the stress-strain curve of concrete and the the compressive stress level in concrete at quasi-stable Limit which is considered to

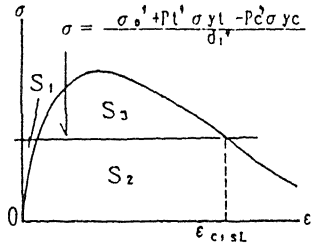


Fig. 1. The strain in the extreme compressive fiber and at stable limit.

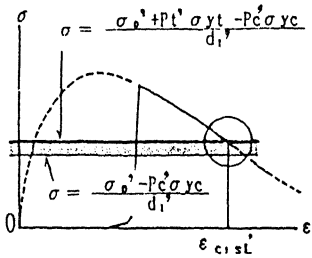


Fig. 2. The strain in the extreme compressive fiber at quasi-stable limit.

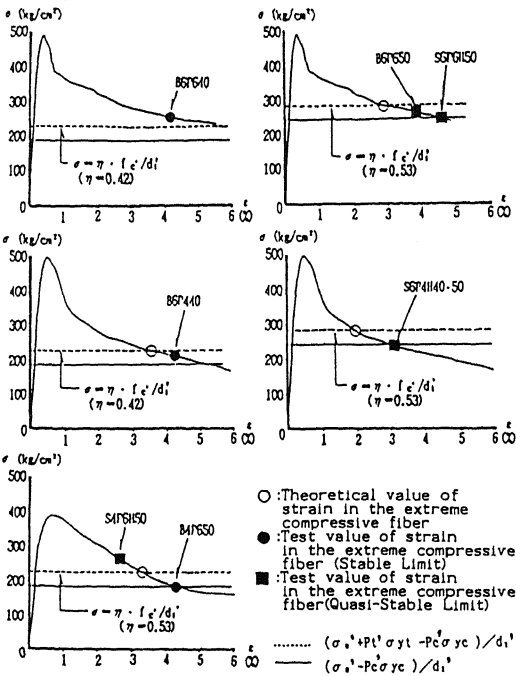


Fig. 3. The measured and theoretical strain in the extreme compressive fiber at stable limit and quasi-stable limit.

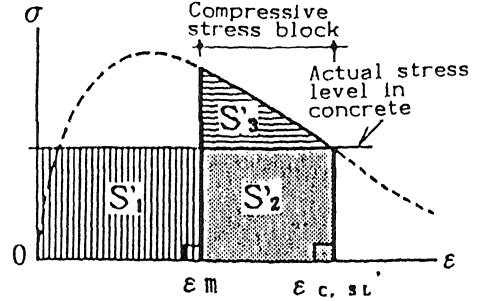


Fig. 4. The areas in Eq.(2).

$\sigma_{yt}$  : Yield stress of tension steel  
 $\sigma_{yc}$  : Yield stress of compression steel

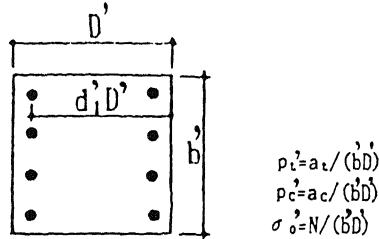


Fig. 5. Column section.

exist between two parallel lines (Fig.2)(Ref.3). The assumed column section is shown in Fig.5. Figure 3 shows the measured strains in the tests and the theoretical strains at stable limit and the quasi-stable limit. The reasonable good agreement is obtained between the measured and theoretical strains.

2.2 The curvature at the stable and quasi-stable limit

The curvature at the stable limit is calculated by Eq.(1), where S1,S2 and S3 are the areas shown in Fig.1 (Ref.3) The curvature at quasi-stable limit is calculated by Eq.(2), where S1',S2' and S3' are the areas shown in Fig.4 (Ref.3).  $\epsilon_m$  in Fig.4 is dependent on the loading history.

$$\phi_{sl} = \frac{(S_2 + S_3)}{(S_1 + S_2)} \times \frac{\epsilon_{c,sl}}{d_1 D'} \quad (1)$$

$$\phi_{sl'} = \frac{(S_2' + S_3')}{(S_1' + S_2')} \times \frac{\epsilon_{c,sl'}}{d_1 D'} \quad (2)$$

The measured curvature in the tests and the calculated curvature are compared on Table 1. The values shaded in Table 1 which expresses the ratio of the measured and calculated curvature indicated that the proposed methods for estimating the

curvature at the stable limit and quasi-stable limit are adequate.

### 3 INFLUENCE OF OVERLAP OF THE INELASTIC STRAIN IN CONCRETE AT THE CRITICAL SECTION

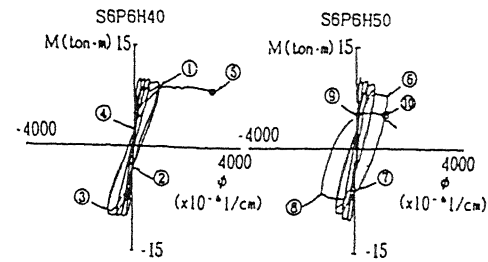
Figure 6 shows the measured moment-curvature relations, the measured axial strain-curvature relations, and the measured strain profiles in the critical section at the peak curvature and the curvature of zero, for the two columns with low and high axial stress ratio respectively. In the column with high axial stress ratio, the deterioration in moment increases with the extending of the region of the overlapped compressive strain. However in the column with low axial stress ratio, no deterioration and no axial shortening are observed because the large compressive strain which is considered to exceed the elastic range of concrete does not overlap each other in both directions.

Figure 7 shows the analytical moment-curvature relation and the analytical axial strain-curvature relation of the concrete section with no steel bars subjected to the cyclic curvature reversals. The used stress-strain curve for concrete is shown in Fig.7(a). When the section is subjected to the successive cyclic curvature with the small amplitude after the large curvature reversal which has resulted in the overlap of the inelastic strain in concrete, the cyclic moment deterioration and axial strain are remarkably accumulated, and finally the section reaches the quasi-stable limit. Figure 8 shows the analytical moment-curvature relation and the analytical axial strain-curvature relation of the same concrete section. The used stress-strain curve for concrete is shown in Fig.8(a). Though no deterioration in moment is observed, the axial strain is accumulated

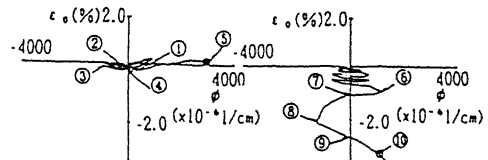
Table 1. The measured and calculated curvature at stable limit and quasi-stable limit.

SPECIMEN	$\phi_{exp}^{1)}$	$cal.1 \phi_{SL}^{2)}$	$\frac{\phi_{exp}}{cal.1 \phi_{SL}}$	$cal.2 \phi_{SL}^{3)}$	$\frac{\phi_{exp}}{cal.2 \phi_{SL}}$
	( $\times 10^{-6} 1/cm$ )	( $\times 10^{-6} 1/cm$ )		( $\times 10^{-6} 1/cm$ )	
B6P6-40	2891	2091	1.38	2532	1.14
B6P6-50	1616	1200	1.35	2508	0.64
B6P4-40	3679	2832	1.30	3036	1.21
B4P6-50	3016	2769	1.09	3249	0.92
S6P6H50	1250	1101	1.14	2757	0.45
S6P4H40-50	472	455	1.04	2208	0.21
S4P6H150	1383	1098	1.26	1635	0.84

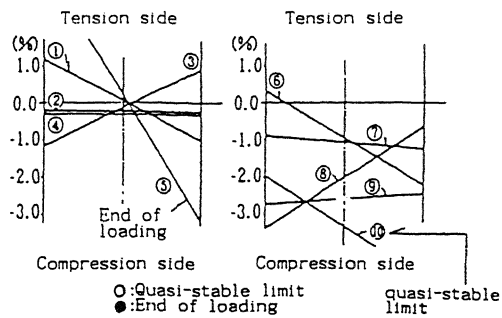
- 1) The measured curvature
- 2) The calculated curvature at quasi-stable limit
- 3) The calculated curvature at stable limit



(a) Moment-curvature relation



(b) Axial strain-curvature relation



(c) Strain profiles of core concrete

Fig. 6. Test results for the two columns with low and high axial stress ratio.

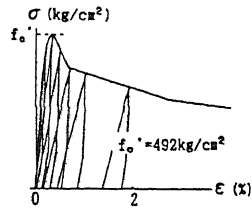
due to the overlap of the inelastic strain in concrete.

### 4 THE AVAILABLE MAXIMUM DRIFT IN SEISMIC DESIGN

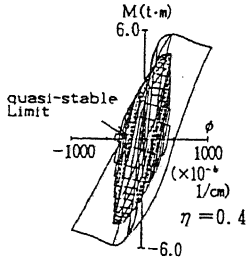
The strain profiles of the critical section with a constant axial load ( $N$ ) subjected to the cyclic curvature reversals ( $\phi$ ) are idealized in Fig.9. The stress-strain characteristics and the hysteresis rule of core concrete is also idealized in Fig.10. In Fig.10,  $f_c'$  is the maximum compression stress of core concrete, and  $\epsilon_s$  is the strain corresponding to  $f_c'$ .  $\alpha$  is a ratio of the stiffness in the falling branch to the initial stiffness of core concrete, and it represents the extent of confinement.

#### 4.1 The drift capacity based on the overlap of inelastic strain in concrete

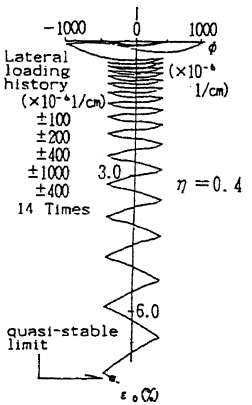
The following two equations are obtained



(a) Stress-strain relation



(b) Moment-curvature relation



(c) Axial strain-curvature relation

Fig. 7. Moment-curvature analysis and axial strain-curvature analysis.

from the condition that the overlapped compressive strain in concrete should not exceed the elastic range of concrete at the peak curvature of cyclic reversals and at the curvature of zero.

$$2 \varepsilon_{o, p} \leq \varepsilon_B \quad (3)$$

$$\varepsilon_{o, o} \leq \varepsilon_B \quad (4)$$

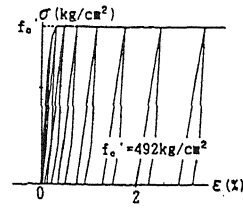
where,  $\varepsilon_{o, p}$  is the strain of the centroid at the peak curvature,  $\varepsilon_{o, o}$  is the strain of the centroid at the curvature of zero (Fig.9). Under the condition expressed by Eq.(3), the relationship between the maximum curvature,  $\phi$ , and the axial stress ratio,  $\eta$ , can be obtained in the following form.

$$\text{for } 1 < \frac{\phi D'}{\varepsilon_B} \leq 1 + \frac{\alpha}{2} \text{ and } \alpha \neq 0$$

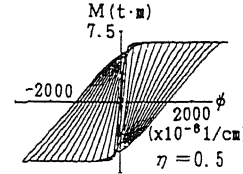
$$\eta = \frac{1}{2} - \frac{\alpha}{8} \left( \frac{\phi D'}{\varepsilon_B} + \frac{1}{\phi D' / \varepsilon_B} - 2 \right) \quad (5-1)$$

$$\text{for } 1 + \frac{\alpha}{2} < \frac{\phi D'}{\varepsilon_B} \text{ and } \alpha \neq 0$$

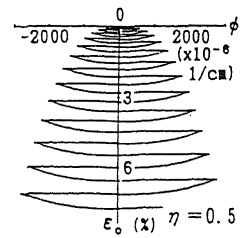
$$\eta = \frac{1}{2} \left( 1 + \frac{1}{\alpha} \right) \frac{1}{\phi D' / \varepsilon_B} \quad (5-2)$$



(a) stress-strain relation



(b) Moment-curvature relation



(c) Axial strain-curvature relation

Fig. 8. Moment-curvature analysis and axial strain-curvature analysis.

$$\text{for } 1 < \frac{\phi D'}{\varepsilon_B} \text{ and } \alpha = 0$$

$$\eta = \frac{1}{2} \quad (5-3)$$

$\eta$ , under the condition expressed by Eq.(4) is also obtained below.

$$\text{for } 1 < \frac{\phi D'}{\varepsilon_B} \leq 1 + \frac{2}{1+\alpha}$$

$$\eta = 1 - \frac{1+\alpha}{4} \left( \frac{\phi D'}{\varepsilon_B} + \frac{1}{\phi D' / \varepsilon_B} - 2 \right) \quad (6-1)$$

$$\text{for } 1 + \frac{2}{1+\alpha} < \frac{\phi D'}{\varepsilon_B}$$

$$\eta = \left( 1 + \frac{1}{1+\alpha} \right) \frac{1}{\phi D' / \varepsilon_B} \quad (6-2)$$

The maximum curvature based on the overlap of the inelastic strain in concrete is determined by smaller curvature in the two curvatures shown in the Eq.(5) and Eq.(6). Figure 11 shows the relationships expressed by Eq.(5) and Eq.(6), taking  $\alpha$  as a parameter.

The test results indicated that the plastic hinge length was almost equal to the depth of the column section. (Ref.2) Therefore, the horizontal axis of Fig.11 is considered to be the drift angle normalized by  $\varepsilon_B$ .

#### 4.2 The drift capacity based on the stable limit

Using Fig.1 and Eq.(1), the relationship between the curvature at the stable limit,

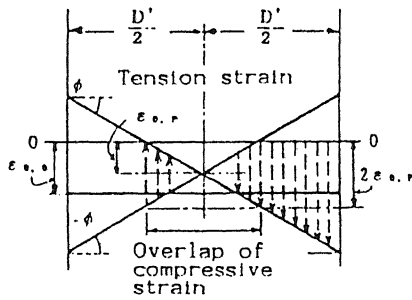


Fig. 9. Idealized strain profiles subjected to cyclic curvature reversals.

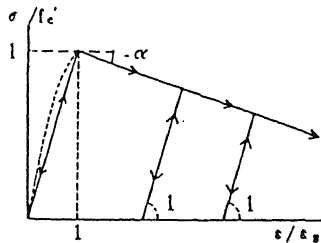


Fig. 10. Idealized stress-strain relations in core concrete.

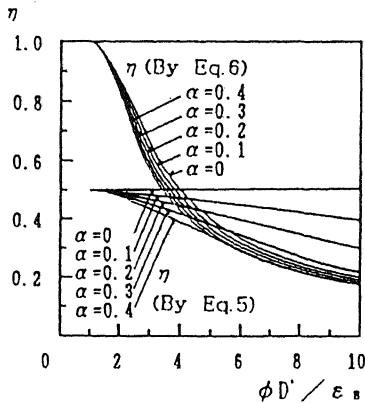


Fig. 11. The drift capacity based on the overlap of the inelastic strain in concrete.

$\phi_{sl}$ , and axial stress ratio,  $\eta$ , is expressed by Eq.(7-1) and Eq.(7-2), and is shown in Fig.12, taking  $\alpha$  as a parameter.

$$\text{for } \sqrt{1 + \frac{1}{\alpha}} < \frac{\phi_{sl} D'}{\epsilon_B} \text{ and } \alpha \neq 0$$

$$\eta = -\alpha \frac{\phi_{sl} D'}{\epsilon_B} + \sqrt{(\alpha \frac{\phi_{sl} D'}{\epsilon_B})^2 + \alpha + 1} \quad (7-1)$$

$$\text{for } 0 < \frac{\phi_{sl} D'}{\epsilon_B} \leq \sqrt{1 + \frac{1}{\alpha}} \text{ and } \alpha \neq 0$$

$$\eta = 1 - \alpha (1 - \sqrt{\frac{\alpha}{1 + \alpha}}) \frac{\phi_{sl} D'}{\epsilon_B} \quad (7-2)$$

The horizontal axis in Fig.12 is also considered to be the normalized drift angle.

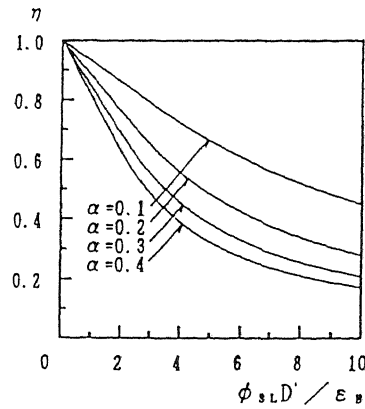


Fig. 12. The drift capacity based on the stable limit.

The available maximum drift angle of columns could be determined by the smaller drift angle in the two drift capacity above, depending on the axial stress ratio,  $\eta$ , and  $\alpha$

## 5 CONCLUSIONS

The following findings were drawn from this study.

1. The proposed methods for estimating the strains and the curvature at the stable limit and the quasi-table limit are adequate.

2. "The overlap of inelastic strain in concrete" has a significant influence on the seismic performance of columns, and it should be considered to determine the maximum available drift in seismic design.

3. Using the simplified stress-strain relation for core concrete, the drift capacity based on "the overlap of inelastic strain in concrete" is obtained, and is shown in equations and figures.

4. Using the same simplified stress-strain relation for core concrete, the drift capacity based on the stable limit is obtained, and is shown in equations and figures.

5. The available maximum drift angle of columns could be determined by the smaller drift angle in the two drift capacity, depending on the axial stress ratio and the extent of concrete confinement.

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