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Recursive system identification in earthquake engineering

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ABSTRACT: This paper looks into the application of a number of system identification techniques to problems of earthquake engineering. A number of recursive techniques for structural system identification have been developed over the past few years. Many of these techniques have been successful at identifying properties of linearized and time-invariant equivalent structural systems. Most of these techniques were verified using mathematical models simulated on the computer.
In this paper, a number of structural identification algorithms are reviewed and applied to the identification of structural systems subjected to earthquake excitations. The algorithms are applied to experimental data. The data pertains to the acceleration records from two building models subjected to various loading conditions. The performance of the various identification algorithms is critically assessed and guidelines are obtained regarding their suitability to various engineering applications.

1 Introduction

This paper presents the results from implementing a number of recursive system identification algorithms to experimental data. Its aim is to shed further light on validating system identification techniques, as well as on implementing techniques that are suitable for monitoring the variation with time of models of structural systems.

Two experiments were performed using scale models of steel and concrete buildings. The data set from each of the experiments consisted of acceleration records measured at various floor levels. Each of these records was analysed using a number of different system identification techniques.

The two system identification techniques which are reported in this study consisted of the recursive least squares and the recursive instrumental variable methods. A variant on the recursive least squares, recommended in the literature by various investigators, was also implemented. It provides for an exponential phasing out of old data. Furthermore, a variant on the recursive instrumental variable technique which resulted in an improved instrumental variable series was also implemented. A comparative study of the performance and the accuracy of these techniques, as well as other non-recursive techniques, was also carried out.

2 Mathematical Models

The class of structures that fall within the scope of the present investigation can be adequately modeled by the following N-dimensional system of equations which describes the motion of the structure,

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{g}[\mathbf{u}, \dot{\mathbf{u}}] = \mathbf{f}(t) + \omega(t) . \quad (1)$$

Here, $\mathbf{M}$ denotes the inertia matrix associated with the structure. $\mathbf{C}$ denotes the corresponding viscous damping matrix and $\mathbf{K}$ the stiffness matrix. Furthermore, the vector $\mathbf{f}(t)$ denotes the externally applied forces, and $\mathbf{g}[\mathbf{u}, \dot{\mathbf{u}}]$ is a vector whose components are nonlinear functions of the structural displacement $\mathbf{u}$ and its first derivative $\dot{\mathbf{u}}$. In the above equation, the term $\omega(t)$ represents errors due to modeling approximations. Obviously, it can also be used to model an additive noise to the excitation process $\mathbf{f}(t)$ in which case, the noise may be attributed to unmeasured environmental factors. The most useful form for this noise process has proven to be a zero-mean stationary Gaussian white noise.

For the purpose of structural identification, measurement devices are placed at certain locations throughout the structure. Their number is usually less than the number of degrees of freedom of the structure. This is due to both the expense associated with additional measurements, as well as to the fact that theoretically, each measured record contains enough information to permit the identification of all the unknown parameters. Furthermore, measurement noise is usually associated with the measurement process, leading to the following observation equation which relates the observation vector at the i^{th} observation time interval to the response vector at that instant.

$$\mathbf{y}_i \;=\; \mathbf{H}\ddot{\mathbf{u}}_i \;+\; \mathbf{e}_i \qquad (2)$$

In the above equation, $\mathbf{H}$ is a matrix which reflects the location of the measurement devices in relation to the structural nodes, and the associated amplification or attenuation factors, and $\mathbf{e}_i$ is a vector denoting the measurement noise and is usually assumed to be a zero-mean Gaussian white noise.

Alternatively, the identification problem can be cast completely in terms of the observed input and output, without any reference to the underlying mechanics or the associated differential equation. This approach provides an algorithm which permits forecasts of the response of the structure that are compatible, in some sense, with measured past input and output data. A general class of models referred to as the prediction error models is obtained using the following equation (Goodwin and Payne, 1977)

$$\mathbf{y}_i \;=\; \mathcal{Y}_i\left(\mathbf{y}_{i-1},\ldots,\mathbf{y}_{i-k},\mathbf{f}_i,\ldots,\mathbf{f}_{i-l}\right) \;+\; \mathbf{e}_i \;. \qquad (3)$$

Obviously, the more complicated the form of the functional $\mathcal{Y}_i$, the more sophisticated the model is, but also the more specialized and less robust it is. In the important case of a linear functional relationship, equation (3) can be conveniently rewritten as

$$\mathbf{y}_i \;=\; \boldsymbol{\theta}_i^T\,\mathbf{x}_i \;+\; \mathbf{e}_i \qquad (4)$$

where $\boldsymbol{\theta}_i$ is a matrix of the coefficients in the linear expansion, and

$$\mathbf{x}_i \;=\; \left[\,\mathbf{y}_{i-1},\ldots,\mathbf{y}_{i-k},\mathbf{f}_i,\ldots,\mathbf{f}_{i-l}\,\right] \;. \qquad (5)$$

Since equations (1) and (3) are mathematical expressions of the same physical problem, an equivalence, in some sense, should be anticipated between them. Depending on the dimension of the observation space, this equivalence can take one of many forms. Also, the extent of the desired equivalence is problem dependent and is usually limited to the equivalence of the predicted output of a linearized version of these equations. Such an equivalence can be achieved by matching the spectral density of the response of a linearized version of equation (1), with that of an appropriate linear difference equation model. Thus the difference equation associated with a scalar observable can be written as

$$\sum_{k=0}^{2N} a_k y_{i-k} \;+\sum_{k=0}^{2N} b_k f_{i-k} \;=\; 0\;. \qquad (6)$$

Equating the transfer function associated with equations (6) and (1) after its linearization, results in the following expressions for the physical parameters in terms of the regression coefficients.

$$\omega_j = \frac{\sqrt{\lambda_j^2+\delta_j^2}}{\Delta t}\;,\quad \xi_j = \frac{\delta}{\sqrt{\lambda_j^2+\delta_j^2}} \qquad (7)$$

where Δt denotes the sampling rate, and

$$\lambda_j = \mathrm{Arg}\left[z_j\right]\;,\delta_j = -\frac{1}{2}\ln|z_j|^2 \qquad (8)$$

In the above equation, ω_j and ξ_j denote the modal frequency and damping ratio of the jth mode, respectively. Also, z_j denotes the jth pole, in the upper half of the complex plane, of the transfer function of the model in equation (6). The equivalence given by the above equations is based on the assumption that modal superposition applies to the dynamical system under consideration. This assumption may lead to spurious results when trying to recover the physical parameters from the regression coefficients. In particular, it is noted that for z_j real, a value of 100% is obtained for the corresponding critical damping ratio ξ_j.

3 The Identification Algorithms

3.1 Recursive Least Squares

The recursive least squares method consists of updating a least squares fit to the available data, as more data is made available. It can be shown that the estimates obtained using a least squares algorithm tend to be biased unless the prediction errors are uncorrelated, which is seldom the case. The bias is generally associated with the propagation of the initial error in the estimates. The effect of this error can be substantially reduced by implementing a process whereby less weight is given to older data. An exponential weighting function has been successfully implemented to this end in a number of investigations. This technique is mathematically based on minimizing the following loss function (Goodwin and Payne, 1977),

$$S_k(\boldsymbol{\theta}_k) \;=\; \alpha S_{k-1}(\boldsymbol{\theta}_k) \;+\; \left(y_k - \mathbf{x}_k^T\boldsymbol{\theta}_k\right)^2\;, \qquad (9)$$

where the second term represents the error associated with the current observation, and $0 < \alpha < 1$. It can be shown that the cost function given by the above equation is equivalent to the cost function given by the equation

$$S_k(\boldsymbol{\theta}) \;=\; \sum_{i=1}^{k}\left(y_i - \mathbf{x}_i^T\boldsymbol{\theta}\right)\alpha^{k-i}\;n \qquad (10)$$

which better explains the role of the parameter α. The prediction equation and the gain matrix $\mathbf{K}$ are given by

$$\hat{\boldsymbol{\theta}}_{k+1} \;=\; \hat{\boldsymbol{\theta}}_k \;+\; \mathbf{K}_{k+1}\left[\,y_{k+1} - \mathbf{x}_{k+1}^T\hat{\boldsymbol{\theta}}_k\,\right]\;, \qquad (11)$$

and

$$\mathbf{K}_{k+1} = \frac{\mathbf{P}_k \mathbf{x}_{k+1}}{\alpha + \mathbf{x}_{k+1}^T \mathbf{P}_k \mathbf{x}_{k+1}} , \qquad (12)$$

respectively. Furthermore, the recursion for matrix $\mathbf{P}$ appearing in the above equation is given by

$$\mathbf{P}_{k+1} = \frac{1}{\alpha} \left[\mathbf{I} - \mathbf{P}_k \frac{\mathbf{x}_{k+1} \mathbf{x}_{k+1}^T}{\alpha + \mathbf{x}_{k+1}^T \mathbf{P}_k \mathbf{x}_{k+1}} \right] \mathbf{P}_k . \qquad (13)$$

In all the subsequent implementation of this algorithm, a zero initial guess for the regression coefficients, and a diagonal matrix with large elements (1000) for the matrix $\mathbf{P}$ were used. Values of α of 0.99 have been recommended in the literature. In the course of the present research, values of α ranging from 0.7 to 0.99 were implemented.

3.2 Recursive Instrumental Variable

The least squares criterion for system identification can be viewed as a minimization of the following norm of the prediction error

$$||e|| = \int e^2 dt . \qquad (14)$$

A useful generalization of this concept is to view the above integral as a weighted residual. It is then apparent that a more flexible criterion for computing the coefficients of the hypothesized model is obtained by using the following norm of the error

$$||e|| = \langle e, f \rangle , \qquad (15)$$

where $\langle \ , \ \rangle$ denotes a suitable inner products and e and f denote either functions or discrete series. The Instrumental variable method is obtained as a special case of the above technique. Specifically, the weighting series is so chosen as to be minimally correlated with the error, while having a large correlation with the output of the system, uncorrupted by the measurement errors. It can be shown that this choice of template function has a number of desirable effects on the statistical properties of the estimates.

The series given by the vector

$$\mathbf{v}_k^T = [\, f_{k-L-l} \cdots f_{k-L} \; f_{k-l} \cdots f_k \,] \qquad (16)$$

has been suggested as an instrumental variable series (Young, 1984). This series consists of two observation blocks of the input separated by a lag of L observations. Assuming the input to be uncorrelated with the observation noise, the above series obviously satisfies one of the requirements for an instrumental variable. Furthermore, the lag parameter L can be so adjusted as to achieve maximum correlation with the output series corresponding to the system response. In this investigation, the parameter L was chosen in such a way that the two observation blocks were adjacent and non-overlapping. The resulting recursion algorithm is quite similar to the one derived for the recursive least squares, and is given by the following equations

$$\hat{\theta}_{k+1} = \hat{\theta}_k + \mathbf{K}_{k+1} \left[\mathbf{y}_{k+1} - \mathbf{x}_{k+1}^T \hat{\theta}_k \right] . \qquad (17)$$

where

$$\mathbf{K}_{k+1} = \frac{\mathbf{P}_k \mathbf{x}_{k+1}}{1 + \mathbf{x}_{k+1}^T \mathbf{P}_k \mathbf{v}_{k+1}} . \qquad (18)$$

and

$$\mathbf{P}_{k+1} = \left[\mathbf{I} - \mathbf{P}_k \frac{\mathbf{x}_{k+1} \mathbf{x}_{k+1}^T}{1 + \mathbf{x}_{k+1}^T \mathbf{P}_k \mathbf{v}_{k+1}} \right] \mathbf{P}_k . \qquad (19)$$

It is important to note that although the recursive least squares can be shown to yield identical results to the non-recursive least-squares, the same is not true for the recursive instrumental variable algorithm.

A more general implementation of the Instrumental variable technique can be achieved by an instrumental variable series having the following form.

$$\mathbf{v}_k^T = [\, h_k \cdots h_{k-l} \; f_k \cdots f_{k-l} \,] . \qquad (20)$$

where h_k is a series so chosen as to maximize the correlation with the output of the system while minimizing the correlation with the measurement noise. One way to achieve this goal is to chose $\{h_k\}$ as the output of an auxiliary system which is a good approximation to the real system. In this case, h_k is given by the following recursive equation

$$h_k = \beta_k^T \mathbf{v}_k . \qquad (21)$$

where β_k denotes the parameters of the auxiliary system. In this investigation, they are obtained from the estimated system parameters through the following algorithm (Young, 1984),

$$\beta_{k+1} = (1-\gamma)\beta_k + \gamma\theta_{k+1} . \qquad (22)$$

Note that for γ equal to 1, the auxiliary system coincides with the real, noise-corrupted, system. Values of γ between 0.03 and 0.1 have been suggested in the literature. In addition to this range of values, values between 0.1 and 1 were also implemented in this study in order to provide a comprehensive assessment of the sensitivity of the algorithm, in the context of earthquake engineering, to variations in γ.

4 Numerical Results

Two sets of experiments provided acceleration time histories for the verification of the above parameter estimation algorithms. The experiments involved a three story steel building model and a five story reinforced concrete model. In both experiments, the model was subjected in turn to a white noise base motion, and to a motion representing the El-Centro earthquake. Also, in both experiments, accelerometers measured the structural response at floor levels. Digital band-pass filters conditioned the acceleration time histories after digital data acquisition. An extensive parametric study was then performed using the collected database. A detailed presentation of the associated results can be found elsewhere (Ghanem et.al, 1991). In this paper, only some of the results pertaining to the five story building model are presented. Typical results for the five-story building model associated with the unmodified recursive least squares are shown in Figure (1). This figure shows the time variation of one of the coefficients of the linear prediction model associated with observations at the fifth floor. The exponential window algorithm was implemented on the above data. The value of α corresponding to this figure is equal to 0.99. Figure (2) shows corresponding values of the coefficients of the prediction model, associated with the same observations as the ones shown in Figure (1). An important observation can be made concerning the results associated with the exponential window. Specifically, it is noted that the effect on the first few observations is a desirable smoothing of the estimates, which deteriorates for later observations. A variant on the algorithm was implemented whereby the exponential window was used only for a fraction of the observations. In this case, one fourth of the data at the beginning of each record was processed through an exponential window with a value for the parameter α equal to 0.99. The effect of this procedure on the stability of the estimates was quite significant as can be seen in Figure (3). The recursive instrumental variable algorithms described earlier were implemented in a fashion similar to that described above for the recursive least squares algorithms. The first algorithm involved an unfiltered instrumental variable series. The coefficients of the linear prediction model identified in this fashion exhibited a pronounced transient behavior which was indicative of a deficient instrumental variable series which was incapable of identifying the parameters of the model. Typical results pertaining to these coefficients are displayed in Figure (4). Figure (5) shows the results corresponding to the filtered instrumental variable technique. This latter technique does not seem fit for on-line identification, since it requires pre-tuning the auxiliary filter to the given data.

As discussed above, the coefficients in a linear prediction model can be associated with the parameters of an equivalent linear differential equation. These parameters can be related to such modal quantities as the natural frequencies and the damping ratios of the structure. In this section, these equivalent modal quantities are obtained which are associated with the coefficients presented in the above. Figures (6) and (7) show typical estimated natural frequency and damping ratio obtained from the five-story building model corresponding to measured data from the fifth floor, and using the unmodified recursive least squares algorithm. In general a monotonic trend is observed even at the end of the estimation period, suggesting that the estimates have not yet reached their final values. This behavior may be attributed to a strong bias associated with the estimates. In other cases, large fluctuations were observed throughout the estimation period. These fluctuations seem to be, in most cases, between the values corresponding to two or three different frequencies. This fact may be attributed to the much smaller contribution to the total motion coming from the fifth mode. It is also observed that poor frequency estimates are associated with poor damping ratio estimates. As to the effect of the input motion on the estimates, it was noted that the effect was minimal, and similar behavior of the estimates was observed for both the El-Centro input motion and the white noise input motion. Figures (8) and (9) show the results corresponding to the least squares estimation using an exponential window. Except for few cases, these estimates are not well-behaved, and are in general poorer than the results without a the exponential window. The processing of only an initial block of the data through the exponential window had a substantial positive effect on the results. As can be seen in Figures (10) and (11). The fluctuations have disappeared from all the estimates. The estimated modal quantities using the instrumental variable technique ranged between well behaved and widely fluctuating. The method, in this form, cannot form the basis for a reliable system identification technique. Typical results are shown in Figures (12) and (13). By filtering the instrumental variable series as indicated above, substantial improvement can be achieved. Figures (14) and (15) show the results corresponding to this case. The well behaved results obtained with this technique bely the difficulty of its implementation. Specifically, only certain values of the parameter γ were found to yield converging estimates for a given record. However, as can be observed, when such a value was found, the estimates exhibited a pronounced improvement over the previous implementation of the instrumental variable algorithm.

5 Conclusions

The emphasis placed throughout this paper on time domain techniques for system identification is justified by the desire to monitor the evolution in time of the identified parameters. This capability has the potential of permitting the synthesis of more

Table 1: Comparison of Identification Algorithms

Identification Techniques	Required Expertise	Numerical Convergence	On-Line Potential	Initial Guess	Reliability of Results
MLE	high	med	low	close	high
Extended Kalman	high	med	low	close	high
RLS	low	high	high	any	med
RLS with Exp Window	low	high	high	any	high
Rec IV	med	high	high	any	med
Rec IV with Filter	high	low	high	any	med

meaningful damage assessment indices, as well as enhancing the reliability of adaptive schemes that may be used for on-line control of structural systems.

The issue of a suitable identification algorithm is compounded with the issue of deciding on an adequate mathematical model for the structure. This issue comes into play when deriving an equivalence between the parameters of the linear prediction model and a set of physical parameters such as modal quantities. Whereas a linear prediction model has a definite interpretation as a linear relationship between the input and output measurements, a differential equation model based on modal superposition involves further assumptions that are likely not to hold under earthquake-type excitations. As a consequence of this, although the linear prediction model can be used to forecast the behavior of the structure with a well understood optimization criterion, the same does not hold for the differential equation model. Therefore, depending on the context in which the identification algorithm is being used, it may be more consistent to use the linear prediction model.

Table (1) summarizes the recommendations from this study while highlighting the issues that were deemed important in assessing the worthiness of each of the identification algorithms.

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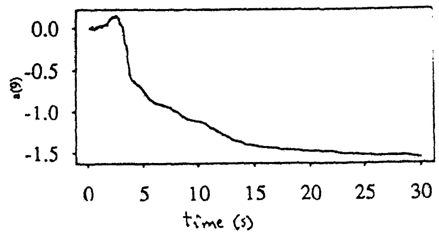

Figure 1: Recursive Least Squares; $\alpha = 1$; El-Centro Input; Ninth Coefficient; Fifth Floor Measurement.

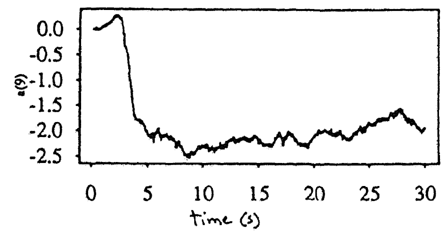

Figure 2: Recursive Least Squares; $\alpha = 0.99$; El-Centro Input; Ninth Coefficient; Fifth Floor Measurement.

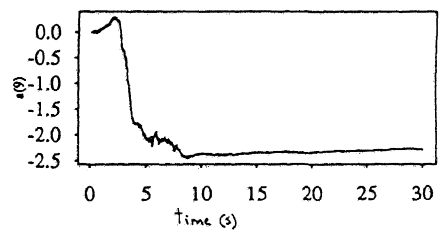

Figure 3: Recursive Least Squares; $\alpha = 0.99$ Only for Initial Block; El-Centro Input; Ninth Coefficient; Fifth Floor Measurement.

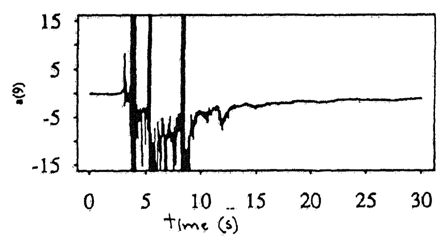

Figure 4: Recursive Instrumental Variable; El-Centro Input; Fifth Floor Measurement.

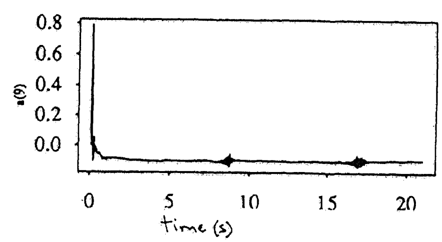

Figure 5: Filtered Recursive Instrumental Variable; $\gamma = 0.13$; White Noise Input; Fifth Floor Measurement.

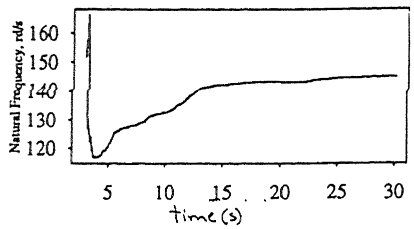

Figure 6: Recursive Least Squares; $\alpha = 1$; Identified Natural Frequency; El-Centro Input; Fifth Floor Measurement.

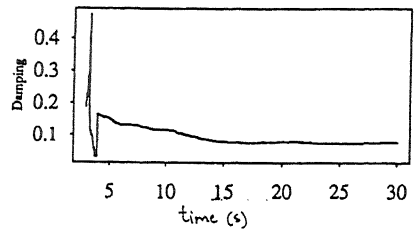

Figure 7: Recursive Least Squares; $\alpha = 1$; Identified Damping Ratio; El-Centro Input; Fifth Floor Measurement.

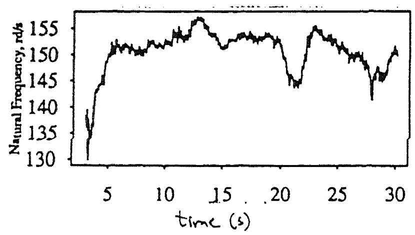

Figure 8: Recursive Least Squares; $\alpha = 0.99$; Identified Natural Frequency; El-Centro Input; Fifth Floor Measurement.

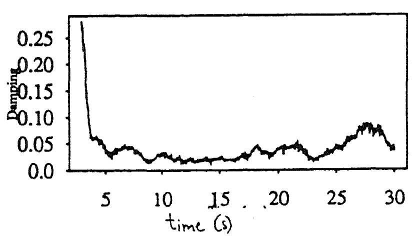

Figure 9: Recursive Least Squares; $\alpha = 0.99$; Identified Damping Ratio; El-Centro Input; Fifth Floor Measurement.

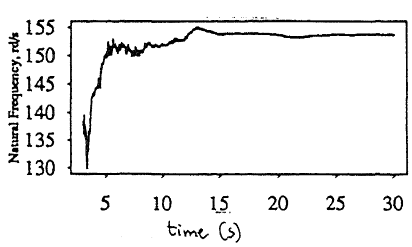

Figure 10: Recursive Least Squares; $\alpha = 0.99$ Only for Initial Block; Identified Natural Frequency; El-Centro Input; Fifth Floor Measurement.

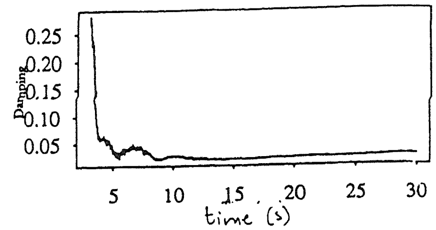

Figure 11: Recursive Least Squares; $\alpha = 0.99$ Only for Initial Block; Identified Damping Ratio; El-Centro Input; Fifth Floor Measurement.

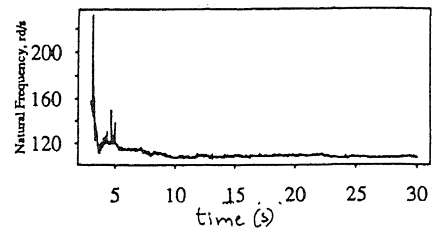

Figure 12: Recursive Instrumental Variable; Identified Natural Frequency; El-Centro Input; Fifth Floor Measurement.

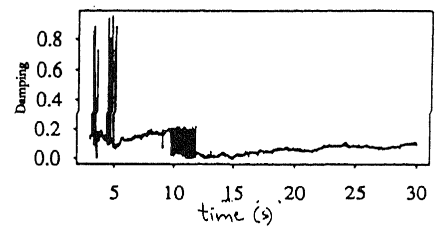

Figure 13: Recursive Instrumental Variable; Identified Damping Ratio; El-Centro Input; Fifth Floor Measurement.

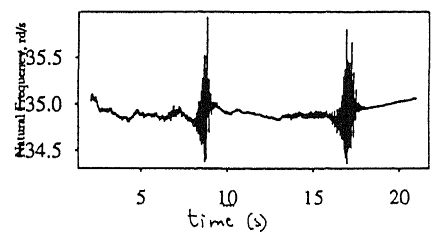

Figure 14: Filtered Recursive Instrumental Variable; $\gamma = 0.13$; Identified Natural Frequency; White Noise Input; Fifth Floor Measurement.

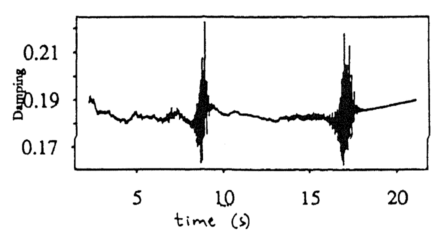

Figure 15: Filtered Recursive Instrumental Variable; $\gamma = 0.13$; Identified Damping Ratio; White Noise Input; Fifth Floor Measurement.