Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

Modelling anchorage slip for dynamic analysis

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ABSTRACT: Reinforced concrete frame structures subjected to strong ground excitations may develop plastic hinges near the adjoining members. Formation of a plastic hinge and penetration of inelasticity in reinforcement into the adjacent member result in anchorage slip within the adjacent member. This produces member end rotations that may be as significant as those due to flexure. Omission of anchorage slip in dynamic inelastic analysis may result in gross inaccuracies in results. A hysteretic model has been developed to incorporate effects of anchorage slip in response history analysis of reinforced concrete structures. A computer program, including the model, has been used to investigate the significance of anchorage slip. The results indicate that overall displacement response increases when anchorage slip is permitted in the analysis. However, ductility demand decreases significantly due to the introduction of an additional energy absorption mechanism.

1 INTRODUCTION

Reinforced concrete frame structures may develop inelastic deformations during strong earthquakes. Inelasticity may occur in beams near beam-column joints, and in columns at column-footing interface. Formation of plastic hinges in these regions results in inelastic strains in reinforcement. Inelastic strains may penetrate into the adjoining member, resulting in extension and/or slippage of reinforcement within the adjoining member. This may be referred to as anchorage slip. Anchorage slip at member ends produces rigid body rotations that are not accounted for in flexural analysis.

Tests of large scale reinforced concrete beams (Viwathanatepa et al. 1979) and columns (Saatcioglu and Ozcebe 1987, Saatcioglu and Ozcebe 1989) under reversed cyclic loading indicate that deformations due to anchorage slip may be as significant as those due to flexure. Although the contribution of anchorage slip to total deformation has been recognized during the last two decades (Otani and Sozen 1972, Filippou, et al. 1983, Morita and Kaku 1984), it has not become a practice to consider these deformations in inelastic analyses of structures. This may be attributed to lack of a suitable analytical model.

An analytical model has been developed by the authors for hysteretic behaviour of anchorage slip (Saatcioglu et al. 1992, Alsiwat and Saatcioglu 1992). A summary of the model, as well as its application to dynamic inelastic response history analysis of a reinforced concrete frame structure are discussed in this paper.

2 HYSTERETIC MODEL

The analytical model for anchorage slip includes primary moment-anchorage slip rotation relationship, and a set of rules describing unloading and reloading branches of hysteresis loops.

The primary curve is established by first conducting a plane section analysis at the critical section. This provides the necessary information on steel strains, neutral axis locations, and bending moments at different load stages. The steel strain is used to establish strain distribution along the length of the bar inside the adjoining member. This region consists of elastic and inelastic portions. The inelastic portion is further divided into yield plateau, strain hardening, and pull-out cone regions. This is illustrated in Fig. 1. The length of each region is computed from equilibrium of forces and assumed bond stress distribution between the steel and concrete. The expression for average elastic bond stress, developed by ACI Committee 408 (1979) is adopted here.

$$u_e = u_{ACI} = \frac{f_y\, d_b}{4\, l_d} \quad MPa \qquad (1)$$

$$l_d = \frac{440\ A_b}{K\ \sqrt{f_c}}\ \frac{f_y}{400}\ \geq 300\ mm \qquad (2)$$

where, f'_c and f_y are concrete compressive and steel yield strengths in MPa, respectively. A_b and d_b are the area and diameter of reinforcing bar. Coefficient K reflects the effects of confinement and bar spacing, and can be taken as $3d_b$. The elastic bond stress, u_e, is used to establish the length of the elastic region, L_e.

$$L_e = \frac{f_s\ d_b}{4\ u_e} \qquad (3)$$

where, f_s, is the maximum steel stress, and is limited by f_y.

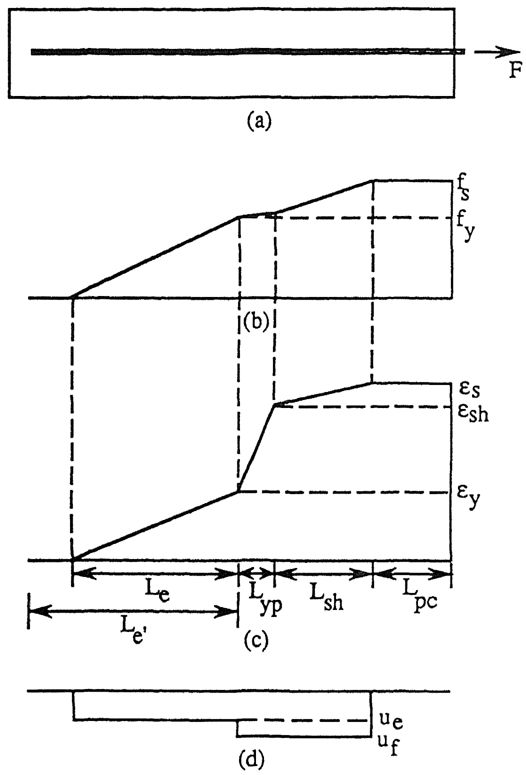

Figure 1. (a) Reinforcing bar (b) Stress distribution (c) Strain Distribution (d) Bond stress

If the available bar length for elastic region is less than that required by Eq. 3, the bar slips as a whole. Slippage of reinforcement is discussed later in this section. If the bar force exceeds the yield force, yielding of the bar takes place at the critical section. Yielding penetrates into the adjoining member, forming the yield plateau region. A bar that experiences inelastic deformations extends significantly, crushing the concrete between the lugs. Therefore, frictional bond stress, u_f, is used for this region as defined by Pochanart and Harmon (1989). The length of the yield plateau region, L_{yp}, is then established from the equilibrium of bar forces.

$$u_f = (5.5\ -\ 0.07\ \frac{S_L}{H_L})\ \sqrt{\frac{f'_c}{27.6}}\ MPa \qquad (4)$$

$$L_{yp} = \frac{\Delta f_s\ d_b}{4\ u_f} \qquad (5)$$

where, S_L and H_L are clear spacing and height of lugs on the bar respectively, and Δf_s is the incremental stress between the beginning and end of the yield plateau region.

Yielding of reinforcement is generally a localized phenomenon. The length of the yield plateau region may approach zero if the post yield stiffness approaches zero. As yielding progresses into the adjoining member the bar may enter into the strain hardening range. In this region bond between the steel and concrete can be taken as the frictional bond specified in Eq. 4. The length of the strain hardening region, L_{sh}, can be computed from Eq. 5 if the corresponding value of Δf_s is used in the equation.

When the anchored bar extends significantly within the adjoining member, the cover concrete of the adjoining member breaks loose. This results in a pull-out cone, and usually occurs when the bar is strained beyond yielding. The onset of yielding may also be taken as the onset of the pull-out cone. The length of this region, L_{pc}, is limited to the net cover of the adjoining member in the direction of the bar. The pull-out cone, however, does not occur if the potential cone region is at the interface of two members, and hence may be reinforced. This may be the case in beam-column and column-footing connections. The pull-out cone commonly occurs in ordinary pull-out tests where either a single, or a group of bars are pulled from a concrete block.

The length of each region discussed above is required to establish the strain distribution. Integration of strains (or the area under the strain diagram) along the length gives the extension of reinforcement within the adjoining member. If the bar is strained to the far end (to the cut-off point), the elastic bond stress given in Eq. 1 is not sufficient to develop the bar force. In such cases the elastic bond will increase, while the bar will slip as a rigid body, in addition to developing deformations due to extension. Increased elastic bond,

u'_e, is determined from equilibrium of forces, based on the available elastic length, L'_e.

$$u'_e = \frac{f_s\, d_b}{4\, L'_e} \quad if \quad L'_e \le L_e \tag{6}$$

The local bond-slip relationship, developed by Ciampi et al. (1981) and Eligehausen et al. (1983) is adopted here to compute bar slip, δ_s. The model is illustrated in Fig. 2.

$$\delta_s = \delta_{s1} \left(\frac{u'_e}{u_u}\right)^{2.5} \tag{7}$$

The behaviour of hooked bars with a 90-degree hook can be simulated by an analytical model (Soroushian et al. 1988) that has the same characteristics as those of the bond-slip model shown in Fig. 2. In this case the vertical axis indicates hook force, and the horizontal axis indicates hook displacement. The ascending branch of the model, as well as ultimate and frictional hook forces are given by the following expressions:

$$P_h = P_{hu} \left(\frac{\delta_h}{2.54}\right)^{0.2} \tag{8}$$

$$P_{hu} = 271\ (0.05\ d_b - 0.25) \tag{9}$$

$$P_{hf} = 0.54\ P_{hu} \tag{10}$$

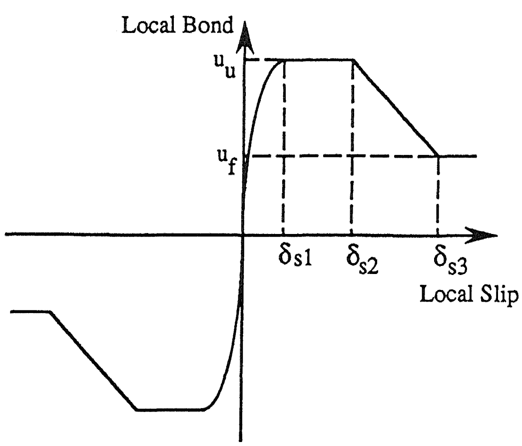

Figure 2. Local bond-slip model (Ciampi et al. 1981 and Eligehausen et al. 1983).

The hook contributes only if the straight lead length of the bar is not long enough to develop the bar force. In this case the additional anchorage is provided by the hook force as shown below:

$$P_h = A_b\, f_s - \pi\ d_b\ L_e\ u_e \tag{11}$$

The force determined by Eq. 11 can be used with the model described by Eqs. 8 to 10 to compute hook deformation.

Beam reinforcement crossing an interior beam-column joint may be subjected to simultaneous pull on one side and push on the other. If the development lengths in tension and compression overlap inside the joint, the compression force contributes to the slippage of reinforcement. Tests indicate that yield penetration in compression is negligibly small (Saatcioglu et al. 1992). Therefore, the compression side of the bar may be assumed to be limited to linear strain distribution. The local bond stress in the elastic portion of the bar, now partly in compression and partly in tension, can be computed using Eq. 6.

The extension and slip of reinforcement (including hook deformation if any) within the adjoining member produces cumulative bar deformation, δ_{total}, at the interface. This results in member end rotation that can be computed as shown below:

$$\theta = \frac{\delta_{total}}{d - c} \tag{12}$$

where, d and c are the effective depth and neutral axis location at the critical section, respectively. Member end rotations computed from Eq. 12, and the corresponding bending moments obtained from the sectional analysis are plotted to obtain the primary curve. The primary curve may be idealized as a bi-linear curve for convenience.

Hysteretic moment-rotation relationship due to anchorage slip is established by following a set of rules, devised on the basis of experimental observations (Saatcioglu et al. 1992). Figure 3 illustrates general characteristics of the proposed hysteretic model. Unloading from the primary curve is established by subtracting the elastic deformation from total, and marking the unrecovered plastic deformation on the deformation axis. When bending moment is accompanied by axial compression, the elastic deformation is recovered prior to complete unloading of bending moment. Subsequent unloading and reloading in the opposite direction are aimed at the previous maximum (or yield point if yielding in the opposite direction has not occurred). The behaviour of asymmetric members with different moment capacities in two directions is also considered. Accordingly, loading in the strong direction results in early crack closure at approximately the same level as the capacity in the weak direction. Therefore two primary curves are used for the strong direction; the inner being identical to the primary curve in the weak direction, and the outer being the primary curve of the strong direction. Reloading branches are then directed towards the previous maximum point on the inner primary curve, followed by a steeper slope equal to the unloading slope to reach the previous maximum on the outer primary curve. This results in pinching of hysteresis loops.

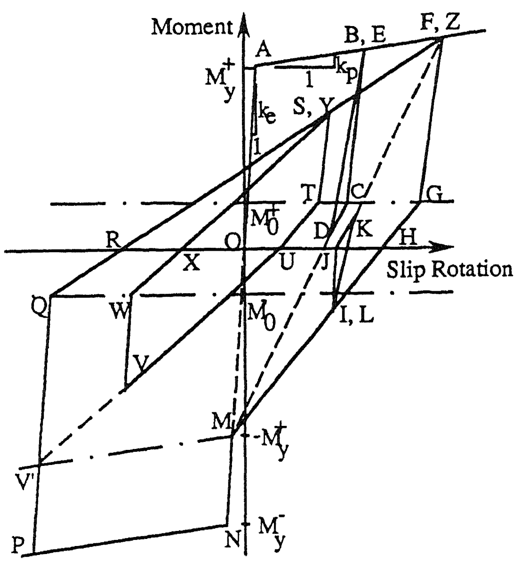

Figure 3. Hysteretic model for anchorage slip.

3 DYNAMIC INELASTIC ANALYSIS

A modified version of computer software DRAIN-2D (Kanaan and Powell 1973) was used to illustrate the significance of anchorage slip on a reinforced concrete frame structure. The program was originally developed at the University of California at Berkeley. Several modifications have been introduced into the program at the University of Ottawa, including the implementation of anchorage slip hysteretic model. DRAIN-2D is a general purpose program for dynamic inelastic analysis of plane structures subjected to earthquake excitation. The structural stiffness matrix is formulated by the direct stiffness method, with the nodal displacements as unknowns. Dynamic response is determined using step-by-step integration based on the assumption of a constant average response acceleration during each time step. Inelasticity is introduced by means of two springs at the end of each member; one for flexure, and the other for anchorage slip. Hysteretic model presented in the previous section is assigned to the anchorage slip spring. The hysteretic model developed by Takeda et al. (1970) was assigned to the flexural spring. The flexural model was idealized to have a bi-linear primary curve, with effective elastic flexural rigidity, EI, equal to 50 % of EI based on gross sectional properties. Shear response was assumed to remain linear, with reduced rigidity to account for shear cracking. The shear rigidity, GA, assigned to each member was equal to 30 % of GA based on gross sectional properties.

3.1 Structure Selected

A 10-storey reinforced concrete frame structure was selected for dynamic analysis. The structure had a square plan with three equal bays in each direction. It was designed according to the requirements and specifications of the 1990 National Building Code of Canada (1990), and the Canadian Standard CAN3-A23.3-M84, Design of Concrete Structures for Buildings (1984). A typical interior frame was considered in the analysis. Figure 4 illustrates geometric details of the frame. Table 1 includes structural and ground motion properties.

3.2 Results of Dynamic Analysis

The results of dynamic analysis are shown in Figs. 5 to 8. The structure experienced inelasticity in the beams and base level columns. While inelasticity in the beams were high enough to strain reinforcement

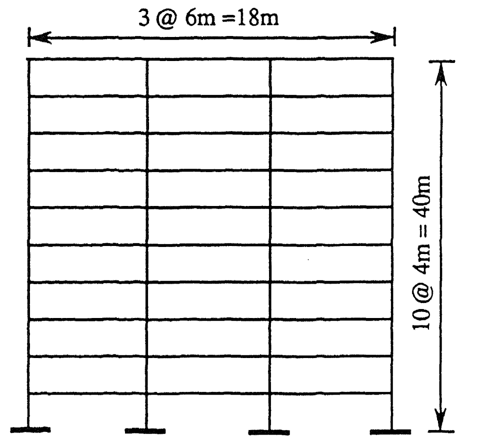

Figure 4. Frame structure considered for dynamic inelastic analysis

Table 1. Structural and ground motion parameters for structure considered.

No. of stories	10
Height	40 m
Fundamental period	2.3 sec.
Girder stiffness parameters:	
0.5 EI	7.4 x 10^4 kN.m^2
0.3 GA	6.2 x 10^5 kN
EA	4.9 x 10^6 kN
Column stiffness parameters:	
<u>Exterior columns</u>	
0.5 EI	4.6 x 10^4 kN.m^2
0.3 GA	6.9 x 10^5 kN
EA	5.5 x 10^6 kN
<u>Interior columns</u>	
0.5 EI	1.0 x 10^5 kN.m^2
0.3 GA	1.0 x 10^6 kN
EA	8.3 x 10^6 kN
Yield moment, M_y:	
<u>Roof girders</u>	
positive moment	127 kN.m
negative moment	186 kN.m
<u>Floor girders</u>	
positive moment	167 kN.m
negative moment	312 kN.m
<u>Columns</u>	
exterior columns	306 kN.m
interior columns	497 kN.m
Mass (DL + 25% LL):	
floors	60,000 kg/floor
roof	45,000 kg/floor
Ground excitation (1940 El-centro N-S):	
peak acceleration (scaled by 1.5)	0.5 g
duration	10 sec.

NOTE: Column yield moments indicated are for base level. For upper floors, strength taper is applied such that the yield level is reduced to 230 kN.m and 323 kN.m for exterior and interior columns at the roof level, respectively.

into the strain hardening region, inelasticity in column steel was limited to the yield plateau region. Therefore, column rotation due to anchorage slip was not significant. However, member end rotations in beams due to anchorage slip were as high as 60 % of the plastic hinge rotations due to flexure.

Another important observation was the effect of anchorage slip on flexural and total displacement ductility demands. Consideration of anchorage slip allowed for the development of an additional energy absorption mechanism, and resulted in up to 40 % reduction in ductility demands as indicated in Fig. 7. This implies that inelastic analysis without the

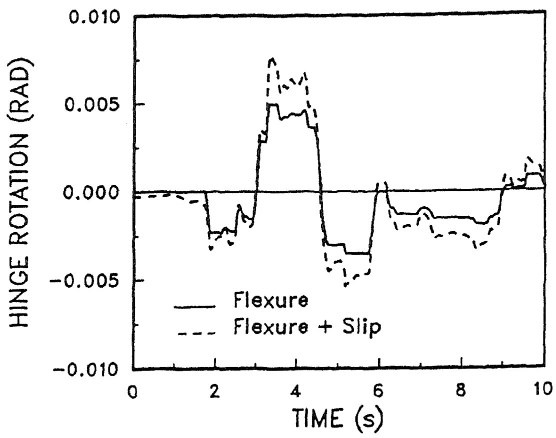

Figure 5. A typical time history of beam hinge rotations

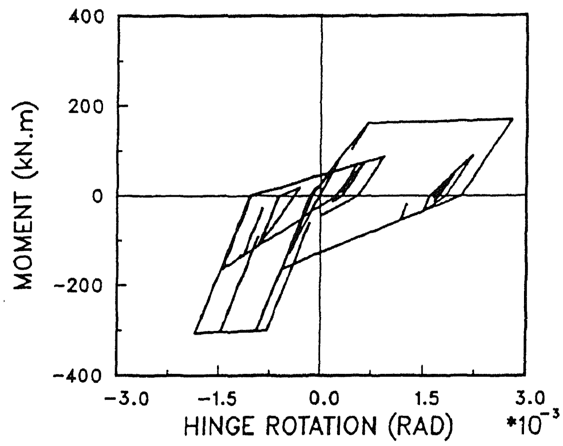

Figure 6. Typical hysteretic relationship due to anchorage slip, recorded at the end of a beam

consideration of anchorage slip may overestimate ductility demands significantly. When anchorage slip in columns is not significant, however, the effect on overall displacement response is small. Figure 8 shows the lateral displacement time histories of roof with and without the consideration of anchorage slip.

4 CONCLUSIONS

The following conclusions can be drawn from the research reported in this paper:

1. Anchorage slip in reinforced concrete structures can be very significant when inelastic deformations are generated, as in the case of strong earthquakes.

2. The effect of anchorage slip under mononotic and cyclic loading can be considered by using the analytical model presented in this paper.

3. The effect of anchorage slip is to increase inelastic member deformations significantly. When reinforcement at the interface of two adjoining members enters into the strain hardening region, member end rotations due to anchorage slip can be as significant as those due to flexure.

4. Consideration of anchorage slip in non-linear structural analysis allows for an additional energy absorption mechanism, thereby reducing flexural ductility demands. This implies that an analysis where this effect is not considered may call for unrealistically high ductility requirements.

5. The overall behaviour of a structure, as reflected by lateral displacements, may not be affected significantly by anchorage slip unless column reinforcement enters into the strain hardening region.

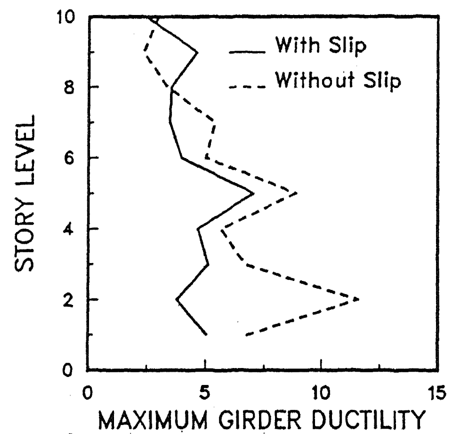

Figure 7. Variation of maximum flexural rotational ductility in beams

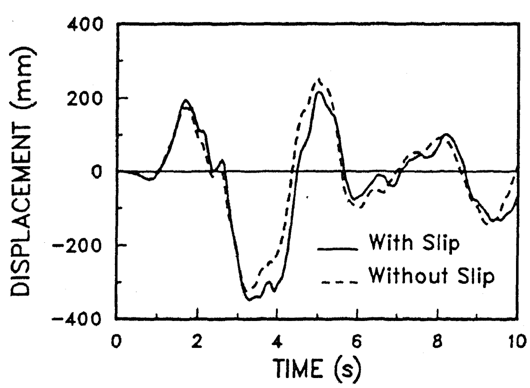

Figure 8. Time history of lateral roof displacement

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