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Rotative behavior at a discrete section in RC members caused by opening and closing behavior of cracks under reversed cyclic loading

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ABSTRACT: Opening and closing behavior of a crack that is an original performance at a discrete section in a RC member was investigated empirically. One hundred one prism type specimens were tested. These are the simplest composite elements at the cracking area in RC members. From the test results, a suitable analytical model that represents the relation between axial force and crack width was proposed. The opening and closing behavior of cracks in RC beams under reversed cyclic loading were represented very well by this model.

Using this crack model, the analytical method was proposed in order to predict the rotative behavior at a discrete section in RC members. Special feature of this analysis is to be able to take into account the non-continuity of curvature in a RC member. The test results of moment-rotation relationships were reproduced very well by this analysis.

1. INTRODUCTION

It is shown experimentally that the behavior of a crack contributes to the elasto-plastic deformations under reversed cyclic loading of RC members. Especially, rotation at a discrete section is caused by the opening and closing behavior of cracks. Therefor, it is very important to develop an effective analytical model for opening and closing behavior of a crack in order to predict the correct load-deflection characteristics of RC structural members subjected to reversed cyclic loading like seismic forces.

The objectives in this research are to clarify the opening and closing behavior of a crack empirically and develop the analytical model. Furthermore, the method to compute the rotative behavior at a discrete section in RC members were proposed. It is an example for application of the crack model.

2. TEST PROGRAM

The shape of a specimen and measuring method is shown in Figure 1. A reinforcing bar was arranged at the center of a concrete rectangular prism in the mainly specimen. This is the simplest composite element including a reinforcing bar, concrete, bond and a crack. This represents the cracking area of RC members. A pre-crack or a notch that causes a crack was set up at the center of the specimen. One hundred one prism type specimens were tested to develop and verify the analytical model. Variables in this tests are shown in table 1. The maximum size of the coarse aggregates used for the concrete mix was 25mm.

Tensile force was applied to the reinforcing bar and compressive force was applied to the concrete face at both ends of the specimen in order to represent the cracking area between tensile cracks. Except some specimens led to fail of concrete, the peak compressive stress was set at half of the concrete compressive strength so as not to cause the damage of concrete conspicuously during the compressive loading.

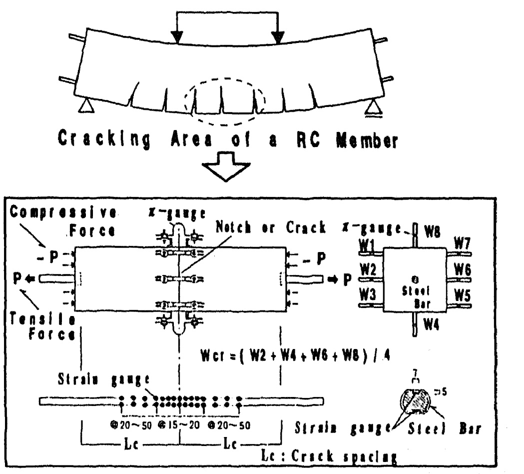

Fig. 1 Shape of prism type specimen

Crack opening and closing behavior was measured by displacement meters or pi-type gauges, which were attached on the lateral face of the specimen with a gauge length of 50mm. The strains in a reinforcing bar were measured by using strain gauges. The strain gauges were stuck at an interval from 15mm to 50mm in the ditch which was excavated around the reinforcing bar so as not to disturb the bond action.

3. TEST RESULTS

The relationships between axial force (P) or stress intensity (P/As) and crack width (Wcr) are shown in Figures 2 and 3.

A jumping phenomenon is observed at the crack initiation in the P-Wcr relations. As the tensile stress of steel bar is higher when the crack initiate, the increase of the crack width is larger. After that, the stiffness of a curve gradually decreases until the reinforcing bar yields at the crossing point to the crack. After the reinforcing bar yields, the curve bends and becomes the straight line that has a low stiffness. The shape of hysteresis loop under reversed cyclic loading shows the similar Bauschinger effect as the hysteresis characteristics of steel bars. Because of the local contact of concrete, the stiffness of a curve gradually increases before the crack closes completely. The crack width at the time of the local contact increases in proportion to the maximum crack width that it has experienced. At the reloading in each loop, every curves aim at the previous turning point (Figure 3).

The influence that each variables gives on the crack behavior was examined. Crack spacing is the most important factor which affected the opening and closing behavior of a crack. As the crack spacing is bigger, the crack width increases and decreases largely. When a crack spacing is small, a splitting crack tends to occur early. The difference of the P-Wcr curves caused by the diameter of reinforcing bar is observed in the stiffness before the reinforcing bar yields. However, the influence of the stress intensity of the reinforcing bar at the initiation of a crack is larger than that of the diameter of the reinforcing bar. The cover thickness is an important factor to decide a crack spacing. Differences of the shapes of hysteresis loops caused by the other variables are not observed.

4 PROPOSE THE CRACK MODEL

A constitutive equation needs to be constructed in such a form that the opening and closing behavior of a crack can be adopted in the numerical analysis in order

Table 1 Variables in the test

1) Section of specimens
10×10, 15×15, 20×20, 25×25, 15×30 cm
(Concrete cover: $t=37.5\sim112.5$ mm)
2) Crack spacing (Lc)
70, 120, 170, 220, 270, 320 mm
3) Diameter of reinforcing bars
13, 19, 22, 25, 29, 32 mm
4) Yield strength of reinforcing bars
$340\sim1000$ N/mm²
5) Shapes of libs of reinforcing bars
screw libs bar :
lateral libs bar:
6) Loading hystelesis
nine kinds of cyclic loading
7) Concrete compressive strength (fc)
$21\sim50$ N/mm²
8) Number (n) and spacing of reinforcing bars
n = 1, 2 (spacing : 60, 120 mm)
9) Lateral reinforcement ratio
0, 0.35, 0.70 %

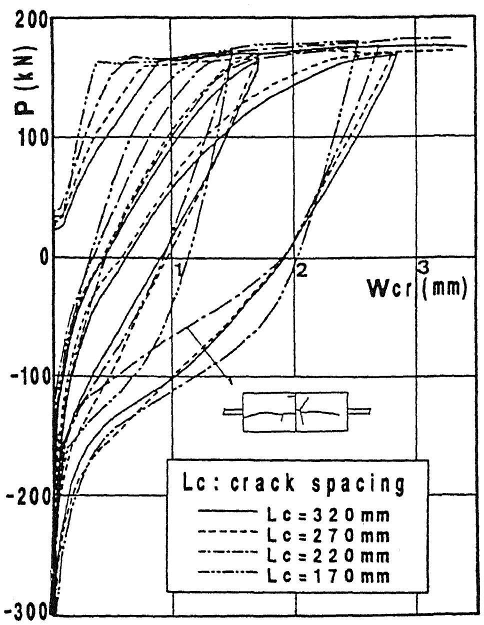

Fig. 2 P-Wcr relationships

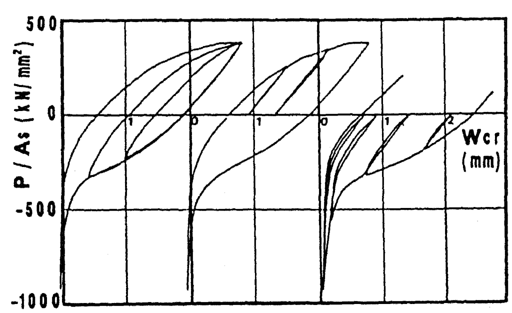

Fig. 3 P/As-Wcr relationships

to investigate the resistance mechanisms at the discrete section in RC members under reversed cyclic loading. The P-Wcr relation that was obtained from the experiments was modeled in numerical formulae. The model curve is shown in Figure 4. In this model, the influences of the deterioration of bond action and the local contact effect of a crack are taken into account. This local contact effect shows the phenomenon that concrete shares compressive force before a crack closes completely. These are the typical phenomena of a RC composite element including a crack.

The each parts of the model curve are derived from the following equations. The main symbols used in this equations are put in NOTATION.

$$\delta ax = Wcr + \delta cc \tag{1}$$

where δax=total axial deformation, Wcr=crack width and δcc=axial deformation of concrete.

① A compressive envelope curve before crack initiation (Wcr=0)

$$\delta cc = \left(\sqrt{\frac{P}{fc \cdot Ae}+1}-1\right) \cdot Lm \cdot c\varepsilon u \tag{2}$$

where $c\varepsilon u$=absolute value of compressive strain of concrete at the point of maximum compressive stress.

② Unloading and reloading curves before crack initiation (Wcr=0)

$$\delta cc = \frac{Lm \cdot (P - Pbs)}{Ec \cdot Ae} + Wbs \tag{3}$$

where (δbs, Pbs) is the turning point for unloading or reloading. At the time of compressive reloading, a curve follows to Eq.(2) when the curve reachs the envelope curve. When a crack opens, axial deformation of concrete is constant as Eq.(4). However, it can be neglected for simplicity when the δre is small.

$$\delta cc = \delta re = \frac{Lm \cdot (Pcr - Pbs)}{Ec \cdot Ae} + \delta bs \tag{4}$$

③ Cracking load (Pcr)

$$Pcr = ft \cdot Ae \tag{5}$$

$$ft = 0.00776 \cdot t \cdot c\sigma t \quad (t < 87.5\ \text{mm}) \tag{6}$$
$$ft = 0.679 \cdot c\sigma t \quad (t \geqq 87.5\ \text{mm}) \tag{7}$$

where ft and $c\sigma t$=tensile strength of concrete when a crack initiate and by material test, respectively, t =concrete cover.

④ Condition where a crack has occured and the reinforcing bar does not yield ($Pcr \leqq P \leqq Py$)

$$Wcr = \frac{0.574 \cdot P \cdot Lc}{As \cdot Es} + 3.25 \times 10^{-7} \cdot Lc \left(\frac{P - Pcr}{As}\right)^{1.45} \tag{8}$$

⑤ An envelope curve after the reinforcing bar yields

$$Wcr = \frac{5.42 \cdot (P - Py)}{t \cdot As} + Wy \tag{9}$$

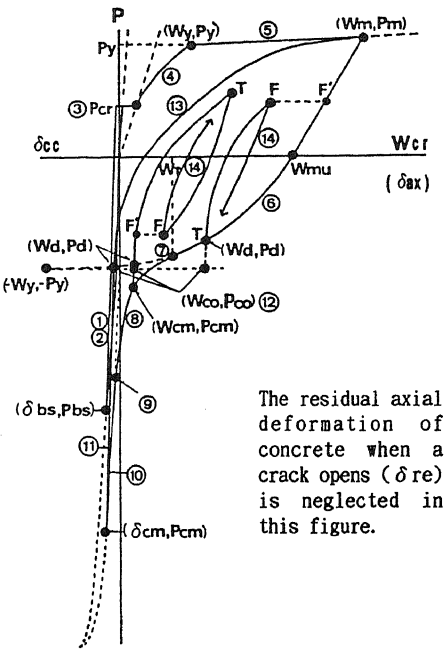

Fig. 4 P-Wcr model curve

in which (Wy, Py) =point that the reinforcing bar yields.

⑥ A curve for decrease of crack width from the point of maximum crack width to the local contact point

$$Wcr = \left(\frac{P + Py}{Pm + Py}\right)^{(1/a)} \times (Wm + Wy) - Wy \tag{10}$$

$$a = 1.12 + 0.408 \cdot Ey / Et \tag{11}$$

where (Wm, Pm) =the turning point that is the maximum crack width. If $Ey/Et > 4.5$ then $Ey/Et = 4.5$. $Et = Pm/Wm$, $Ey = Py/Wy$.

⑦ Crack width of a point that the concrete starting to contact locally (WT)

$$WT = 0.236 \cdot Wmu = 0.236 \cdot \left(\left(\frac{Py}{Pm + Py}\right)^{(1/a)} \times (Wm + Wy) - Wy\right) \tag{12}$$

in which Wmu=residual crack width after unloading from the point of (Wm, Pm).

⑧ Compressive loading after the local contact

$$Wcr = \left(1 - 0.196 \cdot \log\left(1 - 29.1 \cdot \frac{\Delta P}{Ac}\right)\right) \cdot WT \tag{13}$$

$$P = \Delta P + Ps = \Delta P + \left(\frac{Wcr + Wy}{Wm + Wy}\right)^{a} \times (Pm + Py) - Py \tag{14}$$

a) a compressive envelope curve of concrete

$$\delta cc = \left(\sqrt{\frac{\Delta P}{fc \cdot Ac}+1}-1\right) \cdot Lm \cdot c\varepsilon u \tag{15}$$

b) reloading and unloading curves of concrete

$$\delta cc = \frac{Lm \cdot \Delta P}{Ec \cdot Ac} + \delta re \tag{16}$$

where ΔP, Ps is the compressive force sheared by the concrete and reinforcing bar, respectively. Giving the ΔP to these equations, Wcr, δcc, Ps and P can be calculated.

⑨ Closing of a crack completely ($Wcr=0$)

$$\Delta P = -5.67 \cdot Ac \tag{17}$$

⑩ Compressive loading after a crack closed ($Wcr=0$)
δcc follows the Eq.(15) or (16).

⑪ Unloading and reloading after the local contact point

$$\delta cc = \frac{Lm \cdot (P - Pcm)}{Ec \cdot Ac} + \delta cm \tag{18}$$

a) if $\Delta Pcm \geqq -5.67 \cdot Ac$

$$Wcr = Wcm = (1 - 0.196 \cdot \log(1 - 2.85 \cdot \frac{\Delta Pcm}{Ac}))\ WT \tag{19}$$

b) if $\Delta Pcm < -5.67 \cdot Ac$

$$Wcr = Wcm = 0 \tag{20}$$

where (Wcm, Pcm) = the turning point for unloading after the local contact. ΔPcm and δcm = compressive force sheared by the concrete and axial deformation of concrete at the point of (Wcm, Pcm).

⑫ A point that a crack reopens (Wd, Pd)

$$Wd = Wcm \tag{21}$$

$$Pd = \left(\frac{Wcm + Wy}{Wcm + Wy}\right)^a \times (Pm + Py) - Py \tag{22}$$

$$\delta cc = \frac{Lm \cdot (Pd - Pcm)}{Ec \cdot Ac} + \delta cm \tag{23}$$

Ideal starting point of a curve (Eq.25) after a crack reopens (Wco, $-Py$)

$$Wco = \frac{Wd - Wm\left(1 - \left(1 - \left(\frac{Pd + Py}{Pm + Py}\right)^c\right)^{(1/b)}\right)}{1 - \left(1 - \left(1 - \left(\frac{Pd + Py}{Pm + Py}\right)^c\right)^{(1/b)}\right)} \tag{24}$$

where (Wd, Pd) = the turning point before the local contact or reopening point after local contact

⑬ A curve after a crack reopened

$$Wcr = \left(1 - \left(1 - \frac{P + Py}{Pm + Py}\right)^c\right)^{(1/b)} \times (Wm - Wco) + Wco \tag{25}$$

$$b = 0.454 + 0.071 \cdot Ey/Et \tag{26}$$

$$c = 1.06 - 0.139 \cdot Ey/Et \tag{27}$$

If $Ey/Et > 4.5$ then $Ey/Et = 4.5$.

⑭ Cyclic loading inside a cycle

$$Wcr = \frac{(f(P) - Wrt) \cdot (Wrf - Wrt)}{f(Prf) - Wrt} + Wrt \tag{28}$$

in which (Wrf, Prf) = turning point F, (Wrt, Prt) = aim point T, $Wcr = f(P)$: previous curve under same condition of loading, $f(Prf)$ = crack width of the point F' that is on the previous curve and axial force is Prf.

⑮ After the initiation of a secondery crack

$$Wcr = \frac{0.574 \cdot (P - Pcr2) \cdot Lc2}{As \cdot Es} + 3.25 \times 10^{-7} \cdot Lc2 \left(\frac{P - Pcr2}{As}\right)^{1.45} + Wcr2 \tag{29}$$

where $Lc2$, $Pcr2$ and $Wcr2$ = crack spacing, cracking load and crack width when a secondary crack initiates, respectively.

The opening and closing behavior of a crack under reversed cyclic loading is represented by the above mentioned equations. A comparison between this model and the experimental result is shown in Figure 5. It is shown that the behavior of a crack in a pure tensile stress state is reproduced very well by this model.

Six simple supported RC beams were tested under cyclic loading to verify the crack model. A comparison between the model and the test result of a RC beam type specimen is shown in Figure 6. The performance of a crack in the RC beam is reproduced very well by the proposed model. Therefore, this model is confirmed as the suitable analytical model for the composite elements at the cracking area in RC members.

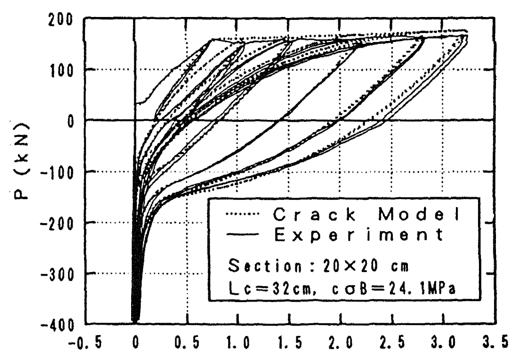

Fig. 5 A comparison of P-Wcr curves (Prism test)

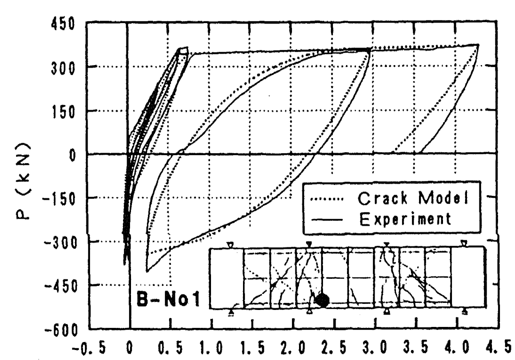

Fig. 6 A comparison of P-Wcr curves (Simple beam test)

5. ANALYTICAL STUDY ON ROTATIVE BEHAVIOR

5.1 Method of analysis

The proposed crack model is used to compute the rotative behavior of a discrete section in RC simple beams subjected reversed cyclic loading. Special feature of this analysis is to be able to take into account the non-continuity of curvature in a RC member. It's an example for methods of application of the proposed crack model.

Arrangement of the discrete section is shown in Figure 7. The ideal prism elements are used at the top and bottom of the cross section of the beam. The concrete covers of the element that are under or over the main steel bars are set the same value. For simplicity in order to decrease the computer time, the concrete except the prism elements is neglected in this study. Once the distribution of crack width (total axial deformation) is given, the axial forces acting on the elements are computed by the crack model. If the equilibrium of the forces is not satisfied, the distribution of crack width must be adjusted until the equilibrium is achieved. An iterative technique is used to calculate points on the moment(M) - rotation (θ) curves.

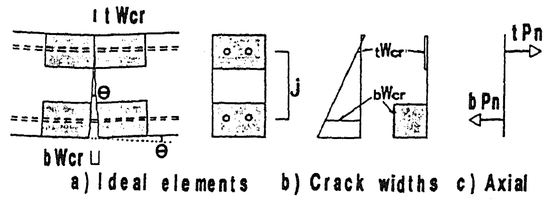

Fig. 7 Ideal element of discrete section

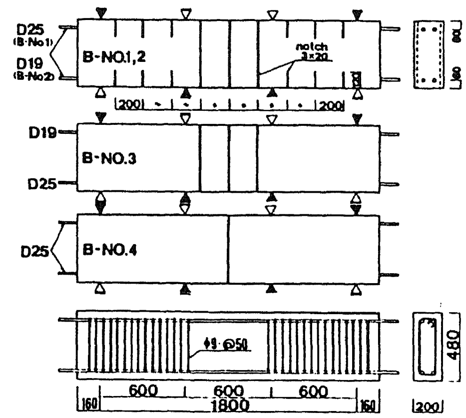

Fig. 8 Dimensions of beam specimens

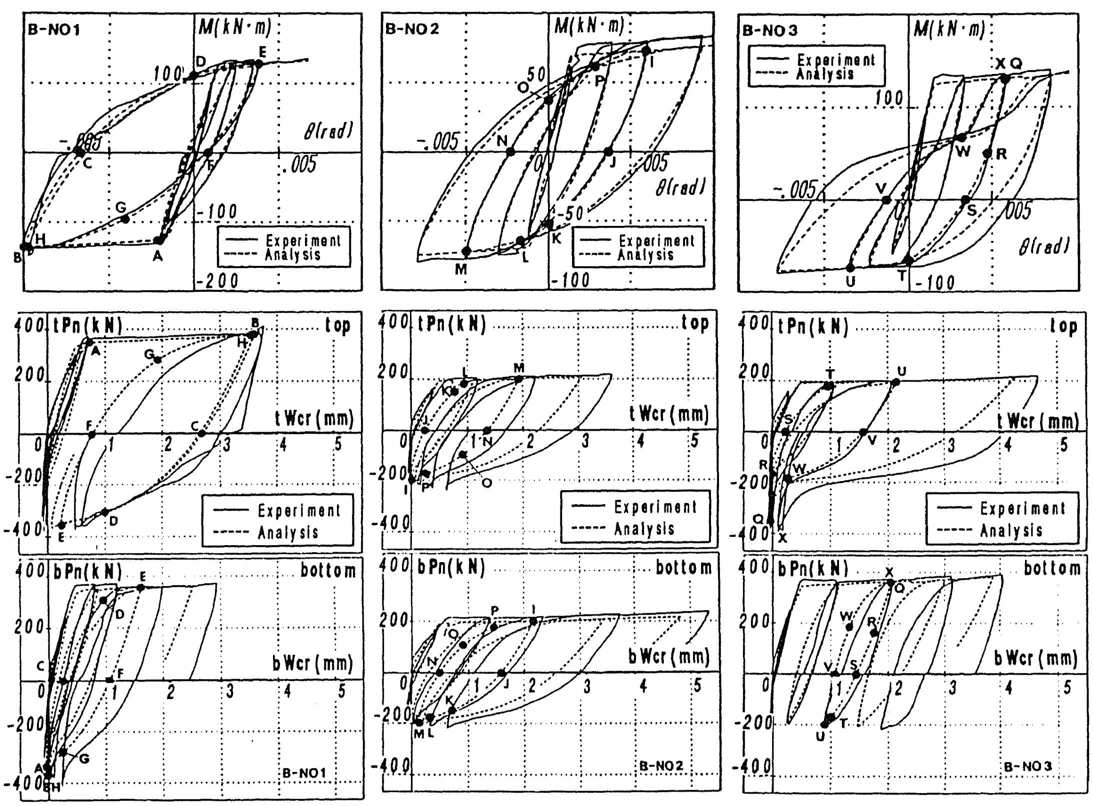

Fig. 9 Comparisons of experimental and analytical rotative behaviors

5.2 Rotative behavior at a discrete section

The RC beam type specimens with constant moment region were tested in order to verify the proposed analytical method. Dimensions of specimens are shown in figure 8. Yield strength of reinforcing bar is 400 N/mm^2 and concrete strength is 25.9 N/mm^2.

Figure 9 shows the comparison of computed and experimental results. Axial forces (P) of experimental curves were calculated from the applied load by using the distance of the top and bottom main bars (j). In spite of some assumptions, the analytical moment-rotation curves at the discrete section and the behavior of cracks compare reasonably well with experiment. In the specimen B-NO3 with different top and bottom steel areas, the stiffness of the pinching curve increases gradually because of the local contact effect. It is marked that this characteristics can be explained by this analysis. In the specimen B-NO1 and NO2 with equal top and bottom steel, the cracks of both elements are open at most part of the moment-rotation curves. It means the behavior of a crack contributes to the rotative performances of a discrete section. From these results, it was confirmed that this analytical method is useful to predict the rotative behavior of a discrete section.

Figure 10 and 11 show the load-deflection relationships of B-NO3 and NO4. Deflection is computed by using the computed rotative behavior and the ideal deflection model shown in figure 11. The agreements between the analytical and experimental results are reasonable. Therefor, it is clarified that the mechanisms of the restoring force characteristics of RC members with flexure are able to explain by the behavior of cracks at the discrete section.

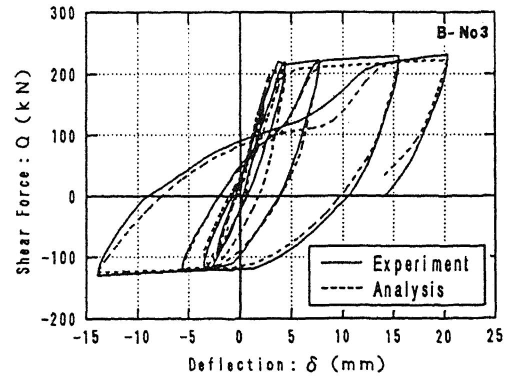

Fig. 10 A comparison of Q-δ curves

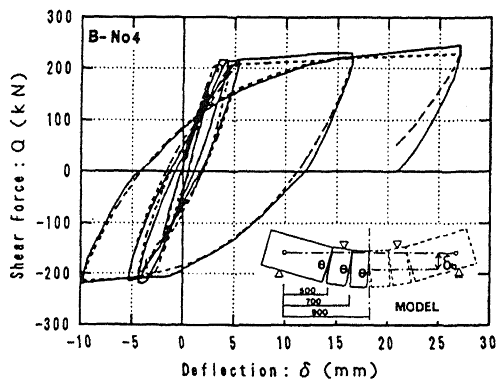

Fig. 11 A comparison of Q-δ curves

6. CONCLUDING REMARKS

The conclusions of this study are summarized as follows.

(1) The effective analytical model that represents the opening and closing behavior of a crack was proposed. This model was confirmed as the suitable analytical model for the composite elements at the cracking area in RC members.

(2) Using this crack model, the analytical method was proposed in order to predict the rotative behavior at a discrete section in RC members. Special feature of this analysis is to be able to consider the non-continuity of curvature in a RC member. The test results of moment-rotation relations were reproduced very well by this analysis.

(3) The mechanisms of restoring force characteristics of RC bending members are able to explain by the opening and closing behavior of cracks at the discrete section.

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NOTATION

The following symbols are used in this paper. (Put the main symbols in here.) (length(mm),force(N),stress(N/mm^2))

Ac and As =sectional area of concrete and reinforcing bar, Ae =Ac+As·Es/Ec, Ec and Es =Young's modulus of concrete and reinforcing bar, fc =absolute value of compressive strength of concrete, Lc =crack spacing, Lm =length of measuring, P =axial force, Py =yield force of reinforcing bar, Wcr =crack width