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A damage model for the seismic analysis of building structures

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ABSTRACT: A procedure to evaluate the damage of reinforced concrete structures subjected to seismic actions is developed. A local damage constitutive model based on Kachanov's theory is developed. The structural model, which includes the local damage constitutive law, is used within a finite element discretization. A global damage index based on potential energy is deduced. Numerical examples are finally given.

1 INTRODUCTION

In the case that a urban area is affected by a strong seismic motion, one of the most important problems to be considered is the safety evaluation of the structures in that area. This evaluation can be done using mechanical and mathematical models to evaluate the damage produced to structures by earthquakes. The damage of reinforced concrete structures will hereafter be defined as the degree of structural degradation that allows conclusions about the future capacity of the structure to withstand other important loadings. A "damage index" is defined as the value of damage normalized to the failure level of the structure, so that a value equal to 1 will reflect complete failure [Park et al. (1987), DiPasquale and Cakmak (1989)]. In recent works Bracci et al. (1989) define a damage index for structural members using a linear combination between a ductility factor and an energy factor. For complex structures the definition of a global structural damage index is generally based on a weighted average of the indices coresponding to the different members of the structure.

This paper develops a procedure of evaluation of the damage in reinforced concrete structures. In order to do this, a local damage constitutive model, based on Kachanov's theory (1958) is used. A structural model which inserts the local damage constitutive theory into beam structures behaviour is developed in the frame of the finite element method. Numerical simulation examples of the proposed analysis procedure are included.

2 DYNAMIC STRUCTURAL MODEL

The structure is modelized using the finite element method. The element used is based on Timoshenko's beam theory completed with a layered formulation (see figure 1). The generalized stresses are solved through C^0 lagrangean finite elements with three degrees of freedom per node in the plane case. The cross-sectional generalized stresses obtained are decomposed point by point, layer by layer, in stress tensors which are treated and corrected by the damage model and afterwards recomposed in the resultant sectional generalized stresses. These last stresses are used then to compute the residual forces, in order to iterate for equilibrium if necessary.

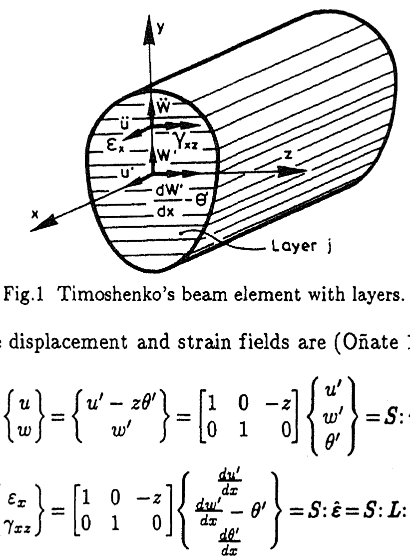


Fig.1 Timoshenko's beam element with layers.

The displacement and strain fields are (Oñate 1992)

$$\boldsymbol{u}=\begin{Bmatrix} u \\ w \end{Bmatrix}=\begin{Bmatrix} u'-z\theta' \\ w' \end{Bmatrix}=\begin{bmatrix} 1 & 0 & -z \\ 0 & 1 & 0 \end{bmatrix}\begin{Bmatrix} u' \\ w' \\ \theta' \end{Bmatrix}=\boldsymbol{S}:\boldsymbol{u}' \quad (1)$$

$$\boldsymbol{\varepsilon}=\begin{Bmatrix} \varepsilon_x \\ \gamma_{xz} \end{Bmatrix}=\begin{bmatrix} 1 & 0 & -z \\ 0 & 1 & 0 \end{bmatrix}\begin{Bmatrix} \frac{du'}{dx} \\ \frac{dw'}{dx}-\theta' \\ \frac{d\theta'}{dx} \end{Bmatrix}=\boldsymbol{S}:\hat{\boldsymbol{\varepsilon}}=\boldsymbol{S}:\boldsymbol{L}:\boldsymbol{u}' \quad (2)$$

where the upper variables have the following meaning: $\boldsymbol{u}$ – displacement vector of a point belonging to a current section; $\boldsymbol{\varepsilon}$ – strain vector of a point belonging to a current section; $\boldsymbol{u}'$ – displacement vector of the beam finite element corresponding to the central axis of the beam; $\hat{\boldsymbol{\varepsilon}}$ – generalized strain vector corresponding to the central axis of the beam; $\boldsymbol{S}, \boldsymbol{L}$ – transformation matrices.

The second order time derivative of equation (1) provides the acceleration field function of the beam central axis acceleration field $\ddot{\boldsymbol{u}} = \boldsymbol{S}:\ddot{\boldsymbol{u}}'$.

On every differential surface dA of a given section of the beam element act two accelerations $\ddot{u}$ y $\ddot{w}$ in the x and z local directions which produce differential inertial forces. Integrating over the cross-section, the inertial sectional forces $\hat{\boldsymbol{f}}_a$ and the generalized stresses $\hat{\boldsymbol{\sigma}}$ are obtained

$$\hat{\boldsymbol{f}}_a=\begin{Bmatrix}N_a\\Q_a\\M_a\end{Bmatrix}=\int_A\begin{Bmatrix}\rho\ddot{u}\\\rho\ddot{w}\\-z\rho\ddot{u}\end{Bmatrix}dA=\int_A\rho\begin{bmatrix}1&0\\0&1\\-z&0\end{bmatrix}\begin{Bmatrix}\ddot{u}\\\ddot{w}\end{Bmatrix}dA \quad (3a)$$

$$\hat{\boldsymbol{\sigma}}=\begin{Bmatrix}N\\Q\\M\end{Bmatrix}=\int_A\begin{Bmatrix}\sigma_x\\\tau_{xz}\\-z\sigma_x\end{Bmatrix}dA=\int_A\begin{bmatrix}1&0\\0&1\\-z&0\end{bmatrix}\begin{Bmatrix}\sigma_x\\\tau_{xz}\end{Bmatrix}dA \quad (3b)$$

where $\boldsymbol{\sigma}=\boldsymbol{C}:\boldsymbol{\varepsilon}$ and $\boldsymbol{C}$ is the constitutive stiffness tensor. Using now the equations (1) and (2), (3a) and (3b) become:

$$\hat{\boldsymbol{f}}_a=\int_A\rho\,\boldsymbol{S}:\ddot{\boldsymbol{u}}\,dA=\int_A\rho\,\boldsymbol{S}:\boldsymbol{S}:\ddot{\boldsymbol{u}}'\,dA=\hat{\boldsymbol{\rho}}:\ddot{\boldsymbol{u}}' \quad (4a)$$

$$\hat{\boldsymbol{\sigma}}=\int_A\boldsymbol{S}:\boldsymbol{\sigma}\,dA=\int_A\boldsymbol{S}:\boldsymbol{C}:\boldsymbol{\varepsilon}dA=\int_A\boldsymbol{S}:\boldsymbol{C}:\boldsymbol{S}:\hat{\boldsymbol{\varepsilon}}\,dA=\hat{\boldsymbol{C}}:\hat{\boldsymbol{\varepsilon}} \quad (4b)$$

From the last relations the generalized density matrix $\hat{\boldsymbol{\rho}}$ and the generalized constitutive matrix $\hat{\boldsymbol{C}}$ are

$$\hat{\boldsymbol{\rho}}=\int_A\rho\,\boldsymbol{S}:\boldsymbol{S}\,dA\;;\qquad\hat{\boldsymbol{C}}=\int_A\boldsymbol{S}:\boldsymbol{C}:\boldsymbol{S}\,dA \quad (5)$$

Equations (5) can be integrated however complex is the distribution of material properties in the section.

Following standard procedures, the equivalent inertial and elastic forces are

$$\boldsymbol{F}_a=\int_\ell\boldsymbol{N}:\hat{\boldsymbol{f}}_a\,ds=\int_\ell\boldsymbol{N}:\hat{\boldsymbol{\rho}}:\ddot{\boldsymbol{u}}'\,ds \quad (6)$$

$$\boldsymbol{F}_e=\int_\ell\boldsymbol{B}:\hat{\boldsymbol{\sigma}}\,ds=\int_\ell\boldsymbol{B}:\hat{\boldsymbol{C}}:\hat{\boldsymbol{\varepsilon}}\,ds \quad (7)$$

where $\boldsymbol{N}$ and $\boldsymbol{B}$ are the shape function and derivative matrices and ℓ is the length of the element.

$$\boldsymbol{u}'=\boldsymbol{N}:\boldsymbol{a}\Rightarrow\ddot{\boldsymbol{u}}'=\boldsymbol{N}:\ddot{\boldsymbol{a}}\;;\quad\hat{\boldsymbol{\varepsilon}}=\boldsymbol{L}:\boldsymbol{N}:\boldsymbol{a}=\boldsymbol{B}:\boldsymbol{a} \quad (8)$$

With these last transformations, the nodal inertial and elastic forces become

$$\boldsymbol{F}_a=\int_\ell\boldsymbol{N}:\hat{\boldsymbol{\rho}}:\boldsymbol{N}ds:\ddot{\boldsymbol{a}}=\boldsymbol{M}:\ddot{\boldsymbol{a}}\;;\;\boldsymbol{F}_e=\int_\ell\boldsymbol{B}:\hat{\boldsymbol{C}}:\boldsymbol{B}\,ds:\boldsymbol{a}=\boldsymbol{K}:\boldsymbol{a} \quad (9)$$

where $\boldsymbol{M}$ and $\boldsymbol{K}$ are the elemental mass and stiffness matrices for the Timoshenko's finite element beam formulation.

3 CONSTITUTIVE MODEL FOR ISOTROPIC DAMAGE

3.1 General concepts

The solution of beam structures subjected to seismic actions beyond the linear behaviour has been usually treated using: (a) Theories based on plastic hinge formation [Massonet and Save 1966]. This approach has the inconvenient of admiting that the damage of a structure point is dominated by bending criteria, which is true only for some very particular structures. (b) Simulation of beam structures based on the concept of plastification bendig moment. This procedure is based on formulating simplified curvature – bending moment constitutive laws [Clough et al. 1965, Aoyama and Sugano 1968].

The last formulations started from representing the behaviour of materials in an approximate form based mainly on experimental studies. Today, it is required that these formulations be thermodynamically sustainable. Between those which meet this latter requirement, the so-called continuous damage theory is generally accepted as an alternative in the most complex constitutive formulations [DiPasquale and Cakmak 1989, Oliver et al. 1990]. Such a model can be seen in Mazars (1991) where a column discretized in plane finite elements, subjected to seismic action is calculated. The damage models have a rigourous but relatively simple formulation strictly based on thermodynamics. They deal with the non-linear behaviour by means of one or more internal variables called damage variables which weight the losing of secant stiffness of the material and are normalized to an unit value which corresponds to maximum damage. Figure 2 shows a unidimensional representation of the behaviour of a point of a damaged material.

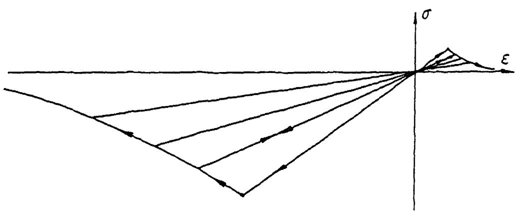


Fig.2 Local damage behaviour.

The model presented herein is a 3D damage constitutive model based on solid mechanics and it has a single internal variable. Therefore it is a local isotropic damage model and it is based on Kachanov's theory (1958). In this model have been included later experiences such as those of Simó and Ju (1987), Lubliner et al. (1989) y Oliver et al. (1990). This formulation is a compromise between the complexity implied in modeling concrete behaviour and the versatility needed when dealing with dynamic problems. This insures accurate results and low cost solutions for the non-linear problems to be solved.

3.2 Characteristics of the model

The model is formulated in the material configuration, for thermodynamically stable problems, with no temperature time variation. For this specific case the subsequent mathematical form for the free energy is suposed, where the non-damaged elastic part is expressed as a scalar quadratic function of tensorial arguments

$$\Psi(\boldsymbol{\varepsilon};d)=(1-d)\Psi_0(\boldsymbol{\varepsilon};d)=(1-d)\left(\frac{1}{2m_0}\boldsymbol{\varepsilon}:\boldsymbol{C}^0:\boldsymbol{\varepsilon}\right) \quad (10)$$

where the strain tensor ε is the free variable of the problem, d $(0 \leq d \leq 1)$ the internal damage variable, m_0 the density in the material configuration and C^0 the stiffness tensor of the material in the initial undamaged state.

For stable thermical state problems the Clasius Planck dissipation is valid, whose local lagrangean form is

$$\Xi_m = \left(\frac{1}{m_0}\sigma - \frac{\partial \Psi}{\partial \varepsilon}\right) : \dot{\varepsilon} - \frac{\partial \Psi}{\partial d}\dot{d} \geq 0 \qquad (11)$$

This expression for the dissipation allows the following two considerations:

a) In order to guarantee the fulfilment of the Clasius Planck inequation, the multiplier of $\dot{\varepsilon}$ which represents an arbitrary temporal variation of the free variable, must be null (Colleman's method). This condition provides the constitutive law of the studied damage problem:

$$\frac{1}{m_0}\sigma - \frac{\partial \Psi}{\partial \varepsilon} = 0 \Rightarrow \sigma = m_0 \frac{\partial \Psi}{\partial \varepsilon} = (1-d)C^0 : \varepsilon = C^s : \varepsilon \quad (12)$$

b) Considering the last equation, the dissipation is now

$$\Xi_m = -\frac{\partial \Psi}{\partial d}\dot{d} = \Psi_0 \dot{d} \geq 0 \qquad (13)$$

The damage yield criterion is defined as a function of the free energy of the undamaged material, expressed in terms of the undamaged principal stresses σ_i^0, as

$$F = K(\sigma^0)\sqrt{2m_0\Psi^0} - 1 = \frac{K(\sigma^0)}{\sqrt{E^0}}\sqrt{\sum_{i=1}^{3}(\sigma_i^0)^2} - 1 \leq 0 \quad (14)$$

where the terms of the above equation have the following meaning:

$$K(\sigma^0) = \frac{r}{\sqrt{(\Psi_t^0)_L}} + \frac{1-r}{\sqrt{(\Psi_c^0)_L}} \; ; \; r = \frac{\sum_{i=1}^{3}\langle \sigma_i^0 \rangle}{\sum_{i=1}^{3}|\sigma_i^0|}$$

$$(\Psi_{t,c}^0)_L = \sum_{i=1}^{3}\langle \pm\sigma_i^0 \rangle \varepsilon_i \; ; \; (\Psi^0)_L = (\Psi_t^0)_L + (\Psi_c^0)_L$$

In these equations $(\Psi_{t,c}^0)_L$ represent the part of the free energy developed when the traction/compression limit is reached and $\langle \pm x \rangle = \frac{1}{2}(|x| \pm x)$ is McAuley's function. Taking into account that the traction/compression strengthes are $f_t = (\Psi_t^0 E^0)_L^{1/2}$ and $f_c = (\Psi_c^0 E^0)_L^{1/2}$ the damage yield function can be written, according to figure 3, as

$$F = \bar{\sigma} - f_c = [1 + r(n-1)]\sqrt{\sum_{i=1}^{3}(\sigma_i^0)^2} - f_c \leq 0 \quad (15)$$

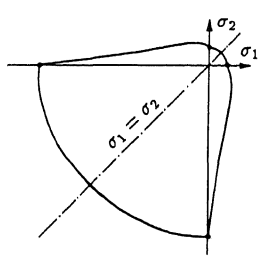


Fig.3 Damage Yield in the principal plane $\sigma_1 - \sigma_2$.

with $n = f_c/f_t$. This damage yield function, expressed in the non-damaged principal stresses space, allows a great choice of distinct solutions. The advantage of the yield criterium (15) is that any yield function F can be used always when homogenous and of first order in stresses, like Mohr-Coulomb, Drucker-Prager, Lubliner et al. (1989), etc. The form given by equation (15) fulfils the above requierements; besides, is simple and satisfactory in results within the work range used with this model and therefore will be used henceforward as the scalar expression defining $\bar{\sigma}$. An expression entirely equivalent to (15) proposed by Simó (1987) with the aim of simplifying the mathematical deduction of the damage variable of the model is the following:

$$\bar{F} = G(\bar{\sigma}) - G(f_c) \leq 0 \qquad (16)$$

where $G(\chi)$ is a scalar monotonic function to be determined.

The following mathematical form is used to deduce the damage internal variable evolution rule:

$$\dot{d} = \dot{\eta}\frac{\partial \bar{F}}{\partial \bar{\sigma}} = \dot{\eta}\frac{\partial G(\bar{\sigma})}{\partial \bar{\sigma}} \; ; \quad \dot{\eta} \equiv \dot{f}_c \qquad (17)$$

Like in plasticity, there is a consistency rule for a point subjected to a damaging process. This is

$$\dot{\bar{F}} = 0 \Rightarrow \frac{\partial \bar{F}}{\partial \bar{\sigma}}\dot{\bar{\sigma}} + \frac{\partial \bar{F}}{\partial f_c}\dot{f}_c = \frac{\partial G(\bar{\sigma})}{\partial \bar{\sigma}}\dot{\bar{\sigma}} - \frac{\partial G(f_c)}{\partial f_c}\dot{f}_c = 0 \quad (18)$$

$$\bar{F} = 0 \Rightarrow G(\bar{\sigma}) = G(f_c) \Rightarrow \dot{\bar{\sigma}} = \dot{f}_c \equiv \dot{\eta} = \frac{\partial \bar{\sigma}}{\partial \sigma^0} : \dot{\sigma}^0 = \frac{\partial \bar{\sigma}}{\partial \sigma^0} : C^0 : \dot{\varepsilon} \quad (19)$$

Substituting this last equation in (16) and afterwards in (13) we obtain the following expressions which formulate the temporal evolution of the damage and dissipation variables:

$$\dot{d} = \frac{\partial G(\bar{\sigma})}{\partial \bar{\sigma}}\frac{\partial \bar{\sigma}}{\partial \sigma^0} : C^0 : \dot{\varepsilon} = \dot{G}(\bar{\sigma}) \Rightarrow d = \int_0^t \dot{d}\, dt \equiv \int_0^t \dot{G}(\bar{\sigma})dt \equiv G(\bar{\sigma}) \quad (20)$$

$$\dot{\Xi} = \bar{\Psi}^0 \frac{\partial G(\bar{\sigma})}{\partial \bar{\sigma}}\frac{\partial \bar{\sigma}}{\partial \sigma^0} : C^0 : \dot{\varepsilon} \qquad (21)$$

The loading/unloading condition is derivated from the relations of Kuhn-Tucker formulated for problems with unilateral restrictions: (a) $\dot{\eta} \geq 0$; (b) $\bar{F} \leq 0$ and (c) $\dot{\eta}\bar{F} = 0$. From these, if $\bar{F} < 0$ then the third condition imposes $\dot{\eta} = 0$ and if $\dot{\eta} > 0$ then the same condition requires that $\bar{F} = 0$.

From the diverse alternatives for defining the function $G(\chi)$, the following was chosen

$$G(\chi) = 1 - \frac{\bar{G}(\chi)}{\chi} \qquad (22)$$

where $\bar{G}(\chi)$ describes a function like the one presented in figure 4, so that it gives for $\chi = \chi^*$ the compression initial yield tension $\bar{G}^*$ and for $\chi \to \infty$ the final strength $\bar{G} \to 0$. Thus, by running all the path, the point will have dissipated an energy equivalent to the specific fracture energy.

In this paper an exponential function obtained by Oliver et al. (1990) was used

$$\bar{G}(\chi) = \chi^* e^{A\left(1-\frac{\chi}{\chi^*}\right)} \; ; \quad G(\chi) = 1 - \frac{\chi^*}{\chi} e^{A\left(1-\frac{\chi}{\chi^*}\right)} \quad (23)$$

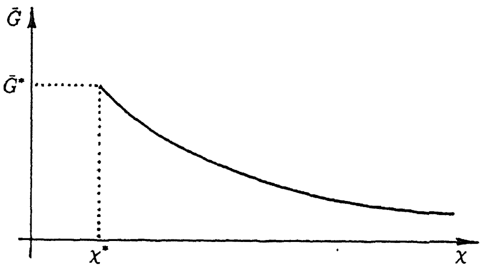


Fig.4 Reprezentation of the chosen $G(\chi)$.

For an uniaxial traction process under monotonically increasing load, the temporal dissipation change is given by (11), with $\bar{\sigma} = n\sigma_t$, $\bar{\Psi}_0 = \frac{1}{2}\varepsilon_t E^0 \varepsilon_t = \frac{(\sigma_t)^2}{2E^0} = \frac{\bar{\sigma}^2}{2n^2 E^0}$. Integrating (11) in time we can calculate the total dissipation at the end of the uniaxial traction process as:

$$\Xi_t^{max} = \int_{\bar{\sigma}^*}^{\infty} \frac{\bar{\sigma}^2}{2n^2 E^0} \frac{\partial G(\bar{\sigma})}{\partial \bar{\sigma}} d\bar{\sigma} = \int_{\bar{\sigma}^*}^{\infty} \frac{\bar{\sigma}^2}{2n^2 E^0} dG(\bar{\sigma}) \quad (24)$$

$$\Xi_t^{max} = \left[\frac{\bar{\sigma}^2}{2n^2 E^0} G(\bar{\sigma})\right]_{\bar{\sigma}^*}^{\infty} - \int_{\bar{\sigma}^*}^{\infty} G(\bar{\sigma}) \frac{\bar{\sigma}}{n^2 E^0} d\bar{\sigma} \quad (25)$$

$$\Xi_t^{max} = \frac{(\bar{\sigma}^*)^2}{n^2 E^0}\left[\frac{1}{2} + \frac{1}{A}\right] \Rightarrow A = \frac{1}{\frac{\Xi_t^{max} n^2 E^0}{(\bar{\sigma}^*)^2} - \frac{1}{2}} \geq 0 \quad (26)$$

Making the same hypotheses for an uniaxial compression process, and postulating that parameter A must be the same in both cases it is deduced that

$$A = \frac{1}{\frac{\Xi_c^{max} E^0}{(\bar{\sigma}^*)^2} - \frac{1}{2}} \geq 0 \Rightarrow \Xi_c^{max} = n^2 \Xi_t^{max} \quad (27)$$

The value of traction maximum dissipation Ξ_t^{max} is an input of the problem and is equal to the fracture energy density g_f, parameter derived from fracture mechanics as $g_f = G_f/l^c$, where l_c is the characteristic length of the finite fractured domain (Lubliner et al. 1989).

From (12), the temporal variation of the stress tensor and finally the unsymmetric tangent constitutive tensor C^d of the damage model can be deduced as

$$\dot{\sigma} = C^s : \dot{\varepsilon} + \dot{C}^s : \varepsilon \; ; \quad \dot{C}^s = \frac{\partial C^s}{\partial d} \dot{d} = -C^0 \dot{d} \quad (28)$$

$$\varepsilon = (C^s)^{-1} : \sigma = \frac{(C^0)^{-1} : \sigma}{1-d} \Rightarrow \dot{\sigma} = C^s : \dot{\varepsilon} - \frac{\dot{d}}{1-d} \sigma \quad (29)$$

$$\dot{\sigma} = \left\{ C^s - \frac{1}{1-d} \frac{\partial G(\bar{\sigma})}{\partial \bar{\sigma}} \left[\left(\frac{\partial \bar{\sigma}}{\partial \sigma^0} : C^0 \right) \otimes \sigma \right] \right\} : \dot{\varepsilon} \quad (30)$$

$$\Rightarrow \dot{\sigma} = C^d : \dot{\varepsilon} \quad (31)$$

3.3 Global damage indices

The starting point for deducing a global structural damage index is equation (10), which relates the damaged part of the free energy Ψ with the non-damaged elastic free energy Ψ_0. In order to find a global index, a similar expression is deduced by integrating (10) over the entire volume of the structure as follows:

$$\Psi = (1-d)\Psi_0 \Rightarrow W_p = \int_V \Psi dV = \int_V (1-d)\Psi_0 dV = (1-D) W_p^0 \quad (32)$$

where D is the global damage index, $W_p^0 = \int_V \Psi_0 dV$ is the total potential energy of the structure if it were undamaged and W_p is the total potential energy corresponding to the actual damaged state. Solving equation (32) for D, the following final relation is obtained:

$$D = 1 - \frac{W_p}{W_p^0} = \frac{\int_V \Psi_0 dV - \int_V (1-d)\Psi_0 dV}{\int_V \Psi_0 dV} = \frac{\int_V d\, \Psi_0 dV}{\int_V \Psi_0 dV} \quad (33)$$

If a damage index for a subvolume of the structure is needed (such as a floor, some columns, etc) the integration will be done only over that specific subvolume.

In a finite element scheme, in the case of a structure discretized with layered beams, the damage index of a beam point D_p (considering the beam as an unidimensional finite element) is given by a similar expression obtained by integrating (10) over the cross-section of the beam, with $\Psi_0 = \frac{1}{2}\varepsilon : \sigma^0$ and $\varepsilon = S : \hat{\varepsilon}$

$$D_p = 1 - \frac{\hat{\varepsilon} : \hat{\sigma}}{\hat{\varepsilon} : \hat{\sigma}^0} \; ; \quad \hat{\sigma} = \int_A S : \sigma \, dA = \int_A (1-d) S : \sigma^0 dA \quad (34)$$

where $\hat{\varepsilon}$ and $\hat{\sigma}$ are the generalized strains and stresses in that beam point, respectively. The global damage index will take the following form:

$$D = 1 - \frac{\sum_e a^{(e)} : \int_{\ell^{(e)}} B^{(e)} : \hat{\sigma}^{(e)} ds}{\sum_e a^{(e)} : \int_{\ell^{(e)}} B^{(e)} : \hat{\sigma}^{0(e)} ds} \quad (35)$$

In this manner a damage index similar to that mentioned by DiPasquale and Cakmak (1989) was obtained.

4 NUMERICAL EXAMPLES

The simulation of the evolution of the damage process in a reinforced concrete plane frame (figure 5) subjected to dynamic loading has been performed.

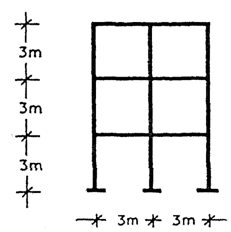


Fig.5 Geometry of the studied frame.

The frame is 9 meters high and 6 meters wide and has three levels. The columns have a 30cm × 30cm cross-section of reinforced concrete with a 4.35% steel ratio. The horizontal beams are 40 cm thick and 30 cm wide with a steel ratio of 5.3%. The structure was discretized in 45 quadratic three-noded beam finite elements having two Gauss points each. Thus, the resulted dynamic model had 87 nodes with three degrees of freedom per node. Each element is one meter long and has the cross-section divided in 20 layers of equal thickness. The 2nd and 19th layer are made of steel and the rest of them of concrete. The steel ratio was controlled by modifing the width of the steel layers. The state of the material is checked at the interface between layers and afterwards interpolated linearly across the layer. This gives 40 check points per cross-section in each Gauss point.

The materials have the following properties: (a) steel - $E = 2.1 \cdot 10^6\,\mathrm{daN/cm^2}$, $\sigma^0 = 4,200\ \mathrm{daN/cm^2}$, $\nu = 0.25$, $\rho = 8\,\mathrm{g/cm^3}$; (b) concrete - $E = 2.0 \cdot 10^5\,\mathrm{daN/cm^2}$, $\sigma^0 = 300\,\mathrm{daN/cm^2}$, $\nu = 0.17$, $\rho = 2.5\,\mathrm{g/cm^3}$.

The equations of motion governing the dynamic behaviour of the structure have been solved using the Newmark step by step algorithm for $\beta = 0.25$ and $\gamma = 0.5$ (Barbat and Canet, 1989). The initial stiffness method was chosen as nonlinear solution scheme due to the negative definition of the tangent stiffness matrix when softening effects occur. The time step used was a thirtieth of the fundamental period of the structure. As the integration of the constitutive law can be done analytically, an explicit formula [equation (20)] was used for the local damage index thus reducing remarkably the solution cost.

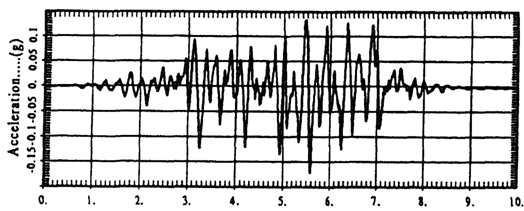


Fig.6 Synthetic accelerogram.

The structure was calculated in two load cases: (a) subjected to a synthetic earthquake accelerogram (figure 6) having a predominant frequency was 4 Hz and a maximum amplitude of 0.175 g and (b) subjected to the same accelerogram with amplitudes multiplied by two. This allows the simulation of the structural behaviour firstly in a less damaged state (figures 8,9,11a) and finally in a generally collapsed state (figures 7,10,11b).

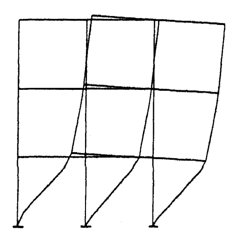


Fig.7 Deformed configuration at collapse.

Figures 9 and 10 show the distribution of the sectional damage as given by formula (34). The damage is located at the joints of the columns with the floors, what is precisely the expected damage localization for this type of structure and load. As the structure is to fail mainly by destruction of the columns at their joint with the base floor, the mentioned diagrams confirm this prognosed behaviour too. Comparing these diagrams with the global damage given in figures 11(a) and 11(b) (the continuous line), it may be seen that it takes values only slightly smaller than the maximum sectional damage at the bases of the columns. This fact ratifies the choice of the global damage index as the ratio between the potential energy which the structure cannot undertake in the damaged state and the potential energy that the structure should undertake if it were undamaged [equation (33)]. The interrupted lines in figure 11 represent the evolution of the floor damages. The first floor damage is practically equal to the global damage of the structure as this floor is the most affected, while the second and third floors follow in decreasing order as the damage reduces with height.

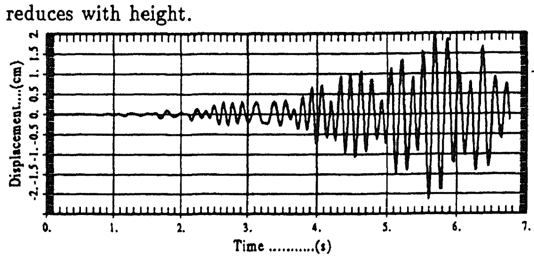


Fig.8 Third floor horizontal displacement evolution.

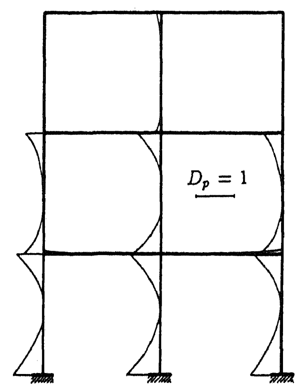


Fig.9 Distribution of sectional damage D_p, case (a).

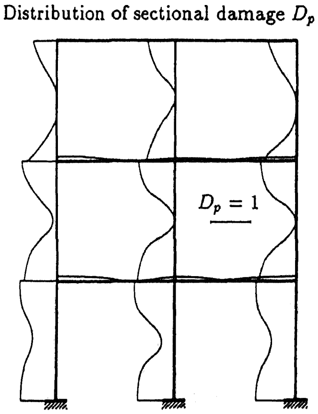
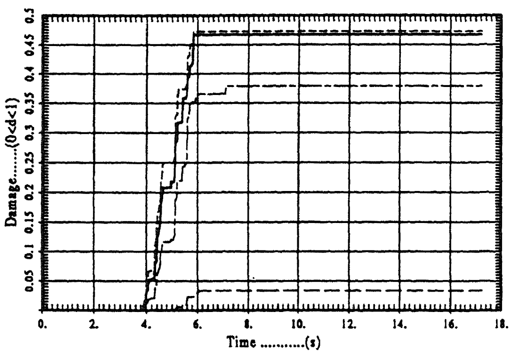


Fig.10 Distribution of sectional damage D_p, case (b).

case (a)



case (b)

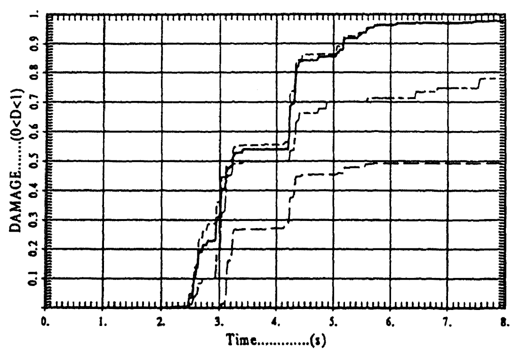


Fig.11 Global and floor damage indices.——— global; -----first floor;----- second floor; — — third floor.

5 CONCLUSIONS

In this paper a damage constitutive model was used, due to its good performances and low solution cost (caused mainly by its property of being analytically integrable), to describe the nonlinear behaviour of reinforced concrete structures under dynamic load. This model was incorporated in a finite element scheme which uses Timoshenko beam elements discretized in layers of concrete and steel in order to approximate the nonlinear behaviour of the beam cross-section. A global damage index was rigorously deduced from the local damage index supplied by the constitutive model. A reinforced concrete building structure subjected to syntetic accelograms was solved and satisfactory results were obtained.

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