*Earthquake Engineering, Tenth World Conference* © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

# A predictive model for hysteretic failure parameters

William C. Stone & Andrew W. Taylor
*National Institute of Standards and Technology, Gaithersburg, Maryland, Md., USA*

ABSTRACT: The ability to analytically predict the inelastic dynamic behavior of reinforced concrete bridge structures during earthquakes is predicated on the availability of an appropriate hysteretic failure model. To date, several such models have been implemented in research computer codes with little or no information available concerning the selection of appropriate coefficients and parameters for the solution of a specific type of reinforced concrete structure. A digital database of load-displacement histories was established for circular, spirally reinforced bridge column tests. A system identification analysis was performed on each specimen in which the error in cyclic absorbed energy was minimized by optimal selection of parameters in a tri-linear hysteretic failure model. Three parameters, comprising a stiffness degrading coefficient ($\alpha$), a strength degrading coefficient ($\beta$), and a pinching coefficient ($\gamma$), were employed in this study. Subsequent multivariable least squares regression analyses were conducted to correlate the variation of the model parameters with the specimens' physical properties. Regression equations were developed for the parameters alpha, beta, and gamma and explanations are offered as to the correlation of the parameters to material and geometric properties of the columns.

## HYSTERETIC FAILURE MODELS

Several computer programs presently exist including SAKE (Otani, 1974), SARCF-II (Rodriguez-Gomez, Chung, and Meyer, 1990), and IDARC (Park, Reinhorn, and Kunnath, 1987), among others, which have, to varying degrees, achieved some measure of success in analytically predicting the dynamic response of reinforced concrete structures, not just to their elastic limits, but to complete failure. All are "research codes" and to date are limited to the analysis of two dimensional structural systems. At the heart of these programs are hysteretic rules to idealize the inelastic behavior of reinforced concrete under dynamic or quasi-static loading. Such models attempt to capture overall behavior of the structural element based on load-displacement (or moment-curvature) predictive algorithms which are calibrated to experimental data. The output of these programs lends itself to the computation of absorbed hysteretic energy and displacement ductility, both of which have been shown to be correlated to a quantifiable measure of the state of damage sustained by the structure following a seismic event (Park, Ang, and Wen, 1984). In the present study a multi-linear hysteretic model, known as a "Three Parameter Model," was employed. This same algorithm is presently used in the program IDARC (Park, Reinhorn, and Kunnath, 1987).

It is common practice to describe the envelope curve of the force-deformation relation of reinforced concrete components by a multi-linear function with three turning points, e.g., cracking, yield, and ultimate strength. The tri-linear approximation to the envelope curve is known as the "skeleton" curve. Procedures have been developed to extract this curve from an equivalent monotonic load-displacement envelope curve that has been fitted to experimental data for columns tested under reversed cyclic loading (Park, Reinhorn, and Kunnath, 1987). A variety of hysteretic properties can be obtained through the combination of the tri-linear skeleton curve and three characteristic parameters, $\alpha$, $\beta$ and $\gamma$. Complete definitions of these variables may be found in Stone and Taylor (1991). Briefly summarizing, the parameter $\alpha$ controls the degradation (softening) in the unloading stiffness that is generally observed in reinforced concrete members as they degrade under reversed cyclic loading. The parameter $\beta$ determines the rate of strength deterioration, and is commonly a function of cummulative absorbed energy and maximum displacement. Pinching behavior is controlled by the parameter $\gamma$. The introduction of such a pinching parameter leads to a reduction in the cyclic energy absorbed by the structural element.

These three parameters, together with the skeleton curve information described above, comprise the variables required for implemention of the hysteretic rule described in Park, Reinhorn, and Kunnath (1987). El-Borgi, White, and Gergely (1991) warn that the proper implementation of this type of model requires calibration of the variables to a specific type of structural element if reliable results are to be obtained. It was for this reason that the initial project scope was restricted to circular, spirally reinforced, single bridge columns. Our first task was thus to assemble a suitable digital database of tests of spirally reinforced, circular concrete bridge columns subjected to reverse cyclic loading combined with axial load. The process of determining the relationship between the hysteretic parameters $\alpha$, $\beta$, and $\gamma$ and the column geometry and materials is known as system identification.

## DIGITAL TEST RECORDS

An extensive literature search identified several sources of bridge column test data. Digitized records were available from Stone and Cheok (1989) and Cheok and Stone (1990); additional records, generally in the form of X-Y plots, were obtained from Ang, Priestley, and Park (1981); Mander (1984); Munro, Park, and Priestley (1976); Priestley and Park (1984); Zahn (1986); Lim, McLean, and Henley (1990); McLean and Lim (1990); Petrovski and Ristic (1984); Wong, Paulay, and Priestley (1990); Ang, Priestley, and Paulay (1985,1989); Davey (1975); Ng, Priestley, and Park (1978); Kenchiku Kenkyu Siryo (No. 2 1975; No. 3 1978); and Watson (1989). All of the latter data was digitized from large scale precision photo enlargments of the analog records.

## SYSTEM IDENTIFICATION

The term "System Identification" is generally associated with the experimental acquisition of such dynamic characteristics of a structure as its mode shapes, frequencies, and damping coefficients. In this paper we use the term to refer to the determination of the three parameters $\alpha$, $\beta$, and $\gamma$, which best characterize the hysteretic behavior of a given test specimen. The system identification procedure used in this study consists of a three dimensional trial and error search with bounded limits on $\alpha$, $\beta$, and $\gamma$ subject to the constraint of minimizing the cummulative error between predicted and experimentally observed differential energy absorption for each point in the experimental database. In equation form, we seek to minimize the function F, where F is defined as follows:

$$F = \sqrt{\frac{\sum_{i=1}^{n} 0.5(\delta_{i+1} - \delta_{i-1})(P_{exp}(i) - P_{theo}(i))^2}{n}}$$

where:

$\delta_{i+1}$ = displacement of next point in load-displacement record

$\delta_{i-1}$ = displacement of previous point in load-displacement record

$P_{exp}(i)$ = experimentally observed load at current point

$P_{theo}(i)$ = predicted load at current displacement, calculated using the multi-linear hysteretic rule and the currently selected values of $\alpha$, $\beta$, and $\gamma$

n = number of data points in the experimental record

Following the initial identification of the parameters $\alpha$, $\beta$, and $\gamma$, the predicted load-displacement record was superimposed graphically, in a different color, on top of the experimental record. Using a high performance graphics workstation it was then possible to interactively adjust any of the three model parameters by means of a dial-box while viewing the results in real time on the graphics screen. This visual-feedback approach proved surprisingly effective in arriving at a "best fit". As a final check, the absorbed energy was calculated for the experimental and predicted data for each cycle and compared in a histogram plot. When the error between the theoretical and experimental sums of the cyclic absorbed energies was reduced to a level of a few percent, the final values of $\alpha$, $\beta$, and $\gamma$ were recorded. The above tasks were carried out using NIDENT 3.0, the NIST graphics-based system identification package, running on a Silicon Graphics 4D-420 workstation.

Examples of the fit between the analytical model and experimental data are shown in Figures 1-2. Several details of these figures bear discussion. The first, as evidenced, for example, in Figure 1, is that as the column begins to fail, there is a tendency for the predicted loads to overshoot the experimentally observed load at the maximum observed displacements for each cycle. This is an expected consequence of using a tri-linear skeleton curve where the post-yielding stiffness for such a model is defined to be positive for reasons of numeric stability. Second, as evidenced in the associated cyclic absorbed energy histograms (see Figures 1-2), there is a tendency for the predicted absorbed energy to overestimate the experimental data for low values of lateral displacement and to underestimate the experimental data for high displacement ductility. Despite these shortcomings, the Three Parameter model is able to generally predict both overall absorbed cyclic energy and displacement ductility to within a few percent error, which was considered acceptable for the puposes of dynamic behaviour studies involving reinforced concrete.

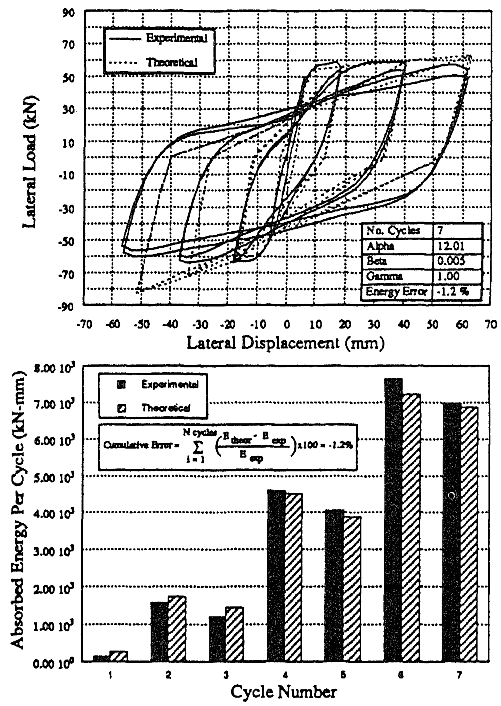


Figure 1. Experimental and analytical results for NIST model specimen N1.

## MULTI-VARIABLE REGRESSION ANALYSIS

Given best-fit values of the parameters $\alpha$, $\beta$, and $\gamma$ for many different column tests, the next objective is to ascertain if there is any correlation between these parameters and the physical properties and dimensions of the test specimens which were modelled. For this objective a stepwise linear regression analysis was conducted. A total of 38 digital test records were used in the present analysis. The regression analyses were conducted using the commercial software package SAS/STAT (SAS/STAT, 1987). $R^2$ (square of the regression correlation coefficient) values of 80%, 80%, and 92% were obtained for the estimates of the parameters $\alpha$, $\beta$, and $\gamma$, respectively. Higher correlations could be obtained, but at the expense of a significant number of additional terms. The regression equations associated with the three parameters are given by:

$$\alpha = 43.73 + \frac{28.86}{\rho_a}(1 - 0.46R_a) - \frac{1.5}{\sigma_o}\left(1 - \frac{8.81}{\sqrt{f'_c}}\right) + 19.12\sigma_o - 17.82R_y - \frac{24.8}{R_y} - 24.04R_s - \frac{1.316}{R_s} + 42.26R_D + 376.3\frac{Q}{P}$$

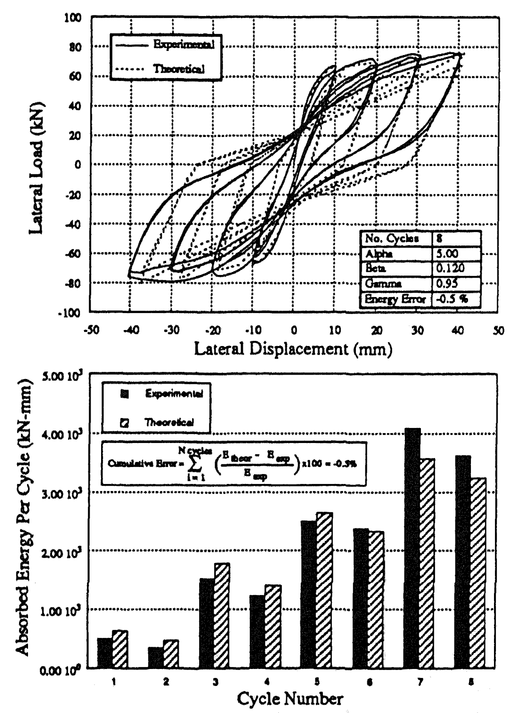


Figure 2. Experimental and analytical results for Specimen 3 of Ng.

$$\beta = -0.789 + \frac{1.52}{\rho_a}\left(1 - 0.2\rho_s - 0.263R_a\right) + \frac{1.64}{\sqrt{f'_c}}\left[\frac{4.308}{R_L} + R_L\left(1 + \frac{f'_c}{50\rho_s}\right) - \frac{348}{f_{ys}} + \frac{\rho_s f'_c}{80}\right] + 0.583R_D - 0.71R_L(0.6)^{\rho_s}$$

$$\gamma = 5.523 - 2.462R_a - 35.01R_f + 20.88R_aR_f - \frac{0.0767}{R_aR_f} + 0.106R_\rho$$

where

$f'_c$ = Concrete cylinder strength, MPa
$f_{ya}$ = Longitudinal reinforcement yield stress, MPa
$f_{ys}$ = Spiral reinforcement yield stress, MPa
$L$ = Length of pier, from base to first point of contraflexure, cm
$D$ = Overall diameter of pier, cm
$D_c$ = Diameter of spiral core (out to out), cm
$A_g$ = Gross cross section area, $cm^2$
$A_c$ = Spiral core cross section area, $cm^2$
$d_s$ = Diameter of spiral bar, cm
$a_s$ = Cross sectional area of spiral bar, $cm^2$
$S$ = Spacing (pitch) of spiral layers, cm
$P_e$ = Axial load on pier, kN
$\rho_a$ = Axial reinforcement content, %
$\rho_s$ = Spiral reinforcement content (volumetric), %

$$\sigma_o = \frac{10P_e}{f'_cA_g} \text{ (kN,MPa and cm}^2\text{)} \qquad P = \frac{2a_sf_{ys}D_c}{S}$$

$$Q = 4\rho_a\sqrt{f'_c}\sqrt{\sigma_o - 0.1} \quad [Q = 0 \text{ if } \sigma_o < 0.1]$$

$$R_s = \frac{d_s}{S} \qquad R_y = \frac{f_{ys}}{f_{ya}} \qquad R_a = \frac{A_g}{A_c}$$

$$R_D = \frac{S}{D} \quad R_L = \frac{L}{D} \quad R_f = \frac{f'_c}{f_{ya}} \quad R_\rho = \frac{\rho_s}{\rho_a}$$

The above equations were subsequently used to derive calculated estimates for each hysteretic model parameter given the geometry and material properties for each test specimen. Scatter plots for each parameter (values determined by system identification on the x-axis; values calculated from the regression equations on the y-axis) are shown in Figures 3-5. A perfect model would be a 45-degree straight line intersecting the origin with all data points lying on

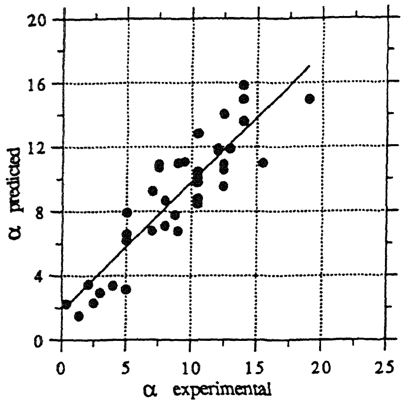

Figure 3. Experimental and predicted values of α

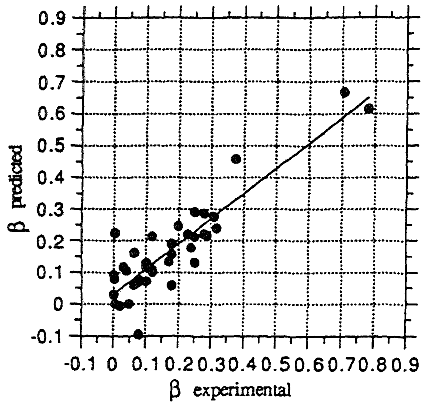

Figure 4. Experimental and Predicted values of β

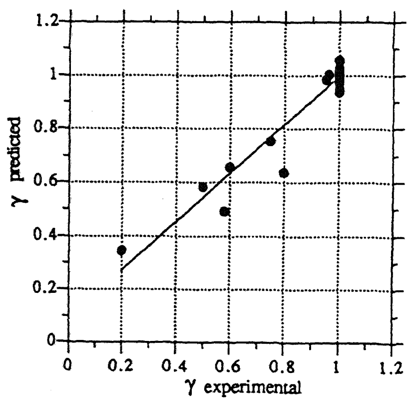

Figure 5. Experimental and predicted values of γ

the line. In actuality there is moderate, but not unreasonable, scatter and the regression lines relating the actual and predicted parameters generally do not have zero intercepts. In the case of the parameter $\beta$ it should be noted that negative predicted values are not valid (and a zero should be substituted). Likewise, predicted $\gamma$ values greater than 1.0 are not valid (and 1.0 should be substituted). We anticipate that the expansion of the column test database will extend the range of applicability of the regression equations. However, as the limits of scatter are likely controlled by the variability of response that is typical of reinforced concrete, it seems unlikely that the quality of correlation between the parameters derived via system identification and those calculated using closed form equations will improve and that the scatter cloud will simply become more dense. For the purposes of conservative inelastic dynamic based design it should be possible to shift $\alpha$ and $\gamma$ downward and $\beta$ upward by some multiple of the standard deviation, much as strength adjustment factors are incorporated in traditional design codes to account for workmanship and material variability.

## DISCUSSION

Stiffness degradation can be shown to be inversely proportional to $\alpha$, with lower values of $\alpha$ leading to more rapid degradation at higher displacements. Such deterioration in stiffness, should, according to the predictive equation for $\alpha$ be retarded by core confinement ratios ($R_a$) close to 1; higher axial stress; use of confining (spiral) steel with a yield stress between 1 and 1.5 times greater than the yield stress for the axial reinforcement; confining steel bar diameters between 0.2 and 0.3 times the spiral pitch; and larger spiral pitch subject to the above constraint on bar diameter. The combination of the latter two variables suggests that the use of larger spiral bar diameters, relative to pitch, will lead to improved energy absorption performance. The latter term, Q/P is a ratio of the shear strength contributions from the concrete and confining steel, respectively. The terms which most strongly affect the correlation are $\sigma_0$, $R_y$, and $R_a$.

Strength deterioration is directly proportional to the parameter $\beta$. Material and geometric characteristics of the column which serve to reduce this parameter are therefore desirable. From the equation for $\beta$ it can be seen that decreasing axial reinforcement; increasing confining reinforcement; and providing a confined core area as close as possible to the gross cross sectional area meet this criteria. Strength deterioration is also strongly affected by concrete strength . Although further study is necessary, it appears that there is a pronounced trend for decreasing values of $\beta$ for increasing values of f'c, until reaching a level of approximately 42 MPa, after which $\beta$ rapidly increases for increasing values of f'c. Strength deterioration may also be reduced by closer spiral spacing; higher spiral reinforcement ratios; and by using higher aspect (L/D) ratios.

The third parameter, $\gamma$, influences the pinching behavior in the hysteresis loops. A smaller value of $\gamma$ leads to a more pronounced pinch and, therefore, less energy absorption during any given cycle. Desirable performance dictates the maintainence of "fat" hysteresis loops. The equation for $\gamma$ indicates that ratios of gross column to confined core area close to 1 are desirable; high values of $R_f$ (concrete strength/axial steel yield stress), and high confining reinforcement ratios (relative to axial reinforcement ratio) also serve to reduce pinching. It is noteworthy that the equation for $\gamma$ contains the fewest independent variables and also has the highest correlation value (0.92).

From the above brief summaries several useful steps appear likely to enhance energy absorbtion characteristics of bridge columns:

a) Increase confining reinforcement ratios relative to axial reinforcement ratios.
b) Use lower yield stress steel for axial reinforcement, relative to spiral reinforcement.
c) Use larger diameter spiral bars relative to bar pitch; maintain the ratio of spiral bar diameter to pitch between 0.2 and 0.3.
d) Use higher strength concretes (up to a maximum of about 42 MPa).
e) There appears to be some benefit in designing for higher axial stresses.
f) Maintain as much of the gross section of the column as possible within the confined core.

## CONCLUSIONS

Within the bounds of the available test data for circular, spirally reinforced concrete bridge columns subjected to cyclic lateral loading, it was found that the Three Parameter model was able to generally predict the observed experimental load-deflection histories, and to produce an estimate of the cummulative absorbed cyclic energy within an error bound of several percent. Preliminary regression equations were developed for the parameters $\alpha$, $\beta$, and $\gamma$ and explanations were offered for the relationship between these parameters and the material and geometric properties of the columns. The availability of closed form equations for these failure model parameters permits an *a priori* inelastic dynamic solution for a large class of bridge columns which employ circular cross-sections and spiral confining steel.

**References:**


Ang Beng Ghee, Priestley, M. J. N. and Park, R., Ductility of Reinforced Concrete Bridge Piers Under Seismic Loading, Report 81-3, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, February 1981.

Ang Beng Ghee, Priestley, M. J. N., and Paulay, T., "Seismic Shear Strength of Circular Bridge Piers," Report 85-5, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, July 1985.

Ang Beng Ghee, Priestley, M. J. N., and Paulay, T., "Seismic Shear Strength of Circular Reinforced Concrete Columns," ACI Structural Journal, January-February 1989, pp 45-59.

Cheok, G.S., and Stone, W.C. (1990), "Behavior of 1/6-Scale Model Bridge Columns Subjected to Inelastic Cyclic Loading," ACI Structural Journal, Nov.-Dec. 1990, Vol. 87, No. 6, American Concrete Institute, Detroit, MI., pp 630-638.

Davey, B.E., "Reinforced Concrete Bridge Piers Under Seismic Loading," Master of Engineering Report, Civil Engineering Department, University of Canterbury, Christchurch, New Zealand, February 1975.

El-Borgi, S., White, R.N., and Gergely, P. (1991), "Analytical Modeling of Pinching and Degrading Hysteretic Systems and System Identification of Parameters: Progress Report to NIST," Department of Structural Engineering, School of Civil Engineering, Cornell University, Ithaca, NY 14853, January 1991, NIST Contract #50SBNB1C6543.

Kenchiku Kenkyu Siryo, "Aseismic Analysis of Building Structural Members: A List of Experimental Results on Deformation Ability of Reinforced Concrete Columns under Large Deflection (No.2), March 1975, Building Research Institute, Ministry of Construction, Japan.

Kenchiku Kenkyu Siryo No.21, "Aseismic Analysis of Building Structural Members: A List of Experimental Results on Deformation Ability of Reinforced Concrete Columns under Large Deflection (No.3), Feb. 1978, Building Research Institute, Ministry of Construction, Japan.

Lim, Kuang Y., McLean, David I., and Henley, Edward H., "Moment-Reducing Hinge Details for the Bases of Bridge Columns," Transportation Research Record, 1275 (no further details for reference).

McLean, David I., and Lim, Kuang Y., "Moment-Reducing Hinge Details for the Bases of Bridge Columns," Report No. WA-RD 220.1, October 1990, Washington State Department of Transportation, Planning, Research and Public Transportation Division.

Mander, J.B. (1984), "Seismic Design of Bridge Piers," Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, Research Report # 84-2, February.

Munro, I. R. M., Park, R., and Priestley, M. J. N. (1976), Seismic Behaviour of Reinforced Concrete Bridge Piers, Report 76-9, Department of Civil Engineering, University of Canterbury, Christchurch New Zealand, February 1976.

Ng Kit Heng, Priesley, M. J. N., and Park, R., Seismic Behaviour of Circular Reinforced Concrete Bridge Piers, Report 78-14, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealnd, February 1978.

Otani, S., "SAKE, a Computer Program for Inelastic Response of R/C Frames Subject to Earthquakes," Civil Engineering Studies, SRS No. 413, University of Illinois, Urbana, November 1974.

Park, Y.J., Ang, A. H-S., and Wen, Y.K. (1984), "Seismic Damage Analysis and Damage-Limiting Design of R.C. Buildings," Civil Engineering Studies, SRS No. 516, University of Illinois, Urbana, October 1984.

Park, Y.J., Reinhorn, A.M., and Kunnath, S.K. (1987), "IDARC: Inelastic Damage Analysis of Reinforced Concrete Frame - Shearwall Structures, (1987) " Department of Civil Engineering, State University of New York at Buffalo, Buffalo, NY 14260, Technical Report NCEER-87-0008, July 20, 1987.

Petrovski, J., and Ristic, D. (1984), "Reversed Cyclic Loading Test of Bridge Column Models," Institute of Earthquake Engineering and Engineering Seismology, University "Kiril and Metodij", Skopje, Yugoslavia. Report # IZIIS 84-164, September.

Priestley, M. J. N. and Park, R. (1984), Strength and Ductility of Bridge Substructures, Report 84-20, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, December 1984.

Rodriguez-Gomez,S., Chung, Y.S., and Meyer, C., "SARCF-II User's Guide: Seismic Analysis of Reinforced Concrete Frames," Technical Report NCEER-90-0027, National Center for Earthquake Engineering Research, State University of New York at Buffalo, September, 1990.

SAS/STAT Guide for Personal Computers, Version 6 Ed., SAS Institute, Inc., Cary, NC, 1987, pp. 773-876.

Stone, W.C., and Cheok, G.C. (1989), "Inelastic Behavior of Full-Scale Bridge Columns Subjected to Cyclic Loading," NIST Building Science Series 166, National Institute of Standards and Technology, Gaithersburg, MD, January 1989.

Stone, W.C., and Taylor, A.W., "System Identification of Spirally Reinforced Circular Bridge Columns Subjected to Cyclic Lateral Loading," Wind and Seismic Effects; Proceedings of the 23rd Joint Meeting, NIST SP 820, National Institute of Standards and Technology, September 1991.

Watson, Soesianawati, "Design of Reinforced Concrete Frames of Limited Ductility," Report 89-4, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, January 1989.

Wong, Yuk Lung, Paulay, T., and Priestley, M. J. N. (1990), "Squat Circular Bridge Piers Under Multi-Directional Seismic Attack," Report 90-4, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, October 1990.

Zahn, F. A., Park, R. and Priestley, M. J. N. (1986), Design of Reinforced Concrete Bridge Columns for Strength and Ductility, Report 86-7, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, March 1986.