Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

A constitutive model for Champlain Sea clay under cyclic load

X.Y.Wu, K.T.Law & A.P.S.Selvadurai
Department of Civil Engineering, Carleton University, Ottawa, Ont., Canada

ABSTRACT: A pseudostatic constitutive model for Champlain Sea clay under cyclic load is developed based on cyclic triaxial tests using incremental plasticity theory. The model is implemented into a finite element code. The results of cyclic triaxial tests show that the void ratio before cyclic loading has a considerable influence on the development of plastic strains and this influence could be as significant as that of initial stress states. It is found that the residual pore water pressure and plastic deviatoric strain during the cyclic loading can be described by a hyperbolic curve. The constitutive model is able to account for the effect of the void ratio as well as the influence of the residual pore water pressure under the undrained condition. In order to simulate the plastic shear deformation occurring within the yield surface, multiple yield surfaces are suggested. The performance of the model is evaluated by cyclic simple shear tests.

1. INTRODUCTION

The analysis of earthquake-induced permanent deformation is of importance to the safe and economic design of earth structures. Much research has been conducted on this topic, but few are using plasticity theory. The research presented in this paper is aimed at establishing a pseudostatic elastoplastic constitutive model for the analysis of earthquake-induced permanent deformation of earth embankments.

A concept of multi-yield-surface model is derived from the observation of the behaviour of a soft sensitive Champlain Sea clay under cyclic triaxial tests. The model is formulated based on the theory of elastoplasticity and is implemented into a finite element code.

The validity of the model is evaluated by comparing the predicted and measured soil behaviour under a cyclic horizontal shear condition.

2. EXPERIMENTAL OBSERVATION

Saturated samples of undisturbed Champlain Sea clay obtained from beneath an existing earth dyke have been tested in a cyclic triaxial machine. The equipment used was described by Law (1985). The samples were consolidated either isotropically or anisotropically before being subjected to vertical cyclic loads with constant amplitude under undrained condition. Hence, in this testing procedure, the effects of static stress-history (over - consolidation ratio) and anisotropic stress state as in existed *in situ* under the earth dyke can be studied. During the tests, the applied vertical cyclic load, and the cyclic response of both excess pore water pressure and axial strain were recorded. The major observation from the tests is as follows:

Effect of Over Consolidation Ratio

The results show that normally consolidated (N.C.) clay samples and over consolidated (O.C.) clay samples fail in different modes. The N.C. clay undergoes significan plastic deformation prior failure, whereas the O.C clay fails abruptly in a characteristic "brittle" manner without the development of appreciable plastic deformation, as shown in Fig.1. This implies that the permanent deformation is of importance in evaluating the seismic behaviour of the N.C. clay and the following discussion will be concentrated on the behaviour of N.C.clay.

Permanent Deviatoric Strain

Fig.2 shows a typical strain response pattern of anisotropically consolidated samples under cyclic triaxial condition. One can see that the permanent deviatoric strain takes place at the very beginning of the loading. Since total volumetric strain is zero under the undrained condition, the deviatoric strain is the only total strain component. Therefore, the deviatoric plastic strain is very significant comparing with plastic volumetric strain.

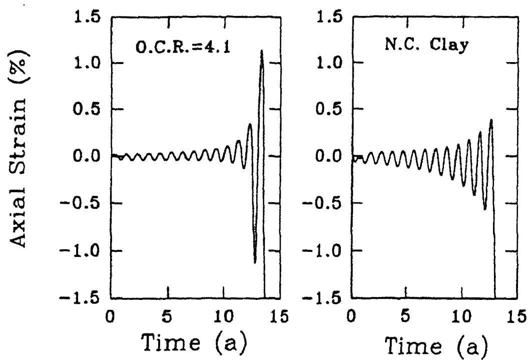

Fig.1 Different axial strain response patterns of O.C. and N.C. clays

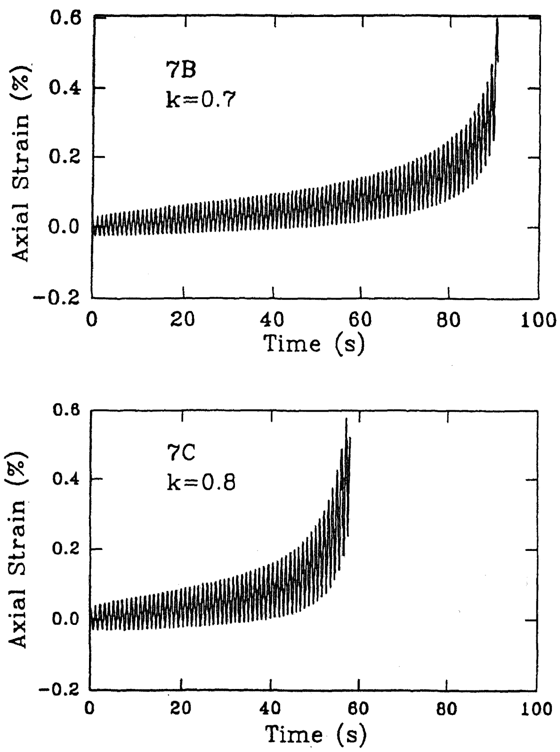

Fig.2 Cyclic Axial Strain Response

Effect of Void Ratio

An important phenomenon observed repeatedly in this series of experiments relates to the influence of the void ratio after consolidation (e) on the development of plastic strain. It is found that in order to describe the permanent deformation, e is a parameter as significant as the initial stress state and the over-consolidation ratio.

Two specimens shown in Fig.2 were prepared from

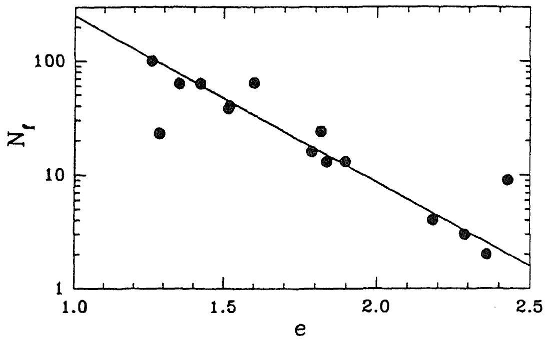

Fig.3: Relation of N_f with e

the same sample block. Before subjected to the same cyclic loading, they were anisotropically consolidated with different consolidation stress ratios ($K_0 = \sigma'_h/\sigma'_v$ = 0.7 and 0.8, respectively) but with practically the same effective mean normal stress p', where $p' = (\sigma'_v + 2\sigma'_h)/3$ Considering that the stress state with lower K_0-value is closer to the strength envelope, one may expect that the specimen consolidated with lower K_0-value should have developed larger deformation and have failed with less number of cycles. Fig.2, however, shows the opposite: the specimen with $K_0 = 0.7$ not only has smaller deformation response but also requires a larger number of cycles to failure (N_f). The reason for this is due to the change in void ratio. The lower K_0-value induced a higher shear stress and caused more volume change which moved the state of the specimen, in terms of void ratio, to a safer one, ie., the soil was more dense. Apparently, the effect of densification in this case is more significant than that of being closer to the strength envelope.

Further study on the effect of e on N_f is shown in Fig.3. This figure shows an approximate linear relation exists between log N_f and e. The specimens presented in this figure were subjected to a similar vertical cyclic stress ($q_d \approx 116kPa$) but had been consolidated under different K_0-conditions before the cycling.

Residual Pore Water Pressure

As a general case, the dynamic response of pore water pressure in a specimen under cyclic loading is consisted of a recoverable part and a residual part. The residual part is the excess pore water pressure cumulated during cycling, which persists at the end of the cyclic loading.

Although the pore pressure caused by the cyclic loading is not high enough to lead to zero confining pressure, it does have significant bearing on the deformation character of the clay soil. It was found that the residual pore water pressure (u^r) is related to the residual axial strain (ϵ_a^p) and their relation can be simulated by a hyperbolic curve

$$u^r = \frac{\epsilon_a^p}{a + b\epsilon_a^p} \quad (1)$$

where both a and b are constants determined from the experimental results. Fig.4 presents a group of six tests, each set of testing data being fitted by a hyperbolic curve. The values of a and b for each curve are presented in Table 1. It can be seen from the Table that the values of a are very close to each other except that of specimen $7D$. The values of b are well related to e. As shown in Fig.5, this relation can be described by:

$$b = -0.035 + 0.037e \quad (2)$$

By taking the average of a-values (ignoring the bad point), Eq. 1 becomes

$$u^r = \frac{\epsilon_a^p}{0.0014 + (-0.035 + 0.037e)\epsilon_a^p} \quad (3)$$

Fig.6 illustrates the reasonable agreement between the predicted curve based on Eq. 1 and those measured from two typical tests.

3. MULTI-YIELD-SURFACE MODEL

3.1 MULTI-YIELD-SURFACE CONCEPT

The experimental observation suggests that permanent deformation is an essential aspect in evaluating seismic behaviour of clay embankments and that void ratio (e) is one of the key parameter affecting deformation character. Therefore, the modified Cam-clay model (Roscoe and Burland 1968) is proposed as a major part of the new model in this study, because it has the advantage to account for the effect of e.

The fact that permanent strain occurs in a normally consolidated soil at the very beginning of cyclic loading (Fig.2) implies that the permanent strain may occur without the company of plastic volumetric strain v^p. Under undrained condition, the build-up of pore water pressure during cyclic loading leads to the decrease of effective mean pressure (p'). When initial stress state is near the p'-axis, any decrease of p' will lead to a stress path away from the current Cam-clay cap yielding surface into the area within the surface; thus, the deviatoric strain occurs without the expansion of the cap surface, ie., it occurs without the increase in v^p. When the initial stress state is not near the p'-axis, an incremental stress vector may cause the expansion of the cap yield surface even though the pore water pressure will decrease the value of p'. For

Table 1: Values of Parameter a and b

Sample	a	b	e
5C	0.001585	0.01853	1.5154
7B	0.001466	0.01993	1.3514
7A	0.001326	0.01079	1.2615
7C	0.001439	0.01679	1.4218
7D	0.0006029	0.02500	1.5748
5D	0.001197	0.01139	1.2860

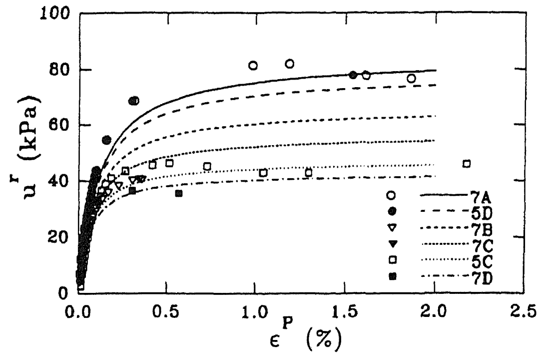

Fig.4: Hyperbolic Relations

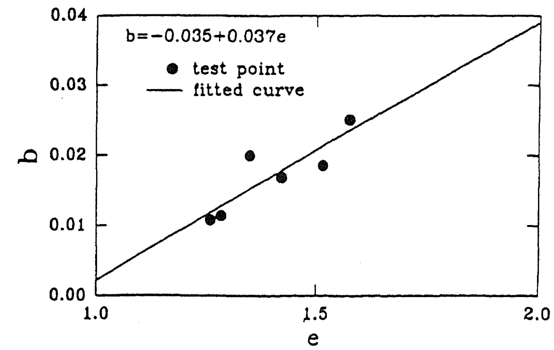

Fig.5: Relation of Paramerter b with e

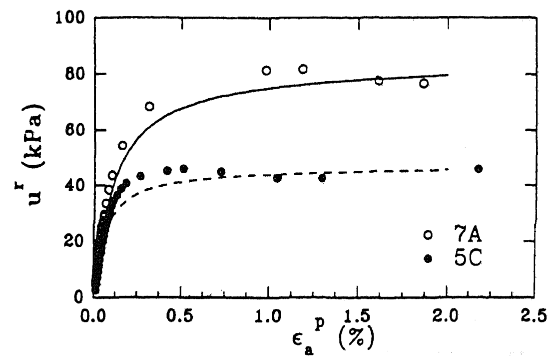

Fig.6:Comparison of Fitted Curve with Test Points

this case, the corresponding plastic deviatoric strain increment ($\Delta\epsilon^p$) will generally contain two components: one occurring within the cap surface and the other as a result of the expansion of the cap hardening surface. Therefore, a deviatoric plastic strain can be resolved into two components: one occurring with the company of v^p and one without v^p.

Based on the above analysis, a new yield condition is required to predict the plastic deviatoric strain which cannot be predicted by the cap yielding condition. Since there is no effect of hydrostatic pressure on this plastic strain, it is appropriate to use Tresca or von Mises-type yield model for the purpose of the prediction. Since the results from both models are similar, the von Mises-type yield surface is used because of its simplicity. The von Mises surface is a cylindrical surface in the stress space, and will be introduced in the study to evaluate the plastic strain caused by pure cyclic distortion effect.

Likewise, the critical state line in $p' - q$ space can be generalized into the 3-d stress space either by the Coulomb's failure criterion, or by the extended Tresca or extended von Mises perfectly plastic criterion. Again, because of its simplicity, the extended von Mises-type yield surface is selected for describing the critical state condition, which is a cylindriconical surface in the 3-d stress space.

The new model, therefore, is a multi-yield-surface model, which is composed of a fixed cylindriconical yield surface, an elliptic cap strain-hardening surface, and a cylindrical work-hardening surface. The multi-yield-surface in $p' - q$ stress space is schematically illustrated by Fig.7.

The problem of singularity will be caused by the undetermined direction of the plastic strain increment due to the undetermined normal of plastic potential function at the corners where one yield surface intersects with the other. This requires a special treatment in formulating the model (Koiter 1953, Chen and Mizuno 1990). The details for the treatment are described by Wu (1992).

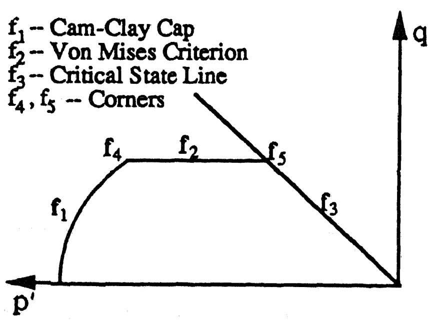

Fig.7 Multi-yield-surface in p'-q plane

3.2 GENERAL FORMULATION

For isotropic materials, the yielding conditions of the new model can be generally expressed as a function of stress invariants and internal variable:

$$f(I_1, J_2, \kappa) = 0 \tag{4}$$

in which

$$I_1 = \sigma'_{ii} \tag{5}$$

$$J_2 = \frac{1}{2}S_{ij}S_{ij}, \tag{6}$$

and κ can be plastic work, plastic volumetric strain, or an equivalent plastic strain.

In two dimensional finite element formulation, the consistency condition can be expressed from Eq. 4 as:

$$df = (\frac{\partial f}{\partial \sigma'})^t d\sigma' + \frac{\partial f}{\partial \kappa}d\kappa \tag{7}$$

in which the superscript t indicates the transposition of matrix and

$$\mathbf{d}\sigma' = \{d\sigma'_x, d\sigma'_y, d\tau'_{xy}, d\sigma'_z\}^t. \tag{8}$$

Eq. 7 assures that the current stress state remains on the current yield surface. Assume:

$$d\kappa = h d\lambda, \tag{9}$$

in which $d\lambda$ is an undetermined proportionality factor in the normality condition. The expression for h can be derived from the normality rule and it has different formation for different internal variables. The stress increment vector, therefore, is:

$$d\sigma' = \mathbf{D}d\varepsilon - \mathbf{D}\frac{\partial Q}{\partial \sigma'}d\lambda, \tag{10}$$

in which $\mathbf{D}$ is the plastic matrix. $d\lambda$ is determined by substituting Eqs 10 and 9 into 7:

$$d\lambda = \frac{(\frac{\partial f}{\partial \sigma'})^t \mathbf{D} d\varepsilon}{(\frac{\partial f}{\partial \sigma'})^t \mathbf{D}\frac{\partial Q}{\partial \sigma'} - \frac{\partial f}{\partial \kappa}h}. \tag{11}$$

Let

$$d\lambda = \frac{l}{A} \tag{12}$$

where A is called the elastoplastic factor:

$$A = (\frac{\partial f}{\partial \sigma'})^t \mathbf{D}\frac{\partial Q}{\partial \sigma'} - \frac{\partial f}{\partial \kappa}h > 0, \tag{13}$$

and l is the function of loading-unloading criterion:

$$l = (\frac{\partial f}{\partial \sigma'})^t \mathbf{D} d\varepsilon = \begin{cases} > 0, & \text{loading,} \\ = 0, & \text{neutral loading,} \\ < 0, & \text{unloading.} \end{cases} \tag{14}$$

Substitution of Eq. 11 into 10 leads to:

$$d\sigma' = \left(\mathbf{D} - \frac{\mathbf{D}\frac{\partial Q}{\partial \sigma'}(\frac{\partial f}{\partial \sigma'})^t \mathbf{D}}{\left(\frac{\partial f}{\partial \sigma'}\right)^t \mathbf{D}\frac{\partial Q}{\partial \sigma'} - \frac{\partial f}{\partial \kappa} h} \right) d\varepsilon \quad (15)$$

$$= \left(\mathbf{D} - \frac{1}{A}\mathbf{D}\frac{\partial Q}{\partial \sigma'}(\frac{\partial f}{\partial \sigma'})^t \mathbf{D} \right) d\varepsilon. \quad (16)$$

Consequently, the elastoplastic constitutive matrix is obtained as:

$$\mathbf{D_{ep}} = \mathbf{D} - \frac{1}{A}\mathbf{D}\frac{\partial Q}{\partial \sigma'}(\frac{\partial f}{\partial \sigma'})^t \mathbf{D}. \quad (17)$$

Based on the above formulation, specific equations for each yield condition are derived and details are described by Wu (1992).

3.3 Undrained Analysis based on Experimental Relation

As discussed in section 2, the residual pore water pressure u^r can be expressed as a function of plastic deviatoric strain ϵ^p. Since ϵ^p can be generalized as plastic octahedral shear strain:

$$\gamma^p_{oct} = \frac{2}{3}\left[(\varepsilon^p_x - \varepsilon^p_y)^2 + (\varepsilon^p_y - \varepsilon^p_z)^2 + (\varepsilon^p_z - \varepsilon^p_x)^2 + \frac{3}{2}(\gamma^p_{xy})^2\right]^{\frac{1}{2}} \quad (18)$$

the incremental relation between the pore pressure and the strain can be expressed as:

$$\Delta u^r = (u^r(\gamma_{oct^p}))'_{\gamma^p_{oct}} \Delta\gamma^p_{oct} \quad (19)$$

where $(u^r(\gamma_{oct^p}))'_{\gamma^p_{oct}}$ is the derivative of the function of $u^r = u^r(\gamma^p_{oct})$ with respect to γ^p_{oct}. Obviously, Δu^r is still a function of γ^p_{oct} and can be expressed as:

$$\Delta u^r = f(\gamma^p_{oct})\Delta\gamma^p_{oct}. \quad (20)$$

By applying the effective stress law, the constitutive incremental relation in the finite element analysis will be in the form of:

$$\Delta\sigma' = \mathbf{D_{ep}}\Delta\varepsilon - \mathbf{e^t}\Delta u^r. \quad (21)$$

4. APPLICATION

To examine the performance of the proposed model, a group of undrained cyclic horizontal shear tests was conducted using a cyclic simple shear apparatus (Law 1988) modified to allow vertical deformation to occur during the undrained cyclic shear. This shear mode is very different from that of the triaxial and consolidation tests. Table 2 shows two such tests on specimens A and B isotropically consolidated to $p' =$ 150 and $200kPa$, respectively. After consolidation, a cyclic horizontal shear stress of $35kPa$ was applied at the top of the specimen under undrained condition. In the computation, one isoparametric element with eight nodal points was used to simulate the geometry of specimen. The parameters listed in Table 2 for the computation were obtained from the static and cyclic triaxial tests and consolidation tests. Table 2 also lists the results of computed and measured maximum vertical displacements at the middle of the top of the specimen during the cyclic horizontal shear test. The results show that the computed values are in good agreement with the measured values. The good agreement therefore supports the use of the proposed model and the method of obtaining the parameters from conventional tests including static and cyclic triaxial tests and consolidation tests to estimate performance of soil under an entirely different shear mode.

Table 2: Comparison of computed and measured displacements

Properties and results	Specimen A	Specimen B
Young's modulus (MPa)	0.20	0.20
Poisson's ratio	0.40	0.40
Slope of critical state line	1.50	1.50
Slope of swelling line	0.43	0.43
Slope of compression line	1.30	1.30
Void ratio	2.025	1.968
Initial hardening parameter (kPa)	450	600
Computed displacement (mm)	0.078	0.062
Measured displacement (mm)	0.067	0.080

5. CONCLUSIONS

The peudostatic multi-yield-surface model is developed using incremental plasticity theory and the results of cyclic triaxial tests. By generalizing the triaxial stress state into a general stress state, the model is coded into a two-dimensional finite element program for estimating earthquake-induced permanent deformation generated under any stress state and bounary conditions including those similar to and those different from the test conditions under which the parameters were obtained. Preliminary test results of cyclic horizontal shear show the model predicts reasonably well the observed behaviour.

ACKNOWLEDGEMENT

The research described in this paper is financially supported by Ontario Hydro and the authors are indebted to the interest and encouragement of Dr. C.F. Lee of Ontario Hydro. The laboratory study was conducted with the conscientious assistance of Mr. T. Hoogeveen in the Institute for Research in Construction, National Research Council of Canada.

REFERENCES

Chen, W. F., and Mizuno, E. 1990, ≪ Nonlinear analysis in soil mechanics≫, *Elsevies*

Koiter, W. T. 1953, "Stress-strain relations, uniqueness and variational theorems for elastic-plastic materials with singular yield surface", *Q. Appl. Math.*, **11**, pp350-354.

Law, K. T., Cragg, C. B. H., and Lee, C. F. 1985, "Seismic soil behaviour under an earth dyke", *Proc. of the 11th int. conf. on soil mechanics and foundation engineering*, San Francisco

Lwa, K. T. 1988, Dynamic shear behaviour of the northumberland till, Report for Public Works Canada

Roscoe, R.J., and Burland, J.B.1968, " On the generalized stress-strain behaviour of 'wet' clay", *Engineering Plasticity*, Cambridge University Press, pp.553-555.

Wu, X. Y. 1992, Stability and permanent deformations of earth embankments under earthquake conditions, Ph.D Thesis, Carleton University