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Soil liquefaction and failure under earthquake loading

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ABSTRACT: The accurate prediction of the behaviour of soil structures and foundations under dynamic loading relies on the ability to reproduce the constitutive behaviour of soils under such conditions. This paper presents a framework within which it is possible to develop simple constututive models for cohesive anf granular soils together with some applications.

Particular attention is paid to the problem of failure under dynamic conditions and the basic requirements to model it properly.

1. INTRODUCTION

The mechanical behaviour of soils is very much dependent on the interaction between the solid skeleton and the interstitial water and air, and this is particulary important in the case of earthquake loading. The main characteristic of soils under cyclic loading is their tendency to compact or densify. Of course, the response can be extremely complex, and it will depend on factors such as material structure, amplitude of strain, induced anisotropy, amplitude of cyclic stress, initial conditions, etc., but the main keyword to be retained is <u>densification under cyclic generalized shear</u>.

If the load is applied rapidly enough, drainage is not allowed or the permeability is very small, the fluid-skeleton coupling will result in a <u>pore pressure increase</u> with a subsequent reduction of soil stiffness.

In addition to the above facts, the highly non-linear behaviour of the soil skeleton when sheared, should be kept in mind.

All of these phenomena make the full constitutive modelling of soil response a rather difficult task, and justifies the development in the past of simplified models, with a limited range of application.

It has been traditionally assumed that the response of soil under dynamic loading was dependent on the amplitude of cyclic strain induced by it, and, consequently, ad-hoc models for "very. small", "small", "medium" or large strain were developed.

Thise definitions proved useful for engineering purposes but presented some severe limitations.

The cyclic shear stress level provides a more exact description of the possible effects of the earthquake on a soil structure, specially when failure or instability phenomena can be triggered.

The relative mobilized stress ratio $\hat{\eta}$ could be a useful parameter for this purpose. It can be defined as

$$\hat{\eta} = \frac{\sqrt{3J_2'} \ / \ I_1'}{M_g(\theta)} \tag{1}$$

where I_1' and J_2' are the first invariants of the deviatoric and the effective stress tensors respectively.

$$\begin{aligned} J_2' &= \frac{1}{2}\, \mathrm{tr}\, (\underset{\sim}{s}^2) \\ I_1' &= \frac{1}{3}\, \mathrm{tr}\, (\underset{\sim}{\sigma}') \\ \underset{\sim}{\sigma}' &= \underset{\sim}{\sigma} - p_w \underset{\sim}{I} \\ \underset{\sim}{s} &= \mathrm{dev}(\underset{\sim}{\sigma}') = \underset{\sim}{\sigma}' - I_1' \underset{\sim}{I} \end{aligned} \tag{2}$$

and M_g is the slope of a line which separates the regions in which the soil delates or contracts.

In above, θ is the Lode's angle, defined as

$$\theta = \frac{1}{3} \sin^{-1} \left(- \frac{3\sqrt{3}}{2} \ \frac{J_3'}{J_2'^{\,3/2}} \right) \tag{3}$$

The usefulness of the relative mobilized stress ratio $\hat{\eta}$ is that it can define a threshold under which the soil will never fail. In the case of cohesive soils, under undrained loading, the existence of an equilibrium line corresponding to final states for values of $\hat{\eta}$ smaller than a critical value was first shown by Sangrey [1].

In the case of sands, $M_g(\theta)$ defines a surface which reduces to a line on the triaxial plane, known as "characteristic state line" or "phase transformation line" [2,3], while for clays it can be identified with the projection of the critical state line on the space of stress invariants.

Failure of normally consolidated clays under cyclic undrained loading is caused by the accumulation of pore pressure which causes the stress path to bend towards the origin, and takes place at the critical state line for amplitudes of cyclic shear stress larger than a critical value.

In the case of sands, the response is largely dependent on their relative density.

Very loose sands exhibit liquefaction under monotonic loading. This phenomenon is characterized by a peak shear stress followed by a dramatic decrease of strength During the process, both the pore pressure and the relative mobilized stress ratio are continuously increasing. If several cycles of loading are applied, the tendency to densify causes a pore pressure grow-up and a shift of the path towards the origin, until the moment arrives in which liquefaction takes place [4]. Therefore, liquefaction under cyclic loading is the same phenomenon that occurs under monotonic loading of very loose sands.

Cyclic mobility can occur in denser sands provided the stress path approaches the characteristic state line. As it happens, unloading results in an important development of pore pressure and the correspondent decrease of effective stress, up to a point in which the soil is very soft. This increase in pore pressure during unloading is therefore, together with the tendency to densify, responsible for cyclic mobility.

All of the above described phenomena should be accounted for if the response of soil is to be modelled accurately. As inmediate consequences, any sand model based on plasticity should be non-associative, and plastic deformations during unloading should be taken into account.

In any case, irreversible deformations within the yield surface are needed.

2 GENERALIZED PLASTICITY THEORY

This simple framework in which constitutive equations for cyclic loading can be developed was initially suggested by Zienkiewicz and Mroz, [5] and applied to soils under dynamic loading by the authors [6-9].

It is based on the assumption that incremental non-linear elastoplastic relations between stress and strain increments can be written as

$$d\underset{\sim}{\sigma}' = \underset{\sim}{D}^{ep} : d\underset{\sim}{\varepsilon} \tag{4}$$

where $\underset{\sim}{C}$ and $\underset{\sim}{D}$ are fourth - order tensors which depend on the state of stress $\underset{\sim}{\sigma}'$, a set of internal variables $\underset{\sim}{\alpha}$, and the direction of the stress increment $d\underset{\sim}{\sigma}'$.

This last condition is fundamental for cyclic loading, because, if not, any infinitesimal cycle ($+\,d\underset{\sim}{\sigma}$, $-\,d\underset{\sim}{\sigma}$) would result in zero accumulated strain.

The set of internal variables $\underset{\sim}{\alpha}$ takes into account both the actual state and the past history of the material. If the soil response does not depend on it, a series of constant amplitude stresses would produce the same incremental accumulated strain, and in the limit, failure would always happen.

To include the dependency on the direction of the effective stress increment, a direction $\underset{\sim}{n}$ is postulated in the stress space discriminating between loading and unloading:

$$\begin{aligned} d\underset{\sim}{\sigma}' : \underset{\sim}{n} &> 0 \quad \text{loading} \\ d\underset{\sim}{\sigma}' : \underset{\sim}{n} &< 0 \quad \text{unloading} \\ d\underset{\sim}{\sigma}' : \underset{\sim}{n} &= 0 \quad \text{neutral loading} \end{aligned} \tag{5}$$

the response of the material will depend on which of the two tensorial zones corresponds to $d\underset{\sim}{\sigma}'$, and therefore can be considered as incrementally bi-linear [10]

This description can be further elaborated by introducing more than one mechanism of deformation:

$$d\underset{\sim}{\varepsilon} = \sum_{m=1}^{M} \underset{\sim}{C}^{(m)} : d\underset{\sim}{\sigma}' \tag{6}$$

for which M unit vectors $\underset{\sim}{n}^{(m)}$ are consequently defined.

The number of tensorial zones is now 2^M. This approach has proven to be very useful for anisotropy and rotation of principal stress axes effects. [11]

Incremental relations for a single mechanism (m) can be written as

$$d\underset{\sim}{\varepsilon}^{(m)} = \underset{\sim}{C}^{L(m)} : d\underset{\sim}{\sigma} \quad \text{(loading)} \tag{7}$$

$$d\underset{\sim}{\varepsilon}^{(m)} = \underset{\sim}{C}^{U(m)} : d\underset{\sim}{\sigma} \quad \text{(unloading)}$$

and continuity between loading and unloading results in

$$\underset{\sim}{C}^{L(m)} = \underset{\sim}{C}^{e(m)} + \frac{1}{H_L} \underset{\sim}{n}_{gL}^{(m)} \otimes \underset{\sim}{n}^{(m)} \tag{8}$$

$$\underset{\sim}{C}^{U(m)} = \underset{\sim}{C}^{e(m)} + \frac{1}{H_U} \underset{\sim}{n}_{gU}^{(m)} \otimes \underset{\sim}{n}^{(m)}$$

where $\underset{\sim}{n}_{gL}^{(m)}$ and $\underset{\sim}{n}_{gU}^{(m)}$ are unit tensors which characterize the direction of plastic flow, $H_{L/U}$ are scalar functions related to soil stiffness and referred to as loading and unloading plastic modulus and $\underset{\sim}{C}^{e(m)}$ is a tensor which defines material reversible behaviour.

The increment of strain can be written as

$$d\underset{\sim}{\varepsilon} = \underset{\sim}{C}^{e} : d\underset{\sim}{\sigma}' + \left\{ \sum_{m=1}^{M} \frac{1}{H_{L/L}} \underset{\sim}{n}_{gL/U}^{(m)} \otimes \underset{\sim}{n}^{(m)} \right\} : d\underset{\sim}{\sigma} \tag{9}$$

where all reversible contributions have been grouped.

Classical plasticity and Bounding Surface [12] theories can be considered as particular cases of the theory above presented.

The material behaviour can be fully characterized by specifying:

(i) Loading and unloading plastic modulii.

(ii) Directions of plastic flow during loading and unloading $\underset{\sim}{n}_g$

(iii) Loading and unloading direction $\underset{\sim}{n}$

All of them may depend on material history and memory of previous loading events has to be taken into account.

Finally, it should to be stated that models based on a single mechanism and formulated in terms of stress invariants can provide enough accuracy for many soil dynamics and earthquake engineering problems, as it has resulted from comparisons between centrifuge tests and numerical/constitutive models [13].

These simple models are frequently casted in terms of stress invariants p' and q given by

$$p' = I'_1 \tag{10}$$
$$q = \sqrt{3J'_2}$$

where I'_1 and J'_2 have been previously defined in (2).

3 A SIMPLE MODEL FOR SANDS

A simple model for sands can be built in a hierarchical manner as follows.

First, we will consider the behaviour of sand under virgin loading conditions, i.e., no previous loading of higher intensity than the initial stress conditions has been applied to the specimen.

The directions $\underset{\sim}{n}$ and $\underset{\sim}{n}_g$ can be taken as [8,9].

$$\underset{\sim}{n}_g = (n_{vg}, n_{sg})$$

$$n_{vg} = d_g/(1 + d_g^2)^{1/2} \tag{11}$$

$$n_{sg} = 1/(1 + d_g^2)^{1/2}$$

$$\underset{\sim}{n} = (n_v, n_s)$$

$$n_v = -\eta/(1 + \eta^2)^{1/2} \tag{12}$$

$$n_s = 1/(1 + \eta^2)^{1/2}$$

where η is the stress ratio $\eta = q/p'$ and $\alpha_f = \alpha_g$, M_f and M_g are parameters of the model.

The dilatance law described by eqn. 11 assumes the existence of the characteristic line $\eta = M_g$ at which dilatence is zero.

The flow rule proposed is therefore non-associated as M_f and M_g will differ. Previous studies have shown [7] that the ratio M_f/M_g can be assumed to be equal to the relative density. Very loose sands will be described by highly non-linear laws while in much denser specimens M_f will be close to M_g.

The proposed law for the plastic modulus is

$$H_{VL} = H_0 p' \left(1 - \frac{\eta}{\eta_f}\right)^4 \left(H_v + H_s\right) \tag{13}$$

where

$$H_v = \left(1 - \frac{\eta}{M_g} \right)$$

$$H_s = \beta_1 \exp(- \beta_0 \xi) \tag{14}$$

$$\xi = \int | d\varepsilon_s^p |$$

$$\eta_f = (1 + \frac{1}{\alpha}) M_f$$

In the above equations, three new parameters have been introduced. It is important to notice that the proposed law predicts that failure will take place at the Critical State Line. In the case of drained loading of a dense sand, the model predicts also the existence of a peak in the stress-strain curve, followed by a process in which the strength decreases (softening).

Experiments show that a shear band develops in the specimen, and therefore, the measurements are not representative of any interior homogeneous state. It should be pointed out, however, that material softening should be present to obtain an overall softening response.

The model above described is able to reproduce most of the basic features of sand under monotonic loading, and the interested reader is referred to previously published papers [6-8] where full details have been given. In particular, liquefaction under monotonic loading is accurately reproduced.

So far we have restricted our attention to virgin loading conditions. To accurately reproduce a cyclic loading process, the model should be able to describe the material behaviour during unloading and reloading.

Here we will considere both as a unique loading process, from a reversal point R (detected in general stress conditions by a change in the sign of $\underset{\sim}{n} : d\underset{\sim}{\sigma}'$) to a point of emergence E in a surface describing the maximum mobilized stress.

Both unit tensors $\underset{\sim}{n}$ and $\underset{\sim}{n}_g$ together with the plastic modulus H will be interpolated between the reversal and the emergence points according to the following laws:

$$n_v = n_v^o + (n_v^E - n_v^o) \cdot F$$

$$n_s = n_s^o + (n_s^E - n_s^o) \cdot f \qquad (15)$$

where n_v^o and n_s^o are the components of $\underset{\sim}{n}$ at the reversal point changing its sign, n_v^E and n_s^E those of $\underset{\sim}{n}$ at the emergence point and f is a interpolation function given by

$$f = \left(\frac{\eta - \eta_o}{\eta_E - \eta_o} \right)^{\gamma} \qquad (16)$$

where γ is a new parameter model.

The values of $\underset{\sim}{n}_g$ are interpolated in a similar manner, but now the volumetric component is assumed to be contractive.

Finally, the plastic modulus is obtained by interpolation between an initial value H_{ou} and the corresponding to the emergence point

$$H = H_{ou} + (H_E - H_{ou}) \cdot f \qquad (17)$$

In this way, all of the unloading and reloading process will be continuous.

To show the predictive capability of the proposed model, we include a first example concerning densification of a medium dense sand under a growing amplitude series of cycles [14] (Fig. 1) where it can be seen how the densification process is adequately reproduced.

Fig. 2 shows the development of cyclic mobility on a medium dense sand. After four cycles with a very small strain amplitude, the stress path crosses the characteristic state line, and large deformations develop as the origin is approached.

4 CONCLUSIONS

Failure of saturated sand under cyclic loading is caused by an accumulation of pore pressure due to the tendency of the soil to densify, followed either by liquefaction or development of cyclic mobility.

In the first case, the phenomenon is the same as the one encountered under monotonic loading of very loose sands, while in the second it is triggered by large plastic volumetric strain during unloading.

A simple model able to deal with densification, liquefaction and cyclic mobility has been developed within the framework of the Generalized Plasticity Theory.

The model is formulated in terms of stress invariants, and can be easily calibrated from laboratory tests.

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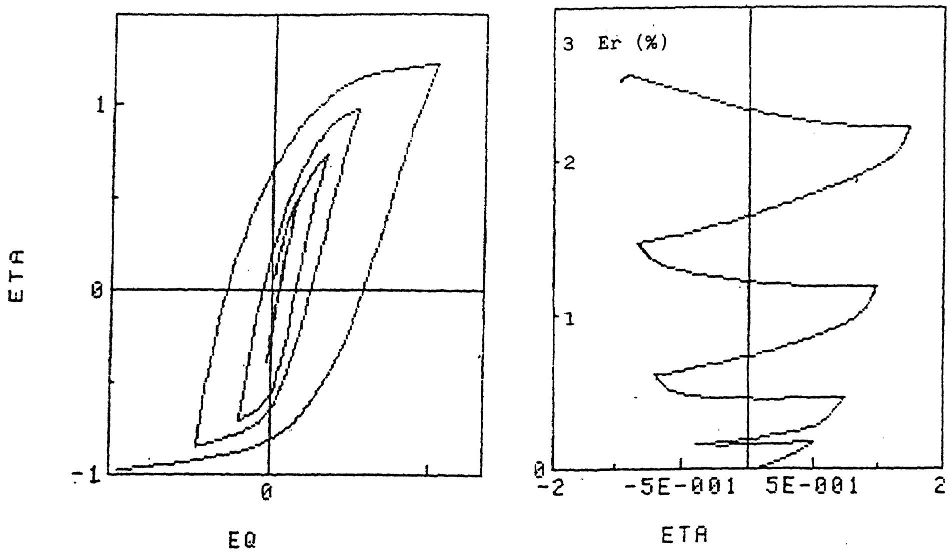
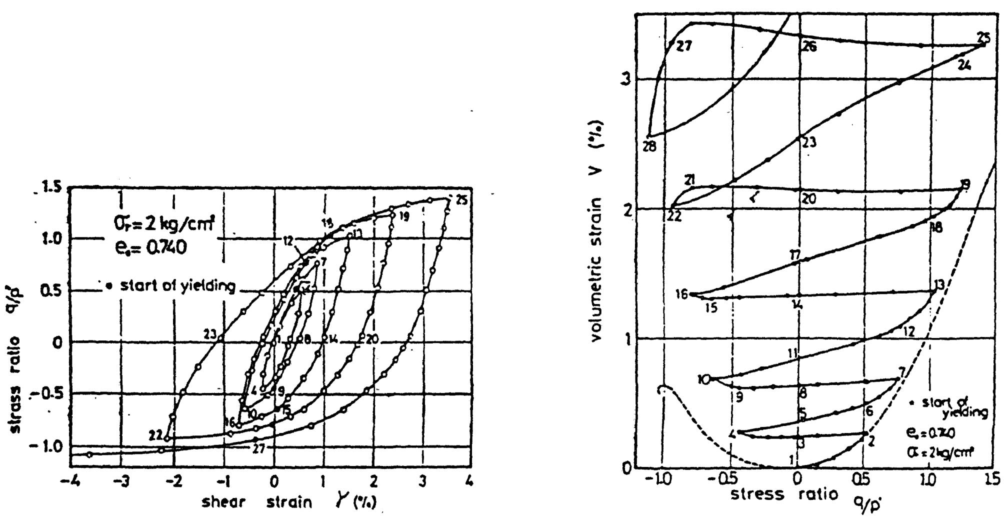


(a)



(b)

Fig. 1 Densification of a medium-dense sand under variable amplitude cyclic loading (Data from Tatsuoka and Ishihara)

(a) Model (b) Experiment

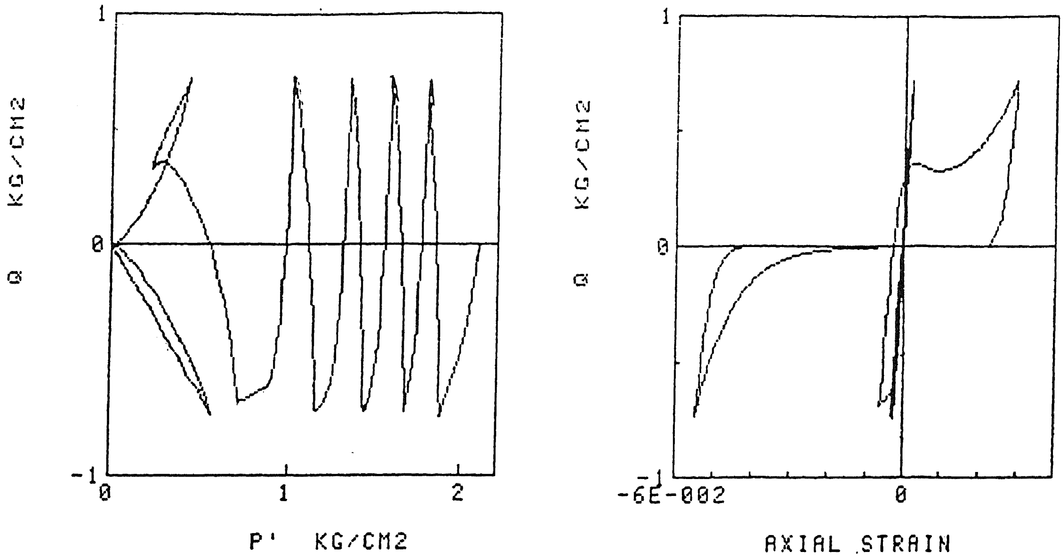


Fig.2 Cyclic mobility of a saturated
medium-dense sand