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A constitutive model for sand based on the non-linear kinematic hardening rule and its application

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ABSTRACT: Constitutive models used in liquefaction analyses should have the potential to simulate accumulations of strain and pore water pressure during cyclic loading. In the present paper, an inviscid constitutive equation is derived with the elasto-plasticity theory along with the concept of the non-linear kinematic hardening rule. A simulation of undrained cyclic triaxial tests is carried out by applying a newly proposed constitutive model. The accumulation of pore water pressure and a reduction in the effective mean stress are well simulated. The constitutive model is then incorporated into a coupled finite element - finite difference (FEM-FDM) numerical method for a liquefaction analysis of a fluid-saturated ground. Based on the numerical results, the effectiveness of the gravel drain method to prevent a sandy ground from liquefaction is discussed.

1 INTRODUCTION

The phenomenon of liquefaction is one of the most important subjects in the earthquake engineering. There are many program codes to simulate the liquefaction phenomenon of a sandy ground. Many constitutive models have been incorporated into finite element codes. The constitutive models used in liquefaction analyses should be simple. They should also have the potential to simulate accumulations of strain and pore water pressure during cyclic loading. To satisfy this condition, the Masing rule has often been used in many elasto-plastic constitutive models (e.g., Shibata et al., 1991). When the Masing rule is used, the hardening function has to be rearranged at the turning over point of the loading direction. This sudden change in the hardening function sometimes causes numerical difficulties. After a change in the loading direction, the strain increment will often keep the same direction for a while. This phenomenon cannot be reproduced by the Masing rule.

To overcome these disadvantages, inviscid constitutive equations are derived in this study with the elasto-plasticity theory along with the concept of non-linear kinematic hardening (Armstrong and Frederick, 1966; Chaboche and Rousselier, 1983). An overconsolidated surface is introduced as the bounding surface (Oka, 1982), and the stress ratio at its maximum compression is calculated by this surface. The non-associated flow rule is used in the derivation of the equations.

A simulation of the undrained cyclic triaxial tests is carried out with the newly proposed constitutive model. The accumulation of pore water pressure and the reduction in the effective mean stress are simulated well. The validity of the newly proposed model is addressed through a comparison of the numerical results and the experimental results (Ishihara and Okada, 1978).

This constitutive model is then incorporated into a coupled finite element - finite difference (FEM-FDM) numerical method for the liquefaction analysis of a fluid-saturated ground. Using a u-p formulation, the numerical method is prepared (Shibata et al., 1991). The finite element method is used for the spatial discretization of the equilibrium equation, while the finite difference method is used for the spatial discretization of the continuity equation. Newmark's β-method is used for the time discretization of both equations.

Finally, the applicability of the proposed constitutive model is examined by means of the liquefaction analysis. Using the numerical results, the effectiveness of the gravel drain method to prevent a sandy ground from liquefaction is discussed. In the analysis, the three-dimensional ground water flow around gravel drains is simulated with a macro-element (Sekiguchi et al., 1986) and the well resistance of the drains is considered (Tanaka et al., 1985). The improvement of the ground permeability is simulated well.

2 CONSTITUTIVE MODEL

Using the concept of the non-linear kinematic hardening rule, Chaboche and Rousselier (1983) proposed some constitutive equations for metal. This theory is also applicable to formulating a constitutive model for the cyclic behavior of soil. Based on Oka's model, a cyclic elasto-plastic constitutive model for sands will be derived in this paper through use of the concept of non-linear kinematic hardening.

The proposed constitutive model for sand is based on the following assumptions:

1. infinitesimal strain;
2. the elasto-plastic theory;
3. the non-associated flow rule;
4. the concept of the overconsolidated boundary surface;
5. the non-linear kinematic hardening rule.

overconsolidated boundary surface

The boundary surface between the normally consolidated region ($f_b \geq 0$) and the overconsolidated region ($f_b < 0$) is given by

$$f_b = \bar{\eta}_{(0)} + M^*_m \ln(\sigma'_m/\sigma'_{mb}) = 0 \quad (1)$$

$$\bar{\eta}_{(0)} = \{(\eta_{ij} - \eta_{ij(0)})(\eta_{ij} - \eta_{ij(0)})\}^{1/2} \quad (2)$$

$$\eta_{ij} = s_{ij}/\sigma'_m \quad (3)$$

where σ'_m is the mean effective stress, s_{ij} is the deviatoric stress tensor and M^*_m is the value of η^* when the maximum compression of the material takes place. σ'_{mb} is generally given by the following rule in the overconsolidated region:

$$\sigma'_{mb} = \sigma'_{mbi} \exp\{(1+e)/(\lambda - \kappa)v^p\} \quad (4)$$

where σ'_{mbi} is the mean effective stress at the end of the anisotropic consolidation, e is the initial void ratio, λ is the compression index, κ is the swelling index and v^p is the volumetric plastic strain.

yield surface

The yield function is given by

$$f = \{(\eta_{ij} - X_{ij})(\eta_{ij} - X_{ij})\}^{1/2} - k = 0 \quad (5)$$

where k is the parameter which defines the elastic region and X_{ij} are the kinematic variables. By introducing the nonlinearity of kinematic hardening, X_{ij} can be written as

$$dX_{ij} = B^*(A^* de^p_{ij} - X_{ij} d\gamma^p) \quad (6)$$

in which A^* and B^* are the material parameters and de^p_{ij} is the increment of the plastic deviatoric strain. The key to this simple model is the second term on the right-hand side, proportional to X_{ij}, and the second invariant of the increment of the plastic deviatoric strain,

$$d\gamma^p = (de^p_{ij} de^p_{ij})^{1/2}. \quad (7)$$

Oka (1982) developed a constitutive equation for granular materials in cyclic loading. In that model, the strain hardening parameter was assumed to be hyperbolic, namely,

$$\bar{\eta}_{(0)} = \frac{M_f G' \gamma^p}{M_f + G' \gamma^p} \quad (8)$$

For a uniaxial case, equations (6),(7) and (8) yield

$$A = M_f \quad \text{and} \quad B = G'/A \quad (9)$$

where $A = \sqrt{3/2}A^*$ and $B = \sqrt{3/2}B^*$. Under one-dimensional proportional loading, equation (7) is explicitly integrable. Figure 1 illustrates the modeling possibilities with equations (7) and (8). For this calculation, $A = M_f = 1.726$, $G' = 600$ and $B = 348$. From Figure 1, it is clear that the initial hardening modulus in the nonlinear hardening rule is higher than that in the hyperbolic hardening rule.

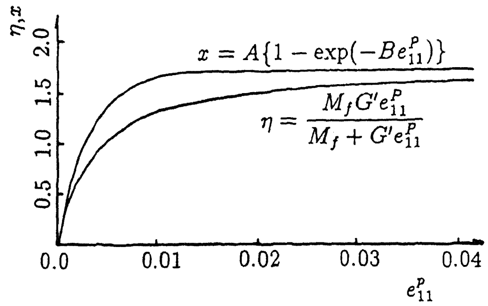


Figure 1 Relationship between hardening parameter and plastic strain.

plastic potential function

Based on the relationship between the stress ratio and the increment of plastic strain, the plastic potential is expressed as follows:

$$f = \{(\eta_{ij} - X_{ij})(\eta_{ij} - X_{ij})\}^{1/2} + \widetilde{M}^* ln(\sigma'_m/\sigma'_{ma}) = 0 \quad (10)$$

where σ'_{ma} is a material parameter and $\widetilde{M}$ is the stress ratio when the maximum compression of the material takes place. In the normally consolidated region ($f_b \geq 0$), $\widetilde{M}$ is kept constant as

$$\widetilde{M}^* = M^*_m. \quad (11)$$

On the other hand, in the overconsolidated state ($f_b < 0$), $\widetilde{M}$ is defined as

$$\widetilde{M}^* = -\eta/\ln(\sigma'_m/\sigma'_{mc}) \quad (12)$$

where the current stress ratio, η, is defined as

$$\eta = (\eta_{ij}\eta_{ij})^{1/2} \quad (13)$$

and

$$\sigma'_{mc} = \sigma'_{mb} exp(\eta_0/M^*_m) \quad (14)$$

in which the initial stress ratio, η_0, is defined as

$$\eta_0 = (\eta_{ij(0)}\eta_{ij(0)})^{1/2}. \quad (15)$$

In this study, we assess the accuracy of the newly proposed constitutive model by a constant stress amplitude test for overconsolidated sand by Ishihara and Okada (1978). The calculated and the experimental results are shown in Figures 2 and 3. From these figures, it is clear that the proposed constitutive model can sufficiently simulate the experimental results.

3 FEM FORMULATION

The governing equations for coupling problems, such as the soil skelton and pore water, are obtained through application of the two-phase mixture theory by Biot (1962). Using the **u-p** formulation (Zienkiewicz et al., 1982), a numerical method for the liquefaction analysis is prepared. The governing equations for the **u-p** formulation are summarized as follows:

Equilibrium equation

$$\rho \ddot{u}_i = \sigma_{ij,j} + \rho b_i \qquad (16)$$

Continuity equation

$$\rho^f \ddot{u}_{i,i} - p_{,ii} - \frac{\gamma_w}{k}\dot{\varepsilon}_{ii} + \frac{n\gamma_w}{kK^f}\dot{p} = 0 \qquad (17)$$

where ρ^f is the density of the fluid, ρ is the density of the total phase, p is the pore water pressure, γ_w is the unit weight of the fluid, k is the coefficient of permeability, n is the porosity, b is the body force and K^f is the bulk modulus of the fluid. Equation (16) will be discretized spatially by the finite element method and equation (17) will be discretized spatially by the finite difference method. Finally, the discretized governing equations at times $t + \Delta t$

Table 1 Material parameters.

M_{mc}	M_{me}	M_{fc}	M_{fe}	λ	κ
1.726	0.900	1.024	0.612	0.017	0.008

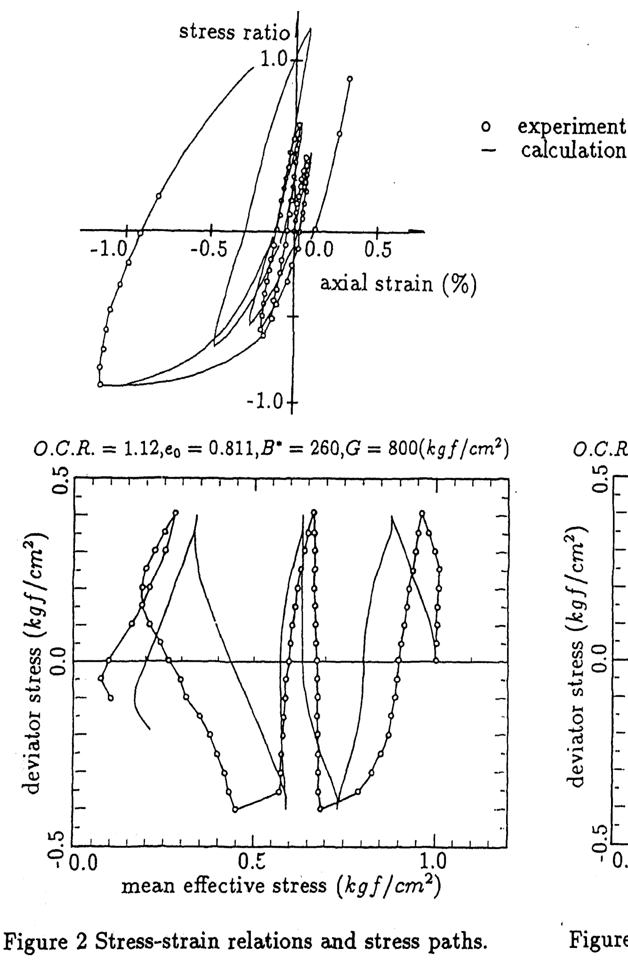


Figure 2 Stress-strain relations and stress paths.

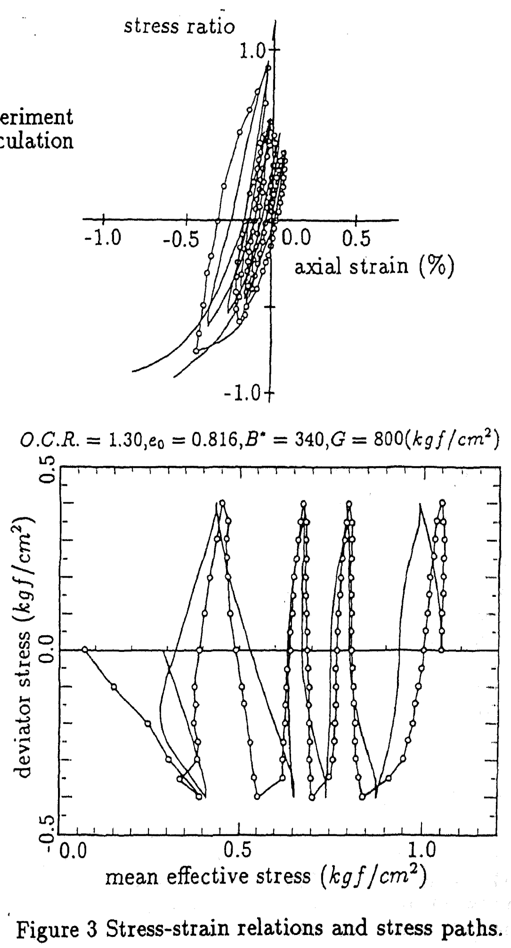


Figure 3 Stress-strain relations and stress paths.

are written as

$$\begin{bmatrix} [M]+\beta\Delta t^2[K]_{|t+\Delta t} & \vec{K}_v \\ \vec{K}_v^T & A-\alpha' \end{bmatrix} \begin{Bmatrix} \vec{\ddot{u}}_{N|t+\Delta t} \\ p_{|t+\Delta t} \end{Bmatrix} + \begin{Bmatrix} 0 \\ \Sigma\alpha_i' p_{i|t+\Delta t} \end{Bmatrix} =$$

$$\begin{Bmatrix} \vec{F}_{|t+\Delta t} - \vec{R}_{|t} - [K]_{|t+\Delta t}(\Delta t \vec{\dot{u}}_{N|t} + (\frac{1}{2}-\beta)\Delta t^2 \vec{\ddot{u}}_{N|t}) \\ \vec{K}_v^T(\vec{\dot{u}}_{N|t} + (1-\gamma)\Delta t \vec{\ddot{u}}_{N|t})/(k(\frac{1}{g} - \frac{\gamma \Delta t}{k})) + Ap_{|t} \end{Bmatrix} \quad (18)$$

where

$$\alpha' = \frac{1}{\gamma_w(\frac{1}{g} - \frac{\gamma\Delta t}{k})}\alpha,\ \alpha_i' = \frac{1}{\gamma_w(\frac{1}{g} - \frac{\gamma\Delta t}{k})}\alpha_i,\ \alpha = \Sigma\alpha_i \quad (19)$$

and

$$A = \frac{1}{\Delta t(\frac{1}{g} - \frac{\gamma\Delta t}{k})}\int_v \frac{n}{kK^f}dv. \quad (20)$$

In the above equations, g is the acceleration of gravity and α_i is defined by both the length of the side of the element concerned and the distance between the gravity center of neighboring element i and that of the element concerned.

4 ANALYSIS OF GRAVEL DRAIN

Various kinds of methods have been proposed to prevent a sandy ground from liquefaction. The use of the gravel drain method, which generates little vibration and noise during construction work, has recently been gaining much popularity. However, design methods (e.g., the determination of the area to be improved) have not yet been established. In this study, a three-dimensional ground water flow around gravel drains during an earthquake is simulated with the macro-element (Sekiguchi et al., 1986) and the well resistance of the drains is considered (Tanaka et al., 1985). The usefulness of the gravel drain method is discussed here and is based on the numerical results. A comparison is made between the behavior of a ground with gravel drains and a ground without gravel drains.

macro-element

Sekiguchi et al. (1986) proposed a macro-element to simulate the radial flow of ground water around a vertical drain under the plane strain condition. If the permeability of the drain material is infinite, the total water flow Q through the drain boundary ($r = r_w$) for Δt is denoted as

$$Q \cdot \Delta t = 2\pi r_w \cdot s_z \cdot k_g \cdot \Delta t \cdot \frac{\partial(p/\gamma_w)}{\partial r}\Big|_{r=r_w} \quad (21)$$

where s_z is the height of the gravel drain and k_g is the permeability of the ground. Using the mean pore water pressure in the macro-element, $\bar{p}$, equation (21) can be rewritten as

$$Q \cdot \Delta t = \alpha_{gd} \cdot \bar{p} \cdot s_z \cdot \Delta t \quad (22)$$

in which

$$\bar{p} = \frac{2\pi\int_0^{r_e} p_w \cdot r \cdot dr}{\pi r_e^2}$$

r_e is the radius of the cylinder of influence of the drain and is determined by the interval between the drains, s_x, in the case of a square arrangement,

$$r_e = \frac{s_x}{\sqrt{\pi}}, \quad (23)$$

α_{gd} is the coefficient and is defined by means of n ($n = r_e/r_w$), namely,

$$\alpha_{gd} = \frac{k_g\Delta t}{\gamma_w}\frac{s_z}{r_e}\frac{2\pi(1-\frac{1}{n^2})}{ln(n) - \frac{1}{4}(1-\frac{1}{n^2})(3-\frac{1}{n^2})} = \frac{k_g\Delta t}{\gamma_w}\xi_{gd}. \quad (24)$$

In the finite element analysis, α in equation (19) is modified as

$$\alpha = \Sigma\alpha_i + \xi_{gd} \quad (25)$$

well resistance

In the formulation of the macro-element, it is assumed that the permeability of the drains is infinite. According to this assumption, therefore, the excess pore water pressure is evaluated to be less than that of the realistic model which considers the finite permeability of gravel drains. Tanaka et al. (1985) introduced the correction factor, C_α, to consider the delay in dissipation of the excess pore water pressure brought about by the finite permeability of the gravel drains. Based on the well resistance coefficient, R, C_α is denoted as

$$C_\alpha = 1 + 1.1848 \cdot R \cdot \frac{1-\frac{1}{n^2}}{\frac{n^2}{n^2-1}ln(n) - \frac{3n^2-1}{4n^2}} \quad (26)$$

$$R = \frac{8}{\pi^2}\frac{k_g}{k_w}(\frac{s_z}{r_e})^2 \quad (27)$$

in which k_w is the permeability of the gravel drains. Considering the well resistance of the gravel drains, a modified ξ_{gd} in the macro-element, ξ_{gd}', is denoted as

$$\xi_{gd}' = \frac{1}{C_\alpha}\xi_{gd} \quad (28)$$

numerical results

Figure 4 shows the finite element mesh used here. The model consists of 5 horizontal layers with 55

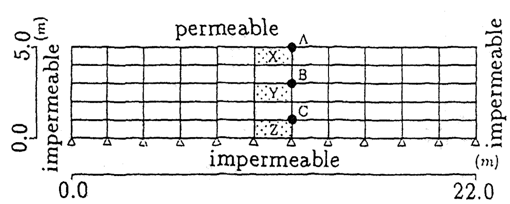


Figure 4 Finite element mesh.

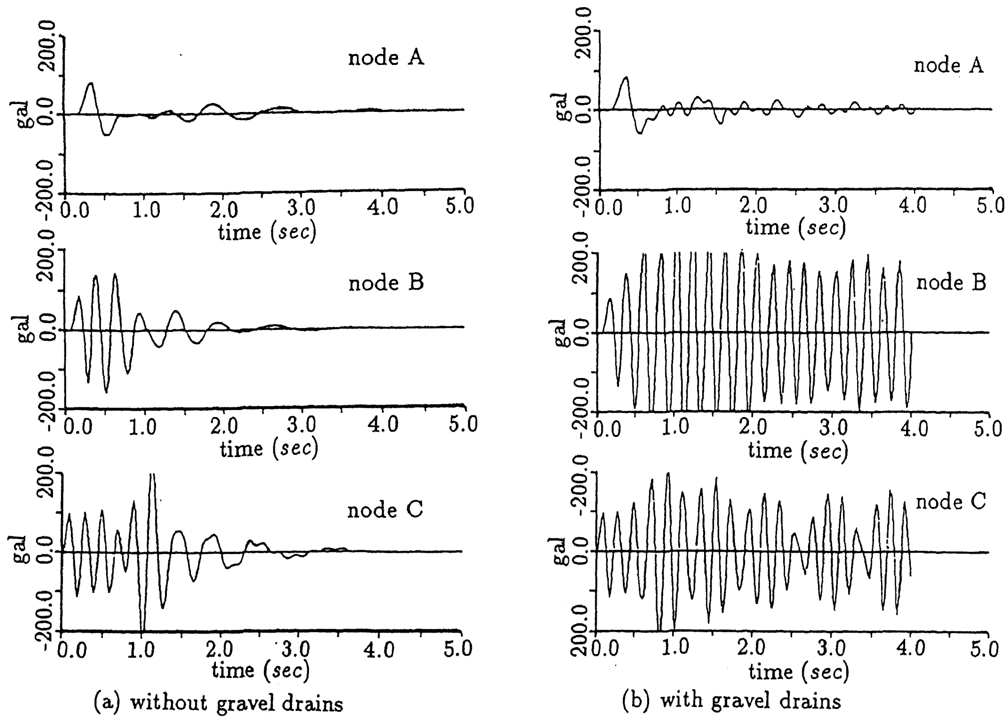


Figure 5 Variation of acceleration with time

Table 2 Material parameters for sand.

initial permeability	k_0	$1.0 \times 10^{-3} cm/sec$
initial void ratio	e_0	1.0
	M_m^*	0.5
	$A^* = M_f^*$	0.7
consolidation index	λ	0.017
swelling index	κ	0.008
O.C.R.		1.05
Poisson's ratio	ν	0.33
	B^*	200

Table 3 Material parameters for gravel.

specific gravity	G_s	2.65
permeability	k_w	$6.0 cm/sec$
diameter of gravel drain	ϕ	$50.0 cm$

elements and 72 nodes. The base line is fixed in both directions and at the undrained boundary. Both sides are free and at the undrained boundaries. The upper surface (the ground surface) is the drained boundary. The material parameters for sand are summarized in Table 2, while the material parameters for the gravel are shown in Table 3. The sinusoidal acceleration data is used as an input wave. The maximum acceleration is 100 gal and the frequency is 5 Hz. In this example, the gravel drains are installed in the square arrangement at an interval of 1 m. Correction factor C_α is based on the wellresistance in this case and is 1.03.

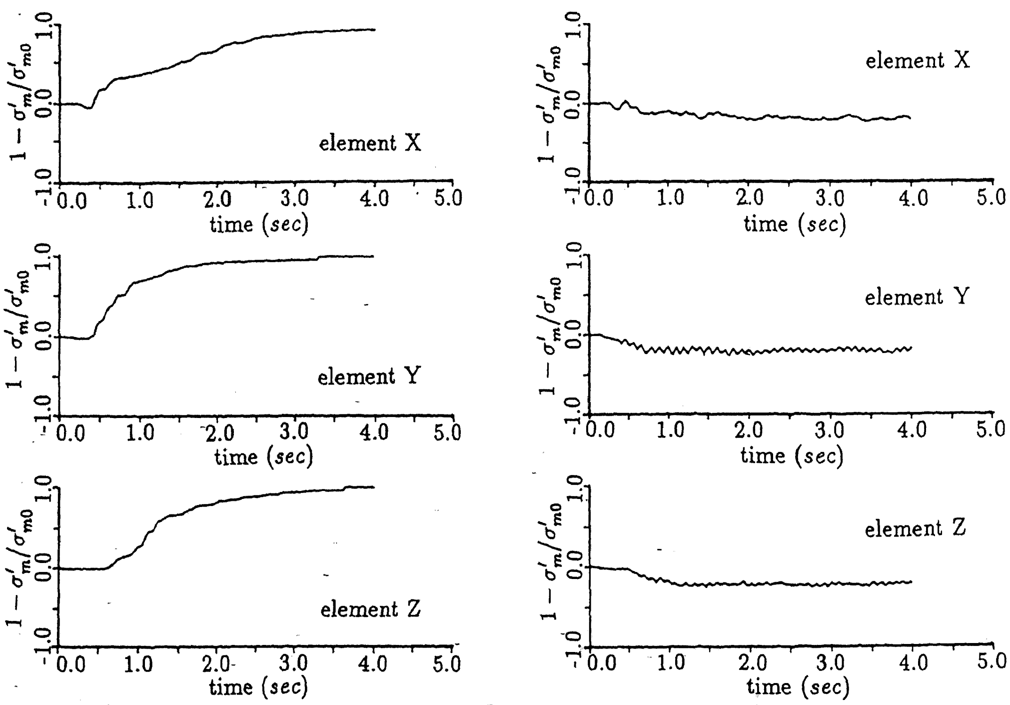
The relationships between acceleration and time at nodes A, B and C for grounds with and without gravel drains are plotted in Figure 5. For a sandy ground with no gravel drains, the acceleration is not transmitted to the ground surface due to the liquefaction of the lower ground.

The time histories of the pore water pressure ratio ($1 - \sigma'_m/\sigma'_{m0}$: σ'_{m0} is the initial mean effective stress) in elements X, Y and Z are illustrated in Figure 6. At the initial state, $\sigma'_m = \sigma'_{m0}$; thus, the pore water pressure ratio is 0.0. On the other hand, once the ground is liquefied, $\sigma'_m = 0$; thus, the pore water pressure ratio is 1.0. For a ground without any gravel drains, liquefaction is observed with an accumulation of excess pore water pressure. On the other hand, for a sandy ground with gravel drains, the dissipation of excess pore water pressure is very fast and does not reach liquefaction.

5 CONCLUDING REMARKS

The main conclusions obtained in this study are summarized as follows:

1. The inviscid constitutive equation was derived using the elasto-plasticity theory along with the concept of the non-linear kinematic hardening.



(a) without gravel drains (b) with gravel drains

Figure 6 Variation of pore water pressure ratio with time

simulation of undrained cyclic triaxial tests was conducted. The proposed constitutive model was found to sufficiently simulate the experimental results.

2. Liquefaction of a loose sandy ground was simulated well by a coupled FEM-FDM analysis with the newly proposed constitutive model.

3. The three dimensional ground water flow around gravel drains during an earthquake was modeled by using a macro-element and the well resistance of the drains was considered in the analysis.

4. Based on numerical results, the effectiveness of the gravel drain method to prevent a sandy ground from liquefaction was discussed. The fast dissipation of excess pore water pressure through the gravel drains was simulated well.

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