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Analytical model for viscoelastic dampers in seismic mitigation application

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ABSTRACT: To mitigate the structure damage resulted from seismic loadings, the viscoelastic dampers with high energy absorption capacity have been applied to structures to enhance the seismic resistance. In order to simulate the material behavior of the viscoelastic damper, methods only to approximate the damping ratio induced by dampers were suggested in the past. Usually, it is difficult to find an adequate modulus and strain to estimate the equivalent damping ratio since the nature of the material is greatly dependent on both the frequency and the strain ratio. These values are are both changing in the presence of the hysteresis loops when material subjected to a dynamic loading. In this paper, a constitutive equation for the viscoelastic damper was developed. This analytical model based on fractional derivatives is able to accurately describe the hysteretic behavior of a viscoelastic damper subjected to cyclic loadings. For the time domain analysis, this model can be applied directly without transformation and inverse transformation to frequency domain, which usually causes computational difficulties. Results obtained by the proposed model were in good agreement with available experimental data.

1 INTRODUCTION

When structures subjected to seismic loadings, tremendous amounts of energy will be input to the structural system. To mitigate the damage and survive the earthquake, it is anticipated that to incorporate mechanical damping devices, which have substantial energy absorption ability, in a structure could improve the dynamic performance of the structure. An ideal device, viscoelastic damper, has been shown to be effective, as is utilized in the World Trade Center in New York City and Columbia Center in Seattle to reduce the vibration induced from wind-loadings.

Numerous experiments for testing the material properties have been carried out (Mahmoodi 1972, Bergman 1986, Lin 1988, and Chang 1991) and showed that the viscoelastic dampers could be a promising material in terms of mitigating the seismic damages. According to the testing data, it can be observed that the hysteresis loops, taken from the shear force and the respected deformation, have a generally elliptical shape. This implies that substantial energy input, equivalent to an area within the hysteresis loops, could be absorbed by the viscoelastic damper.

Although the results for the feasibility testing for the structures incorporated with viscoelastic dampers is quite encouraging, some questions are still remained. The essential one is what the constitutive equation of the material is. Usually, an equivalent damping method has been utilized to perform the structural analysis when the structural system has incorporated viscoelastic dampers (Zhang 1989 and Chang 1991). As indicated in this method, a constant area within the hysteresis loops under a constant testing frequency and strain ratio is used to compute an equivalent damping ratio. The question is what value of modulus under what frequency and what strain will be appropriate to be used to estimate the equivalent damping ratio. Besides, without actually knowing the local behavior of the dampers added to the structural system, the results obtained by using the estimated equivalent damping method might be just reverse to what to be expected.

Therefore, it is the purpose of this study to develop a reliable analytical material model, by which a time domain analysis that can detect the local behavior of the structure could be performed.

2. APPLICATION OF FRACTIONAL DERIVATIVES

For most viscoelastic materials, a linear viscous damping is usually assumed. This means that the loss factor is linearly proportional to the strain rate. This is not always true for the viscoelastic materials. To overcome these difficulties, a fractional derivative model for the elastomer damper was

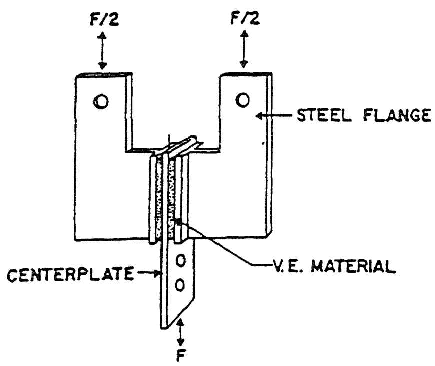

Fig.1 Typical viscoelastic damper in study

developed by Bagley in 1979. A theoretical basis drawn from the molecular theory for these new constitutional relationships was also established (Bagley 1983).

This constitutional relationship having a fractional derivative form is presented as

$$\tau(t) = G_0\gamma(t) + G_1 D^\alpha[\gamma(t)] \tag{1}$$

where τ and γ is the shear stress and shear strain respectively, G_0 and G_1 represent the shear modulus corresponding to the storage and the loss energy. The fractional derivative is accordingly presented as:

$$D^\alpha[\gamma(t)] = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t \frac{\gamma(\tau)}{(t-\tau)^\alpha}\,d\tau, \qquad 0 < \alpha < 1 \tag{2}$$

where $\Gamma(1-\alpha)$ is the gamma function.

To apply this constitutional relationship into the equation of motion for dynamic analysis, usually a transformation into the frequency domain needs to be carried out. Therefore, a time domain analysis is limited in general, particularly when a non-linear analysis is required.

3. ANALYTICAL MODEL FOR VISCOELASTIC DAMPER

The proposed analytical model is based on Bagley's fractional derivative viscoelastic model and modified to account for the modulus degradation. It is observed from the experimental results that in the high frequency and high strain test, the shear modulus is deteriorated with respect to the increase of the testing cycles. The proposed formula for describing the material behavior of the viscoelastic damper subjected to arbitrary loadings is given as:

$$G_0 = G_1 = A_0\{1 + B_0 \times e^{-\beta(\int \tau\, d\gamma)}\} \tag{3}$$

where B_0 and β are coefficients to account for the energy absorption ability of the material, and A_0 is the coefficient corresponding to the shear modulus of the material. All of these unknown coefficients are material dependent and to be determined from the experimental data. This formula indicates that the shear modulus G_0 and G_1 decay with the increase of the strain energy.

To apply the fractional derivative model to the time domain analysis without transforms or inverse transforms, a numerical scheme by using the finite element method is proposed. According to Leibnitz's rule, the derivative of an integral can be written as:

$$\frac{d}{dt}\int_{\phi_1(t)}^{\phi_2(t)} \frac{\gamma(\tau)}{(t-\tau)^\alpha}d\tau = \int_{\phi_1(t)}^{\phi_2(t)} \frac{d}{dt}\left[\frac{\gamma(\tau)}{(t-\tau)^\alpha}\right]d\tau + \frac{\gamma(\phi_2)}{(t-\phi_2)^\alpha}\frac{d\phi_2}{dt} - \frac{\gamma(\phi_1)}{(t-\phi_1)^\alpha}\frac{d\phi_1}{dt} \tag{4}$$

Assuming that a linear variation of the shear strain exists between two time steps, $(n-1)\Delta t$ and $n\Delta t$, the shear strain could be written as

$$\gamma(\tau) = (n - \frac{\tau}{\Delta t})\gamma[(n-1)\Delta t] + [\frac{\tau}{\Delta t} - (n-1)]\gamma(n\Delta t),$$
$$(n-1)\Delta t \le \tau \le n\Delta t \tag{5}$$

Substitution of equation (5) into (1) and (2) yields the constitutional law for the viscoelastic damper at time step $\tau = n\Delta t$; that is

$$\tau(N\Delta t) = \left[G_0 + \frac{G_1(\Delta t)^{-\alpha}}{(1-\alpha)\,\Gamma(1-\alpha)}\right]\gamma(N\Delta t) + F(N\Delta t) \tag{6}$$

where the previous effect is defined as

$$\begin{aligned} F(N\Delta t) = {} & \frac{G_1}{(1-\alpha)\,\Gamma(1-\alpha)}\{\{(N-1)[(N-1)\Delta t]^{-\alpha} \\ & + (-N + 1 - \alpha)[N\Delta t]^{-\alpha}\}\gamma(0) \\ & + \sum_{n=1}^{N-1}\{-2(N-n)[(N-n)\Delta t]^{-\alpha} + (N-n+1)[(N-n+1)\Delta t]^{-\alpha} \\ & + (N-n-1)[(N-n-1)\Delta t]^{-\alpha}\}\gamma(n\Delta t)\} \end{aligned} \tag{7}$$

It should be noted that the first term on the right hand side of (7) representing part of the initial condition is equal to zero at the first time step, N=1, and that the fifth term is equal to zero while n is equal to N-1. There is no singular problem in the above equation even though the initial conditions are involved.

4. VISCOELASTIC DAMPERS UNDER STUDY

As is shown in Fig.1, it is a typical viscoelastic (VE) damper being added to the structure and tested in the experiments. This damper is a production of 3M Company, which consists of two VE layers bounded between three steel plates. When the central steel plate moves relative to the outer two plates, the VE layers deform in shear. Several dimensions of these VE dampers have been tested. Dampers to be studied in this research are: (1) 2 in.x in. of area, 0.24 in. of thickness and 0.1 in. of max. displacement from Lin's test; (2) 1.5 in.x in. of area, 0.20 in. of thickness and 0.1 in. of max. displacement from Chang's test. The environmental temperature for the testing is 20°C, which is an essential factor due to the temperature sensitivity of the VE material.

According to the experimental data in the cyclic loading test, the hysteresis loops showed some interesting characteristics. At

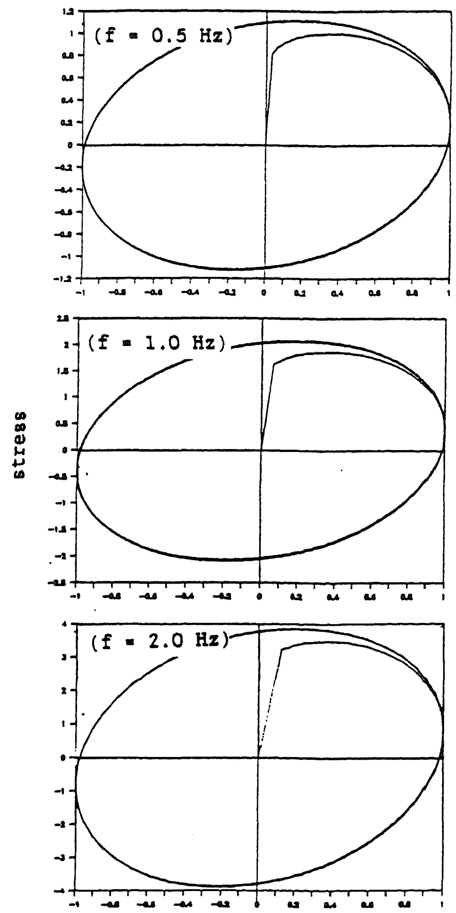

Fig.2. Hysteresis loop

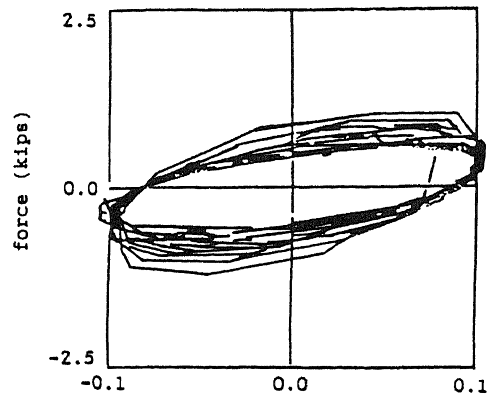

Fig.3.a Experimental result
(Expt.1: f = 0.5 Hz)

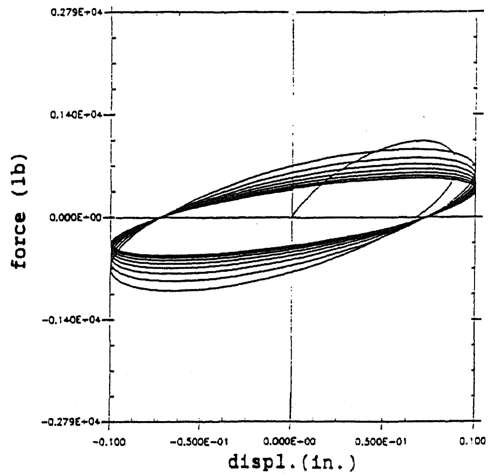

Fig.3.b Analytical result
(Expt.1: f = 0.5 Hz)

first, the stress induced by the displacement input with the same strain ratio increased with respect to the increase of input frequency. It can be observed from the variation of the force in part (a) of Fig.3 to Fig.5, and also part (a) of Fig.6 to Fig.9, in which two sets of strain ratio were applied. This phenomenon was explained such that the shear modulus was varied with the input loading frequency. Actually, as is shown in Fig.2 from (a) to (c), although the strain input has the same ratio, when the harmonic input frequency varied from 0.5 Hz to 2 Hz the stress has an increase without change of the shear modulus. The modulus was remained as $G_0=0$ and $G_1=0.402$ in the analysis.

The other phenomenon is that the degradation of shear modulus becomes more significant when the input frequency, or more precisely the strain rate, is higher. It is observed in part (a) from Fig.3 to Fig.5, where the frequency varied form 0.5 Hz to 2 Hz. This typical degradation, however, is not that substantial when we examine Fig.7.a and Fig.9.a although a higher frequency, 3.5 Hz, was used in the test. This is because a much lower strain ratio, 5 % and 20 % compared to about 40 % in the first test, was used in the later test. Therefore, it is believed that the mechanical properties of the VE material is in fact dependent upon the strain rate instead of frequency. This can also be obtained by comparing Fig.9.a with Fig.7.a, where although input frequency is identical, 3.5 Hz, the degradation in the higher strain test is slightly more significant.

This is resulted from the fact that the energy accumulates faster during a loading with higher strain rate, and the heat transformed from the strain energy thus softens the material and reduces its energy absorbing capacity. This degradation could be taken into account by the proposed formula, in which the variation of the shear modulus is in terms of a form of the accumulation of strain energy.

Table 1. Coefficients for the model

Coeff.	α	β	A_0 (psi)	B_0
Expt.1	0.75	0.01	48.0	3.0
Expt.2	0.75	0.01	5.60	3.0

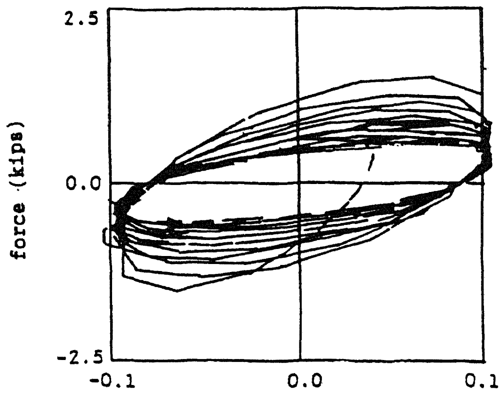

Fig.4.a Experimental result
(Expt.1: f = 1.0 Hz)

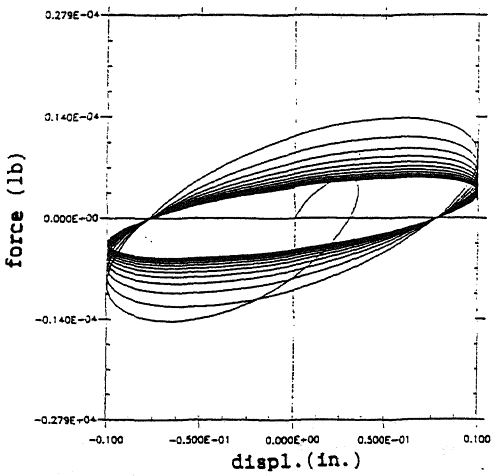

Fig.4.b Analytical result
(Expt.1: f = 1.0 Hz)

5. ANALYTICAL RESULTS AND EXPERIMENTAL VERIFICATION

The fractional viscoelastic model associated with the proposed degrading formula for the shear modulus was calibrated by using the test data. By applying the indicated numerical scheme and the least square method, the required coefficients in the model could be obtained.

There are two sets of coefficients corresponding to VE dampers in two tests, which are shown in Table 1.

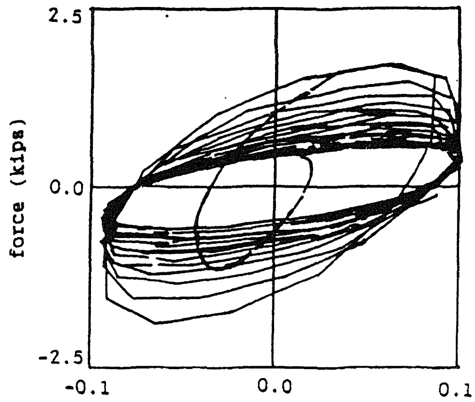

Fig.5.a Experimental result
(Expt.1: f = 2.0 Hz)

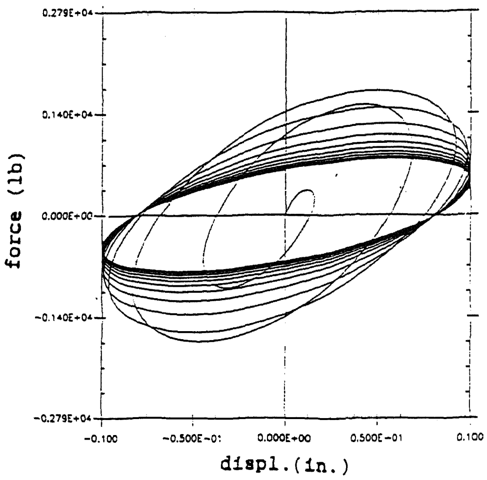

Fig.5.b Analytical result
(Expt.1: f = 2.0 Hz)

Part (b) of figures 3 to 10 are analytical results corresponding to the testing indicated in the figure tags. In general, very good agreement is obtained between the analytical and the experimental results. No variation of shear modulus is shown in either case of simulation of the damper even though the frequency is varied. Similarly, no change of the modulus is needed to input to the model when the damper subjected to a high frequency, high strain cyclic loading. The model simulates the degradation behavior automatically.

6. CONCLUSIONS

It is concluded that very good agreement exists between the experimental data and the analytical results. It is encouraging that this model could describe the viscoelastic material behavior in a great accuracy. In terms of the strain rate sensitivity of the

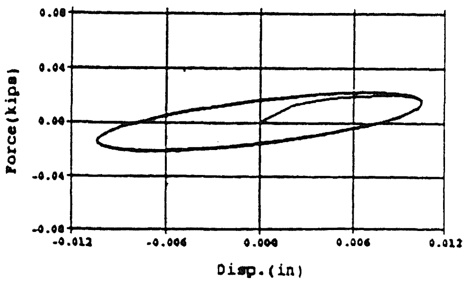

Fig.6.a Experimental result
(Expt.2: 5 % strain, f = 1.0 Hz)

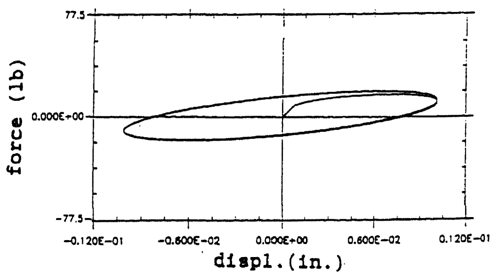

Fig.6.b Analytical result

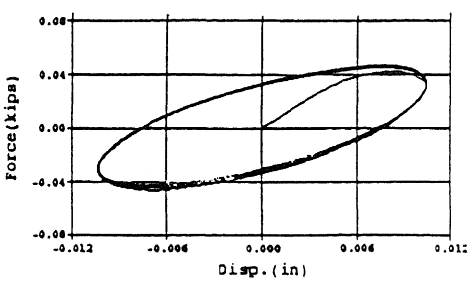

Fig.7.a Experimental result
(Expt.2: 5 % strain, f = 3.5 Hz)

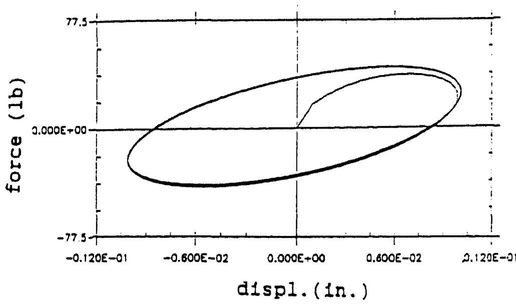

Fig.7.b Analytical result
(Expt.2: 5 % strain, f = 3.5 Hz)

material, there is no need to assign a bunch of shear modulus to the model, which may cause difficulties for dynamic analysis performed in equivalent damping method for the

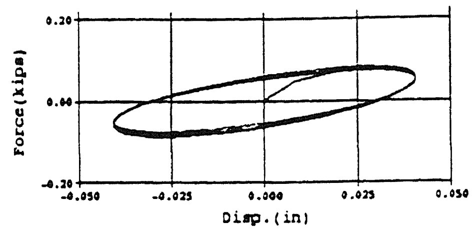

Fig.8.a Experimental result
(Expt.2: 20 % strain: f = 1.0 Hz)

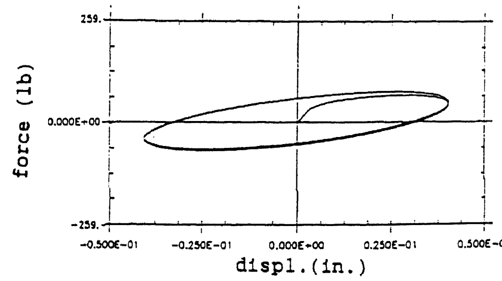

Fig.8.b Analytical result
(Expt.2: 20 % strain: f = 1.0 Hz)

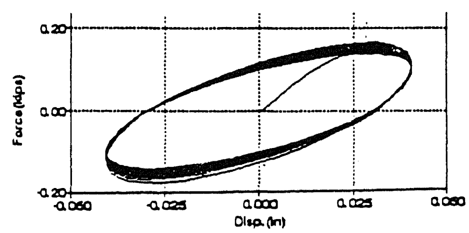

Fig.9.a Experimental result
(Expt.2: 20 % strain: f = 3.5 Hz)

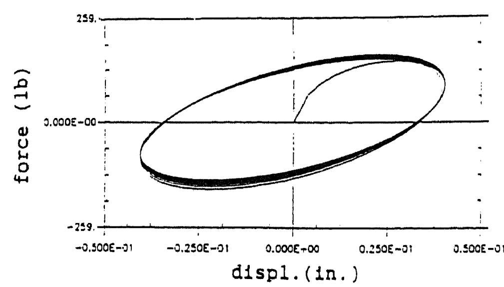

Fig.9.b Analytical result
(Expt.2: 20 % strain: f = 3.5 Hz)

structures combined with added VE dampers. For different frequency of loading with varied strain, the shear modulus is remaining unchanged for the same material.

To have all of these coefficients including the thermal effect caused by the environmental temperature, which is currently under study, corresponding to each category of viscoelastic material, it is believed that a more reliable dynamic analysis for the structures incorporated with viscoelastic dampers could be carried out confidently.

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