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Implications of shaking table tests in the analysis and design of base isolated structures

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ABSTRACT: Within a large R&D program in progress in Italy on seismic isolation, snap-back and seismic excitation tests have been performed on a 394 kN isolated mock-up on the MASTER shaking table of ISMES Dynamic Laboratory. Three Italian earthquake records having different frequency contents have been used in the tests. The most significant response data are presented and discussed. A mathematical non-linear model of the isolation system is developed, and the model is then validated by comparing the simulated response with the measured experimental data.

1 INTRODUCTION

To promote the development and applications of the base isolation technique, in 1989 a considerable research effort was undertaken in Italy by ENEA, ENEL, ISMES and ALGA. The work completed up to now includes experimental and numerical studies on elastomeric materials, isolators and isolated structures (Martelli *et al.* 1991a).

The attention of the research team was focused on isolation systems based on high damping steel-laminated rubber bearings (HDLRB). These devices are manufactured by vulcanization bonding of layers of high damping rubber to thin steel reinforcing plates, thus merging in a single mechanism both the frequency filtering and energy dissipation capacities necessary to achieve an effective isolation action. The choice of HDLRBs was due to the very promising features shown by these isolators, and to the fact that they have been adopted in the construction of the first group of large base isolated buildings in Italy: the SIP Administration Center in Ancona. The experimental work (Forni *et al.* 1991) included tests on rubber specimens, in-situ experiments on isolated buildings, as well as tests at ISMES laboratories on single bearings and isolated structure mock-ups.

The bearings tested at ISMES were of the HDLRB type and of four different sizes, all having a circular cross section and a shape factor of 10.4. Their diameters were 500, 250, 166.5 and 125 mm, corresponding to the full (1:1) and reduced (1:2, 1:3 and 1:4) scale of the bearings used in the Ancona SIP Buildings, respectively. Two isolated structure mock-ups were also tested. The first one consisted of the inertial mass of the ISMES multi-excitation rig, which weights 9,500 kN, supported at the base on six 500 mm diameter bearings (the vertical load acting on each bearing was thus very close to the prescribed design value of 1600 kN). The mock-up was subjected to snap-back tests, in which the mass was pushed to an initial displacement up to 85 mm, and then released using a collapsible device.

A second mock-up was also fabricated. It weighted 394 kN and was supported on four 125 mm diameter bearings, thus providing a vertical load on each isolator close to the design value of about 100 kN (for the 1:4 scale bearing the vertical load design value is 1/16 of that prescribed for the full scale device). The mock-up was subjected to both snap-back and forced vibration tests on the 6-dof MASTER shaking table of ISMES Structural Testing and Survey Department. All the tests were performed at different shear strain levels in the rubber, up to about 100%. Forced vibration tests included: a) 1D harmonic excitation at varying frequencies in the horizontal, vertical and torsional directions aimed at identifying the natural periods of interest and the related damping values; b) seismic tests with 1D horizontal, 2D horizontal and 3D both horizontal and vertical simultaneous excitations using three Italian earthquake records corresponding to rigid, medium and soft soil conditions.

The mock-ups used in the above mentioned laboratory tests have a very large stiffness above the isolation interface, and thus are able to only reproduce the mass of an actual isolated structure, but not its deformability in elevation. However, this corresponds to neglecting the higher modes above the rigid body modes characteristic of an isolated structure, and this is usually allowed because they have a small influence on the global structural response.

The general purpose of the tests on the isolated mock-ups was to experimentally evaluate the actual global behavior of an isolated system. More specifically, the seismic tests on the MASTER table have been performed in order to analyse the influence of the excitation level and of the frequency content of the earthquake, due to the different soil flexibility, as well as to investigate the effect of the simultaneous application of the seismic excitation in two or three directions. Another objective was to compare the measured response data to the results of single bearings tests, in order to evaluate to which extent the dynamic behavior of an isolated structure can be estimated based on the results of such tests. Finally, it was intended to develop a mathematical non-linear model of the isolation system capable of predicting the mock-up response measured during the tests. Such a model would be of great usefulness in the design and analysis of an isolated construction.

In this paper, after a further brief description of the 394

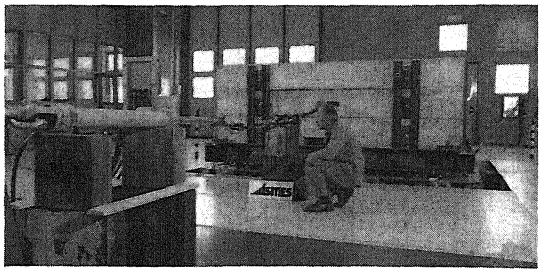

Figure 1. View of the 394 kN mock-up on the table.

kN isolated mock-up and of the instrumentation used to measure its dynamic response, the most significant response data obtained during the snap-back and seismic tests are presented and discussed. A mathematical model capable of taking into account the non-linear behavior of the isolation system is then proposed, and the response predicted through such a model is then compared to the measured experimental data.

2 TESTS DESCRIPTION

Figure 1 shows a view of the 394 kN isolated mock-up on the shaking table. The mock-up is formed by three reinforced concrete blocks, tightly connected to one another and to a very rigid steel frame, to which the upper bearing restraint plates have been soldered. The three concrete blocks measure 3.5×3.5 m^2 in plan and have a total height of 1.2 m.

Each of the isolation bearings is formed by 11 high damping rubber layers of 3 mm height interposed by 10 steel shims 0.75 mm thick. Another steel plate and an external layer of rubber 1.5 mm thick is present on the upper and lower base of the isolators, thus giving a total rubber height of 36 mm. As mentioned before, these isolation devices correspond in the 1:4 scale to the bearings used in the Ancona SIP Buildings. This was the largest scale compatible with the dimensions and capacity of the shaking tables operating in Italy and to the need of driving the tests up to a 100% shear strain in the rubber, i.e. to a horizontal displacement equal to the total rubber height.

Snap-back tests were performed with the shaking table kept steady. The mock-up was pulled toward a fixed stiff frame located outside the table, visible in Figure 1, by means of a hydraulic jack acting at an elevation corresponding to the mass center of gravity. After reaching the desired displacement, the mass was released using a mechanical uncoupling device. Three snap-back tests were performed, at an initial displacement approximately equal to 9 mm, 18 mm and 36 mm, which correspond to a 25%, 50% and 100% shear strain in the rubber, respectively.

The instrumentation network installed to record the mock-up response during the snap-back tests and the location of the single transducers was designed so as to be able to completely describe the 3D motion of the mass, considered as a rigid body. Three displacement transducers to detect the motion in both horizontal and in the vertical direction were placed at each bottom corner of the mass on the same side of the applied pulling force. A seventh displacement transducer in the vertical direction was positioned on the opposite side. Furthermore, a three-axial accelerometer was located at the center of the top side of the mass, for a more complete description of the motion and to check the reliability of the displacement signals. The pressure in the jack, and thus the applied force, was recorded during the entire pulling phase.

Three complete sets of acceleration records obtained at different recording stations during recent Italian earthquakes have been used for the seismic tests. They are: the San Rocco and Tolmezzo records of the 1976 Friuli earthquake, and the Calitri record of the 1980 Campano-Lucano earthquake. These records have a different predominant period, mainly because of the different soil flexibility conditions at the recording site. During the tests, the shaking table was controlled through accelerations instead of displacements, because the correction procedure gives an acceleration time history which is more reliable than the displacement one. Despite of this, and due to the excellent control system of the MASTER table, the actual table acceleration time histories were always very close to those prescribed, particularly in the low frequency range of interest.

To conserve the ratio between the inertia forces acting on the mock-up and the restoring forces in the isolators corresponding to a structure isolated using the full-scale bearings, the time scales of the acceleration records were contracted by a factor of 2, equal to the square root of the geometric scale factor (Schuring 1977). The accelerograms have been applied in one horizontal, and also in both horizontal and both horizontal and vertical directions simultaneously, to obtain data regarding the bi- and tri-axial interaction effects on the isolation bearings. Moreover, to study the non-linearity of the response, each test was repeated more times scaling the applied records at different levels (from -20 up to 15 dB).

For the seismic tests, the recording instrumentation already used during the snap-back tests was improved. Three displacement transducers and a three-axial accelerometer were added to record the motion of the table itself, i.e. below the isolators. Another displacement transducer and four more acceleration channels were placed on the mass, so as to be able to describe its 3D motion not only in terms of displacement but also of acceleration. The transducers located above the isolators detected the displacements relative to the table, while the accelerometers measured absolute accelerations.

3 ANALYSIS OF THE RECORDED RESPONSE

Figure 2 shows the displacement time history in the direction of the applied pulling force recorded during the most severe snap-back test, i.e. the one from an initial displacement of 36 mm, corresponding to a 100% shear strain in the rubber. The signal has been obtained taking the average of the two displacement time histories recorded in the same direction at the two bottom corner of the mass, which were anyhow very similar one to the other. This fact, together with the observation that the displacements in the other horizontal direction were almost negligible, indicates that the mock-up practically moved only in the same direction of the applied pulling force after mass release. Analysis of the displacement signals in the vertical direction indicated a very small amount of rocking around a horizontal axis normal to the loading direction.

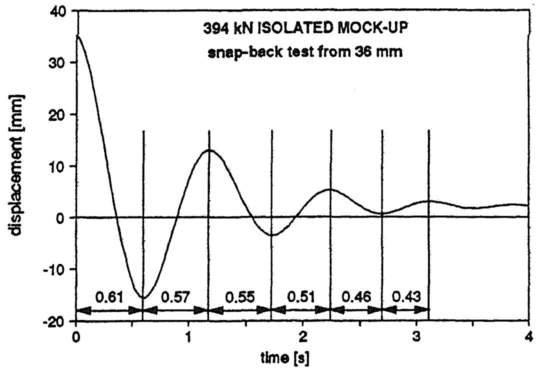

Figure 2. Displacements in the 36 mm snap-back test.

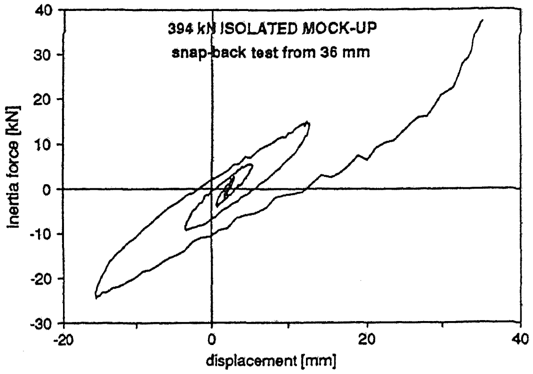

Figure 3. Hysteresis loops in the 36 mm snap-back test.

As expected, the curve in Figure 2 indicates a free damped response, with a significantly high value of damping. The motion ceases a few seconds after mass release, and only three or at most four complete oscillations are clearly visible. The figure shows a decrease of the time interval between two subsequent peaks during the oscillation, and this is due to the non-linear behavior of the isolators, which display an increasing horizontal stiffness with decreasing displacement. Another effect immediately apparent is the incomplete recovery of the initial deformation at the end of the oscillations: immediately after the test the mock-up does not come back to the position corresponding to zero displacement, but a residual displacement is observed. This has to be attributed to the creep phenomenon in the rubber. It is worth to mention that in the first hours after the end of the test, the residual displacement decreases, and seems to tend asymptotically toward a value greater than zero. In other words, recovery of the initial deformation is not complete, while a complete recovery is usually observed in a sample of natural rubber only.

In Figure 3 the inertia force acting on the mock-up in the direction of the applied force is plotted as a function of the horizontal displacement. The inertia force is obtained from the acceleration signal, and corresponds to the total restoring force developed by the four bearings. The shape of the hysteresis loops is very similar to that obtained during the cyclic tests performed on the single bearings by applying a harmonic horizontal displacement, and the increase of stiffness with the decrease of the maximum displacement in each cycle is also immediately apparent. Results very similar to those shown in Figures 2 and 3 have been obtained for the other snap-back tests from a lower level of initial deformation.

Like in the snap-back tests, also in the 1D horizontal seismic tests the mock-up practically moved only in the excitation direction. The shaking table and the mass acceleration time histories obtained by applying the 1D San Rocco NS record at 0 dB (unscaled acceleration values) are shown in Figure 4. The mass acceleration has been obtained by averaging the two acceleration signals recorded on the mock-up in the excitation direction, which were almost identical. A significant reduction of the motion through the isolation system is observed: peak acceleration changes from 0.667 m/s^2 on the table to 0.102 m/s^2 on the mock-up, with a diminution of 84.7%. In the figure, the frequency filtering effect caused by the isolators is also immediately apparent: the San Rocco record has a very short predominant period, but the mock-up responds according to its free vibration period.

Similar results are obtained for the 1D Tolmezzo WE record at 0 dB (Figure 5). Despite the longer predominant period of the input signal, a large reduction of the motion amplitude is still present, and the peak acceleration is reduced from 3.85 m/s^2 on the table to 0.425 m/s^2 on the mock-up, corresponding to a 90.0% diminution. Figure 5 also shows the mock-up acceleration time history experienced in the same direction under the 2D Tolmezzo

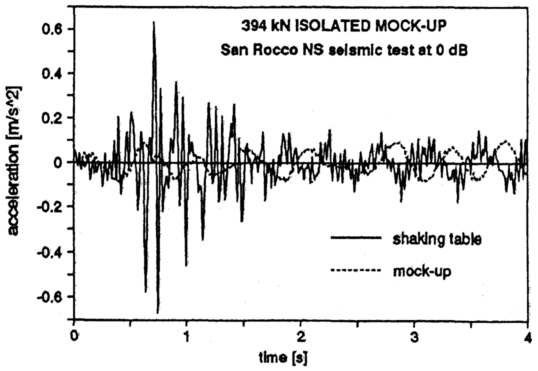

Figure 4. Accelerations in the San Rocco NS test at 0 dB.

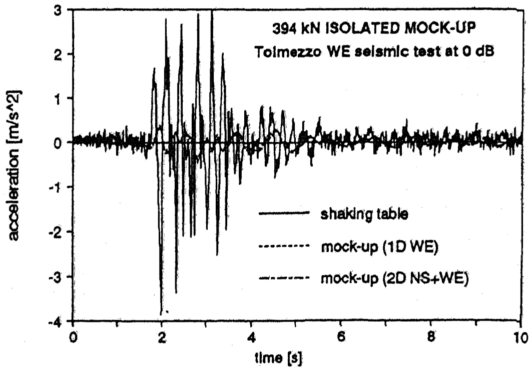

Figure 5. Accelerations in the Tolmezzo WE test at 0 dB.

NS+WE record at 0 dB, i.e. by applying the two horizontal earthquake records to the shaking table simultaneously. The mass response is very close to the one obtained during the corresponding 1D test, and this indicates that the bi-axial interaction effects on the isolation bearings are very small. The same result has been obtained in all the other 2D and 3D tests, and represents an important outcome of the experimental campaign. As an immediate consequence of this finding, it is correct to say that a 3D earthquake analysis of a structure isolated through HDLRBs, in absence of an eccentricity between the superstructure center of gravity and the center of stiffness of the isolation system, may be more easily reduced to three 1D analyses, one for each of its principal directions.

The Calitri records are characterized by longer predominant periods, because of the flexible soil condition at the recording station. When this earthquake is applied to the table, the isolation system is less effective in reducing the input motion transmitted to the structure. For the 1D Calitri WE record at 1 dB (acceleration values amplified by factor 1.12), the mock-up responds with a peak acceleration of 0.851 m/s^2, versus a peak table acceleration of 2.09 m/s^2. This corresponds to a 59.3% reduction, which is smaller than those obtained for the other two earthquake records, but still significant.

As said above, each seismic tests was repeated more times scaling the applied record at different excitation levels. Despite the typical non-linear behavior of the isolation bearings, comparison of the mock-up motion under the same earthquake record, with acceleration values scaled by different factors, has shown that the peak values of the response varies almost linearly with the scaling factor.

4 A MATHEMATICAL NON-LINEAR MODEL

The dynamic response of an isolated structure is strongly influenced by the mechanical properties of its isolation system. HDLRBs exhibit a non-linear behavior particularly at small level of deformation, while the structure above the isolation interface is usually designed so as to remain linear elastic under the design earthquake ground motion. It is therefore of primary importance to devise a mathematical non-linear model of an isolation system formed of HDLRBs, capable of predicting the complete structural response. The availability of the experimental data regarding the mock-up response during snap-back and shaking table tests, as well as tests on single bearings, represents an excellent opportunity for the thorough validation of such a model.

The dynamic sinusoidal tests performed at ISMES laboratories on single bearings have shown that the energy dissipated in each cycle depends on the maximum deformation, but is almost independent from the rate of strain application (Martelli *et al.* 1991b). This observation is in agreement with the results obtained by other researchers in USA and in Japan on similar type of bearings, but it must be said that is valid only in the frequency range characteristic of an isolated structure, as a certain influence of the strain rate has been noticed in rubber specimens when wider frequency ranges have been investigated.

Because of the above observation, a hysteretic damping behavior has been assumed for the isolators. The high damping elastomer used for the bearings is a carbon black-loaded natural rubber vulcanizate. Dynamic shear tests performed on small rubber specimens (Martelli *et al.* 1991a) show hysteresis cycles which are perfectly superimposed during repeated cycles, with practically no material degradation. The tangent shear modulus of the material G_t decreases with increasing strain γ, rapidly in the small deformation range from the initial value G_o, and then tends asymptotically to a value G_∞. Based on this experimental evidence, the variation of G_t with γ is expressed by the two equations, valid for a loading and an unloading curve:

$$\text{loading:} \qquad G_t(\gamma) = \frac{d\tau}{d\gamma} = G_\infty + a\,e^{-b(\gamma-\gamma_{\min})} \tag{1}$$

$$\text{unloading:} \qquad G_t(\gamma) = \frac{d\tau}{d\gamma} = G_\infty + a\,e^{-b(\gamma_{\max}-\gamma)} \tag{2}$$

where $G(0) = G_o = G_\infty + a$. At large shear strains, say above 100%, an increase of the shear modulus has been observed, due to the crystallization under deformation of the natural rubber matrix. The formation of the crystals is extremely rapid, and they disappear as soon as the strain is reduced. This phenomenon has not been accounted for in the mathematical model, because of the lower strain levels reached during the tests. Integrating (1) and (2), the shear stress is obtained for the loading and unloading curves:

$$\tau(\gamma) = \tau(\gamma_{\min}) + G_\infty(\gamma-\gamma_{\min}) + \frac{a}{b}\left[1 - e^{-b(\gamma-\gamma_{\min})}\right] \tag{3}$$

$$\tau(\gamma) = \tau(\gamma_{\max}) - G_\infty(\gamma_{\max}-\gamma) - \frac{a}{b}\left[1 - e^{-b(\gamma_{\max}-\gamma)}\right] \tag{4}$$

When a laminated elastomeric bearing is displaced horizontally by a quantity x, a uniform state of pure shear is added in the rubber, and the additional strain is equal to $\gamma = x/t_r$, where t_r is the total rubber thickness. Denoting with A the cross-sectional area of the isolator, the horizontal restoring force acting in the bearing is then given by:

$$F_R(x) = F_R(x_{\min}) + \frac{G_\infty A}{t_r}(x-x_{\min}) + \frac{aA}{b}\left[1-e^{-\frac{b}{t_r}(x-x_{\min})}\right] \tag{5}$$

$$F_R(x) = F_R(x_{\max}) - \frac{G_\infty A}{t_r}(x_{\max}-x) - \frac{aA}{b}\left[1-e^{-\frac{b}{t_r}(x_{\max}-x)}\right] \tag{6}$$

for the loading and unloading curves, respectively. The above equations neglect the effects of bending deformation and of the vertical compressive load, but can be successfully applied if the shape factor of the bearings is sufficiently high and the axial force is well below the critical load, and both conditions are verified in our case. When more that one bearing act in parallel, as it happens in an isolated structure which moves horizontally without torsion, the total restoring force is equal to the one generated in each isolator multiplied by the number of bearings.

From equations (5) and (6), it is easy to derive the secant stiffness of an isolator subjected to hysteresis cycles of maximum amplitude $\overline{x}$:

$$K_s(\overline{x}) = \frac{G_\infty A}{t_r} + \frac{aA}{2b\overline{x}}\left(1 - e^{-\frac{2b}{t_r}\overline{x}}\right) \tag{7}$$

as well as the energy dissipated in each cycle:

$$E_d(\overline{x}) = \frac{aA}{b}\left[2\overline{x}\left(1 + e^{-\frac{2b}{t_r}\overline{x}}\right) - \frac{2t_r}{b}\left(1 - e^{-\frac{2b}{t_r}\overline{x}}\right)\right] \tag{8}$$

and the corresponding equivalent viscous damping ratio:

$$\nu(\overline{x}) = \frac{1}{2\pi}\frac{E_d(\overline{x})}{K_s(\overline{x}) \cdot \overline{x}^2} =$$

$$= \frac{1}{2\pi}\frac{\frac{aA}{b}\left[2\overline{x}\left(1 + e^{-\frac{2b}{t_r}\overline{x}}\right) - \frac{2t_r}{b}\left(1 - e^{-\frac{2b}{t_r}\overline{x}}\right)\right]}{\frac{G_\infty A}{t_r}\overline{x}^2 + \frac{aA}{2b}\overline{x}\left(1 - e^{-\frac{2b}{t_r}\overline{x}}\right)} \tag{9}$$

5 VALIDATION OF THE MODEL

The values of the rubber parameters G_∞, a and b which allow the best prediction of the dynamic behavior observed during the laboratory tests on single bearings, as well as the snap-back and seismic tests, are:

$$G_\infty = 0.631 \text{ MPa} \quad a = 1.61 \text{ MPa} \quad b = 6.30$$

Figure 6 shows the comparison between the hysteresis cycles obtained during a dynamic sinusoidal test on a 1:4 scale bearing at a shear strain amplitude of 50% and the cycle predicted through the mathematical model. The first loading analytical curve has been obtained from the cyclic loading curve through a similarity transformation of ratio 0.5, thus following Masing's rule. Although the agreement between the experimental and analytical curves is excellent - and results at the same level of accuracy have been obtained for the dynamic sinusoidal tests on bearings of other scales adopting the same values for the rubber parameters - it must be noticed that the model slightly underestimates the bearing tangent stiffness at the very beginning of a loading or unloading curve. As a consequence of this, the energy dissipation capacity of the model at small values of deformation is less than the one observed experimentally. This fact is confirmed if the model $\nu(\overline{x})$ function given by equation (9) is compared to the experimental results: the agreement is good for cycles of strain amplitude equal or larger than 20%, but $\nu(\overline{x})$ decreases and tends to 0 when $\overline{x} \rightarrow 0$, while an almost linear increase of the equivalent viscous damping ratio is observed experimentally with decreasing shear strains. The problem of having no energy dissipation at zero strain is common to all the possible hysteretic models (Wen 1976), but may be attenuated if a constitutive law more complex than the one indicated in (1) and (2), and thus with more than three parameters, is assumed for the rubber.

A computer program to compute the non-linear response of a single-dof system with a hysteretic restoring force given by equations (5) and (6) subjected to free or forced excitation has been written. Computation of the response is performed numerically by step-by-step integration of the equation of motion. The numerical scheme adopted by the program is the linear acceleration method, in which the system total acceleration is assumed to vary linearly, while the tangent stiffness is considered constant, during the time increment. When the system skips from a loading to an unloading curve, or viceversa, large changes of the tangent stiffness occur, which may result in substantial equilibrium violations if a constant time increment is used. To avoid the problem, the program automatically reduces the time interval of integration when the velocity sign changes, so as to achieve the desired level of accuracy in satisfying the dynamic equilibrium equation.

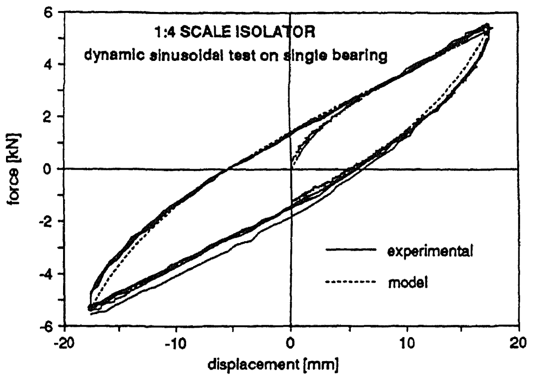

Figure 6. Experimental and model hysteresis loops in a dynamic sinusoidal test on a single bearing.

The mock-up displacement response during the snap-back test from 36 mm computed by the program is compared to that actually recorded in Figure 7. A constant residual displacement equal to the one observed just after the end of the oscillations has been considered in the numerical simulation. The model is able to predict the change in stiffness with decreasing displacement as well as the level of energy dissipation, except in the final portion of the response curve because of the already mentioned insufficient damping capacity of the model at very low levels of deformation. Furthermore, the amplitude of the first peak is overestimated in the simulation: this was observed also when the snap-back response of the 9,500 kN mock-up was simulated by means of an equivalent linear and a viscously damped non-linear model for the isolation system (Serino 1991), and may be explained considering that the residual displacement should not be assumed constant over the entire duration of the test. Because of the creep behavior of the elastomer, it is reasonable to suppose that the residual displacement is slightly larger in the very first instants after the mass release, generating a restoring force on the mock-up smaller than the one computed by the model and thus producing a first excursion of smaller amplitude.

The analytically computed hysteresis loops which represent the total inertia force as a function of the displacement during the same snap-back test are provided in Figure 8. The cycles may be almost perfectly superimposed to those observed experimentally, already shown in Figure 3. Similar results have been obtained in the simulation of the other two snap-back tests.

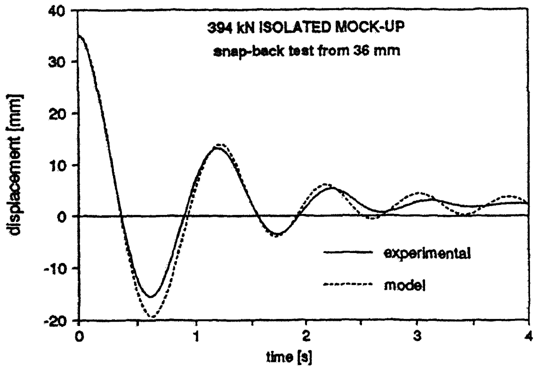

Figure 7. Experimental and model displacement time histories in the 36 mm snap-back test.

The acceleration time history of the mock-up subjected to the 1D Tolmezzo WE record at 0 dB computed by program is compared to the one actually recorded in Figure 9. The agreement between the two curves is very good. Correlations of the other response quantities show that the model is able to accurately predict the kinetic energy, energy absorption and total energy input, as well as the displacements relative to the table, although it slightly overestimates the peak displacement values.

6 CONCLUSIONS

Snap-back and seismic excitation tests performed on a 394 kN isolated mock-up have confirmed that HDLRBs allow a very effective reduction of the earthquake response of a structure. The non-linear behavior of the isolators, already noticed in laboratory tests on single bearings, was clearly revealed also in the snap-back tests, in which the motion was rapidly reduced because of the intrinsic high damping capacity of the isolators.

Significant reductions of the peak acceleration - up to 90% - have been observed during the seismic tests, which have also demonstrated the small influence of the bi- and tri-axial interaction effects in the isolation bearings. The effectiveness of the isolation system was of course strongly dependent on the frequency content of the input earthquake. Despite the typical non-linear behavior of this type of isolators, the peak response values varied almost linearly with the excitation.

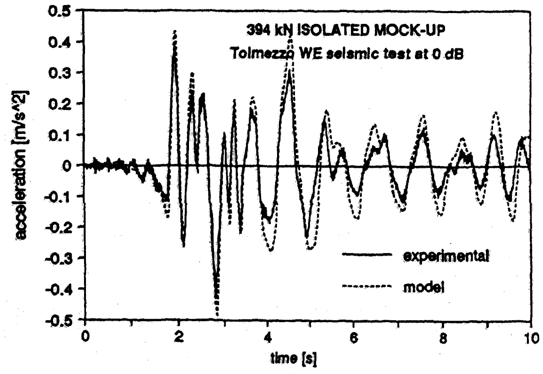

Figure 9. Experimental and model acceleration time histories in the Tolmezzo WE seismic test at 0 dB.

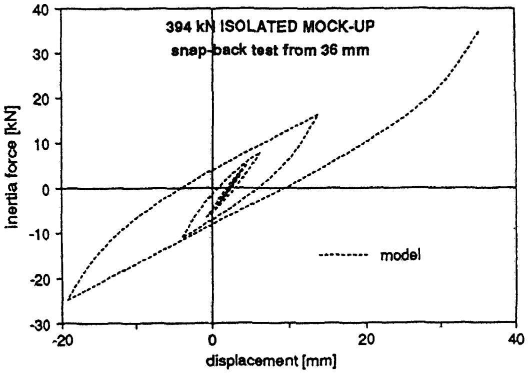

Figure 8. Simulated hysteresis loops in the 36 mm snap-back test.

The proposed mathematical non-linear model of the isolation system is able to predict with very good accuracy the response observed during the snap-back and seismic tests, as well as the hysteresis loops obtained during the laboratory tests on single bearings. However, the model underestimates the energy dissipation capacity of the isolators at very low level of deformation, but this deficiency has practically no consequences in the simulation of the dynamic response under the earthquake levels usually considered in the analysis and design of an isolated construction. The model requires the correct selection of three parameters only, which on the other hand can simply be obtained from tests on rubber specimens.

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