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Spectrum analysis of nonlinear isolated structures

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ABSTRACT : In this paper, nonlinear behavior of isolated shear structures which rest on lead-rubber bearings against seismic loads is studied. To do this, an equivalent linear system which one degree of freedom, which has a maximum response approximately equal to the actual system, is introduced. In order to reach such a linear system, period of the nonlinear system, before yielding of the lead plug, should be multiplied by a greater than one factor and then a proper damping ratio should be found. A spectrum is used that from which for various degrees of freedom, period shift factor and related damping may be determined. Because the system has one degree of freedom, using the above values and design earthquake spectrum, maximum reaction of the actual nonlinear system can be simply evaluated. Using this method, dynamic analysis of a nonlinear isolated system, turns to spectrum analysis of a linear SDOF oscillator which can be done easily. For comparison, some graphs consisting of exact nonlinear analysis and equivalent linear analysis are added to show their agreements.

INTRODUCTION

Behavior os superstructure of an isolated structure is linear even under major earthquakes, but elastomeric bearings on which superstructures rest, have commonly bilinear behavior. The bearings are made of successive layers of rubber and steel with a lead plug which causes bilinear behavior in violent earthquakes (Fig. 1).

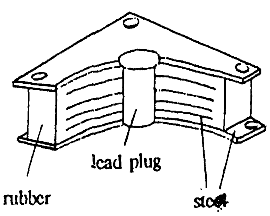


Fig.1 - bilinear bearing

While the shear force of earthquake is less than the yielding limit of lead plug, bearings will remain in their first linear region. But, when the force exceeds a particular value, the lead plug yields and lateral stiffness of bearings reudces greatly. Plastic behavior of the lead core causes an increase in the area of hysteresis loop and diminishes a large part of the destructive energy of earthquake. This, accompanied with decrease in lateral stiffness of the system, cause a great loss in the exerted force of earthquake on the structure and also base displacement.

As a consequence of lateral flexibility, frequency of the isolated structure shifts out from effective frequency range of many earthquakes. The other advantage is that all of plastic deformations accumulate on bearings, and no damages are exerted on structural elements of the building even in violent earthquakes. With respect to Fig. 2, before yielding of lead, bearing stiffness is almost equal to shear stiffness of lead plug, K_1, and after that, equals to lateral stiffness of rubber layers, K_2. Such a bearing is called bearing with *bilinear behavior , or simply, bilinear bearing.*

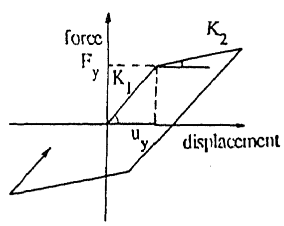


Fig.2 - bilinear behavior

In practice, bilinear bearings are designed for yielding in the force F_y = 0.05 W,(W = Structure weight), the ratio of secondary to primary stiffness : K_1/K_2 = 0.15 and T_b = 1 sec or $\omega_b = 2\pi$ based on Constantinou & Kneifati (1987) in which :

$$\omega_b^2 = K_1/M = K_1 g/W \tag{1}$$

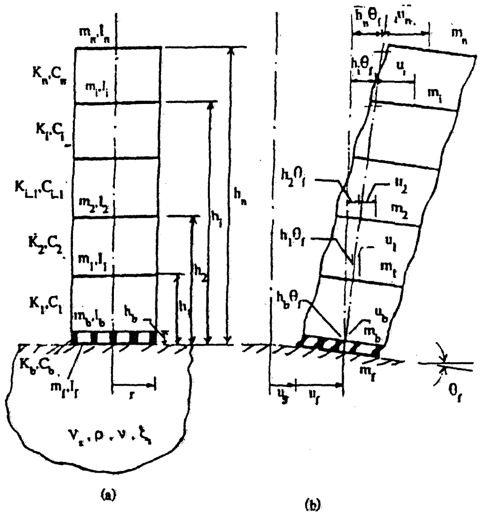
$$Y = F_y/W = F_y/Mg \tag{2}$$

$$u_y = Yg/\omega_b^2 \tag{3}$$

Due to analysis of a nonlinear energy-absorbing system like the above mentioned, the best method is step by step integration in time domain, noting that frequency domain analysis of the nonlinear system is not possible. With this method, time history of structure reaction could be determined. But for design purposes, this is not needed, it is only sufficient to know maximum and minimum values in order to design the structure properly. In spectrum analysis of earthquake resistnat structures, maximum values of reaction are obtained from an elastic design spectrum. Hence, to calculate the response spectrum of a nonlinear isolated system , an equivalent elastic system (from an spectrum viewpoint) should be found and its dynamic characteristics (equivalent frequency $\tilde{\omega}$ and equivalent damping ratio ξ) should be evaluated. Presenting a linear elastic system which is equivalent to the bilinear system, forms the basis of this paper.

DETERMINATION OF EQUIVALENT LINEAR SYSTEM PARAMETERS

So far, some methods, to present dynamic characteristics of a linear system that is equivalent to a nonlinear system in spectrum viewpoint has been demonstrated by Tsia & Kelly (1989) and Iwan & Gates (1979),. In all of these methods, the system has been considered single degree of freedom and soil-structure interaction has not been considered. So using the above methods, there are no concepts of effects of number of inelastic system dgrees of freddeom or soil flexibility on its behavior. On the other hand, exact analysis and determining time history of reaction of a multi-degree-of-freedom system with interaction of soil and structure effects would be very difficult and time consuming. Here, to overcom these problems, we use the method presented by Behnamfar (1990). Basis of this method is shown in Figs. 3 and 4.



(a) (b)

Fig3 - System considered ;
(a) isolated n-story building
(b) displacement against earthquake

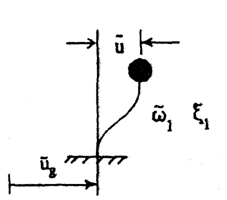


Fig.4- Equivalent one degree of freedom system

Fig. 3 shows the behavior of an n-story isolated shear building which rests on flexible soil during earthquake sttack. In these Figures, structure foundation is a rigid circular mat with radius r, depending on surface of a homogeneous halfspace of density ρ shear modulus G , poison ratio ν , and V_s is shear wave velocity in half space.

Fig. 4 demonstrates a single degree of freedom system with frequency $\tilde{\omega}_1$ and damping ratio ξ_1 which equivalent ground displacement at its base is $\ddot{u}_g$. Value of this movement depends on our aim in every stage of calculations. In other words, we can conduct it so that maximum displacement of the SDOF mass in each step of loading would be equal to one of the soil or structure displaements. For this reason, an equivalent excitation coefficient is defined, which must be affected on the ground acceleration. Applying system equilibrium relations of Figs. 3 and 4 and their comparisons, can be shown as :

$$\tilde{\omega}_1=\frac{\tilde{\omega}}{\sqrt{1+(\tilde{\omega}r/V_s)^2(M/\rho r^3)[(2-\nu)/8+[3(1-\nu)/8\times(h/r)^2]]}} \qquad (4)$$

where :

$$\tilde{\omega}=\sqrt{M/2m_b\times(\omega_*^2+\omega_b^2)[1-\sqrt{1-4m_b/M\times(\omega_*\omega_b/(\omega_*^2+\omega_b^2))^2}]} \qquad (5)$$

$$h=\frac{\sum_{i=1}^{n} m_i\Phi_{i1}h_i}{\sum_{i=1}^{n} m_i\Phi_{i1}} \qquad (6)$$

$$m=\frac{(\sum m_i\Phi_{i1})^2}{\sum m_i\Phi_{i1}^2} \qquad (7)$$

$$\omega_*=2\pi/0.1n \qquad (8)$$

$$M=m+m_b \qquad (9)$$

$$\omega_b=\sqrt{k_b/(m+m_b)} \qquad (10)$$

In above equations, $\tilde{\omega}_1$ is the frequency of equivalent system of Fig. 4; n, number of stories (Fig.3) ; m, effective mass of superstructure of Fig. 3 in first vibration mode; k_b, total of shear elastic stiffness (K_1 in Fig. 2) of bearings in horizontal direction; Φ_{i1} , modal displacement of the ith story in the first mode (Fig.3) and h is distance between applying point of inertial forces during earthquake in the first mode of vibration and center of foundation. Note that in isolated structures, the first mode is predominant (Behnamfar (1990)). For preliminary design of a system, it is necessary to estimate the structure movements.

Now, we attempt to find a linear system which has maximum seismic reaction values, almost equal to the same values of a system with bilinear behavior. Because yielding of lead plug is equivalent to decrease in bearing stiffness and increase in its period, the thought that comes to mind is to multiply a factor by structure main period before yielding of bearings. In other words, structure period should be shifted and then for analyzing it, a proper damping ratio should be found. It is evident that this period and damping ratio should be so that, as spectrum point of view, reactions would be identical in actual and equivalent systems. Relation between period increasing factor and damping ratio for elastoplastic systems with various degrees of freedom in given by Srivastav and Nau (1988) and we have used that spectrum in this paper (Fig. 5).

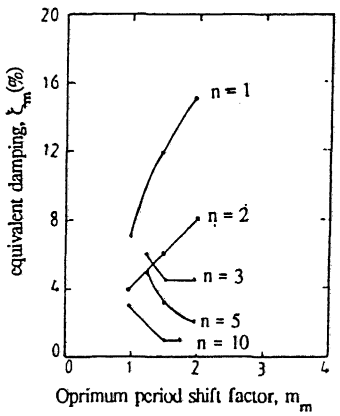


Fig.5 - Optimum period shift factor and damping for equivalent lindarized systems.

Behavior of bearing shown in Fig. 3 is bilinear, not elastoplastic. But because the system ratio of secondary to primary stiffness is about 0.15, it can be estimated that considering above spectrum will not contain great errors. Every curve on Fig. 5 is related to a particular number of stories. The three points on every curve, from the left side, are related to ductility factor (μ) 2, 5 and 10, respectively.

The Figure clearly shows that the ductility factor is proportional to the value of m. This result proves that inelastic behavior tends to have an increase in structure main period. Consequently, a factor to the main period of soil-structure system should be applied (this period could be evaluated, by considering that structures are linear and elastic) and its damping ratio due to number of stories and expected ductility of bearings should be chosen. Thus, analyzing a nonlinear system may be done in a linear way and getting the same spectrum values which could be obtained from exact solution of the nonlinear system.

Now ,with respect to Fig. 4, the equivalent linear structure may be changed into an equivalent oscillator with single degree of freedom, by a simple solution which may be done by hand. Therefore calculation algorithm should be as follow :

Nonlinear n-degree-of-freedom system → equivalent linear system with n degrees of freedom → equivalent linear system with single degree of freedom.

ALGORITHM :

The following is the summary of the above calculation method :

1. For considered isolated structure, superstructure main frequency (ω) from Eq. (8), and h from Eq. (6) and the ratio of the total mass to base mass, or M/m_b should be evaluated.

2. From Eq. (1), a value for ω_b should be chosen . An optimum value for ω_b is about 2π. From ω_b , T_b is obtained ($T_b \cong 1$ sec). Then the ratio of the yielding force of bearings to the structure weight (Y in Eq. 2) should be selected. The best values are in the range of 0.05 to 0.10.

3. u_y from Eq.3 is obtained.

4. $\tilde{\omega}$ from Eq.5 is evaluated.

5. Shear wave velocity of soil (V_s) with respect to soil type should be determined.

6. $\tilde{\omega}_1$ from Eq.4 is obtained and $\tilde{T} = 2\pi/\tilde{\omega}_1$.

7. A value for μ is assumed and m and ζ due to number of stories of the structure from Fig. 5 is evaluated.

8. Knowing $\zeta_e = \zeta$ and $\tilde{T}_e = m\tilde{T}$, maximum response of system (u_{max}) is obtained by using a design spectrum of single degree of freedom system.

9. Evaluate $\tilde{\omega}_e = 2\pi/\tilde{T}_e$ and equivalent excitation factor (EIF) for computing u_b (Fig.3) which is given by :

$$EIF = (\tilde{\omega}_e / \omega_b)^2 \tag{11}$$

10. Multiply maximum response of the system (u_{max}) by equivalent excitation factor in order to get maximum base displacement (u_{bmax}).

11. From the below equilibrium, new μ is computed :

$$\mu = u_{bmax}/u_y \tag{12}$$

12. With the new μ, redo the calculation from step 7 until difference between two successive μ's is negligible. At this time, the last u_{bmax} is maximum displacement of the base.

13. Using relations between displacements of the stories, the base shear of structure could be determined by the reference by Behnamfar (1990).

Example :

In this part the above algorithm is applied and for two samples, 5-stroy and 10-story buildings, the results are compared.

5-stroy building data :

$$\left\{\begin{array}{l} n=5 \quad \omega_s=2\pi/(0.1n)=2\pi/0.5=12.57 \text{ rad/sec} \\ M/m_b=6m/m_s=6,\ T_b=1 \text{ sec} \quad \omega_b=6.28 \text{ rad/sec} \end{array}\right. \Longrightarrow \tilde{\omega}=5.70 \text{ rad/sec}$$

$$u_y=Yg/\omega_b^2 \quad \left\{\begin{array}{l} Y=0.05 \quad u_y=0.05\times 981/(2\pi)^2=1.24 \text{ cm} \\ Y=0.10 \quad u_y=0.10\times 981/(2\pi)^2=2.48 \text{ cm} \end{array}\right.$$

$$EIF=(\tilde{\omega}_e/\omega_b)^2=(2\pi/\tilde{T}_e.\omega_b)^2=1/\tilde{T}_e^2$$

10-story building data :

$$\left\{\begin{array}{l} n=10 \quad \omega_s=6.28 \text{ rad/sec} \\ M/m_b=11,\ T_b=1 \text{ sec} \quad \omega_b=6.28 \text{ rad/sec} \end{array}\right. \Longrightarrow \tilde{\omega}=4.49 \text{ rad/sec}$$

$$\left\{\begin{array}{l} Y=0.05 \quad u_y=1.24 \text{ cm} \\ Y=0.10 \quad u_y=2.48 \text{ cm} \end{array}\right. \Longrightarrow EI\,F=1/\tilde{T}_e^2$$

Now , using exact solution from Constantinou & Kneifati (1987), and values from Behnamfar (1990) are shown on Fig. 6 and compared with exact values. Horizontal axis of these graphs shows the type of soil which is identified with the parameter σ :

$$\sigma = 2\pi V_s/\omega_s h \tag{13}$$

From the graphs, maximum error occurs in 5-story building and equals 40% which considering the use of elastoplastic spectrum, it is an acceptable result.

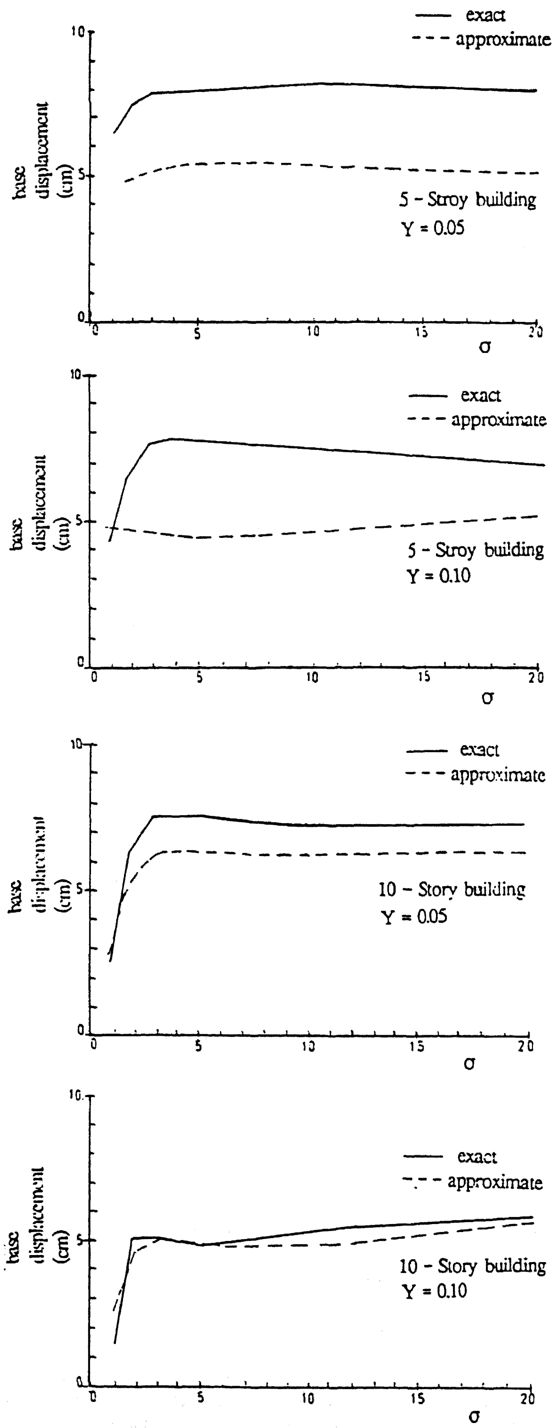


Fig.6 - Spectral dispalcement of the base of the nonlinear isolated structures with exact and approximate solutions

CONCLUSION

Applying isolation system in isolated structures, results in large displacement in the bases of buildings. Therefore bearings with lead plug are used in order to increase energy absorption level and decrease in base displacements. Behavior of these bearings are bilinear and to analyze the system in this condition, nonlinear analysis should be used. But in this paper, a simplified method for spectrum analysis of such systems is introduced which consists of two steps. First, defining an equivalent linear system for the actual nonlinear system. Second finding an equivalent linear SDOF oscillator for the recent linear system.

First step, introduced the spectrum which relates period shift factor to damping ratio for nonlinear systems with various degrees of freedom. The factor is applied on the primary period (elestic behavior region) of the isolated structure to gain an equivalent linearized system with the above damping ratio.

In the second step, and oscillator with one degree of freedom equivalent to the linear system is determined. Here, two systems having identical displacement spectrums by equivalence is meant. Therefore by this method, a single degree of freedom vibrator is gained, which has similar displacement spectrum to the actual nonlinear system. Finally, the actual system can be analyzed by using single degree of freedom systems design spectrum. Examples showed that using bilinear bearings instead of bearings with linear behavior, reduced the base displacements to less than 10 cm, and so greatly decreases the practical problems of the isolation systems.

Although in the first step of the above calculations, spectrum of elastoplastic systems was used, accuracy was still acceptable. To gain better results, a good research job can be, " finding bilinear systems spectrum ". Such a spectrum relates shift period factor to damping ratio for structures with various numbers of stories.

REFERENCES

F. Behnamfar 1990. Study of approximate and exact methods for dynamic analysis of base isolated structures. MSc. Thesis Faculty of Engineering, Tehran University.

M.C. Constantinou & M.C. Kneifati 1987. Dynamics of soil-base-isolated structures systems; 3.Nonlinear systems. Report to NSF .

Iwan & Gates 1979. The effective period and damping of a class of hysteretic structures. Earthquake Engineering and Structural Dynamics, Vol.7; 199-211.

Srivastav & Nau 1988. Seismic analysis of elastoplastic MDOF structures. Journal of Structural Engineering; Vol. 144; 1339-1353.

Tsia & Kelly 1989. Dynamic parameter identification for non-linear isolation systems in response spectrum analysis. Earthquake Engineering and Structural Dynamics, Vol. 18; 1119-1132.