Bases of engineering analysis of elastometric seismic insulators

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ABSTRACT: Bases of engineering analysis of heavy-loaded layered seismic-and-vibration insulators (SVI) have been developed, that allow for nonlinearity, rheological effects, a form and number of SVI layers. Besides it allows to determine deformations and frequencies of natural vibrations under load and optimize SVI dimensions. Both good static and dynamic correlations between the analysis and experimental results are observed.

I DISCUSSION AND OBSERVATION

The most effective means of seismic and vibration isolation of highly responsible structures and people in them is the application of various structures based on high polymers, rubbers, in particular, that are placed between the structure and foundation (Tajiran F.P., et al., 1990; Derham C.J., et al., 1989). Effective computer-based engineering analysis has been in parallel developed. In comparison with sufficiently awkward numerical methods, i.e. finite-and-boundary elements methods, a proposed engineering method of analysis (Dashevskiy M.A., 1997; Dashevskiy M.A. et al., 1990) still keeping high accuracy apparently demonstrates the effects of elastic rubber constants, elements shape, and the number of layers; also it permits to consider nonlinearities in the analysis and rheological effects under high stress compression. Optimization of SVI dimensions may be produced with an aid of the proposed method based on simple relationships.

II METHODS OF ANALYSIS

2.1 Basic assumptions of phenomenological method of analysis:

I. The process of deformation is considered one-dimensional; shape coefficient $K_f = 1 + \gamma F/F_0$ is used to take account of the partial character of stress state, providing

$$E = E^M K_f$$

$E$ - modulus of elasticity of a product;

$E^M$ - material modulus of elasticity.

2. As a measure of deformation value $S = \Delta e/(H_o - \Delta x)$ is used, where $\Delta x$ is a settlement under load at t moment, that allows to make use of relationship of Hooke's law type $\sigma = E^o S$ for the domain of nonlinear deformations (Grigorjev Ye.T., 1960).

3. Rheological rubber properties with allowance for new measure of deformation $S$ are described either by the relation for Kelvin's standard body

$$K_f E^o / \gamma + K_f E^M S = \gamma \sigma_o + \sigma'$$

or by exponential nucleus of creeping by Rehanitsin A.R.

$$K(t) = n \gamma E^{-\beta t}$$

where $\sigma_o$ is compression stress $E^o$ and $E^M$ - an instantaneous and equilibrium moduli of elasticity, $\gamma = E^M / E^M$ - initial coefficient of shape, $n = n \gamma \sigma_o / (E^o K_f)$.

4. Frequencies of SVI natural vibrations under load are determined for the compressed state of SVI with corresponding shape coefficient $K_f^2$.
2.2 In the assumption that rheological processes have ceased \( (t \to \infty) \) SVI stiffness with vertical vibrations is defined by the expression:

\[
C = \frac{2P}{A} = \frac{F_o E_o}{Ho} \frac{H_o}{(H_o - \Delta t)}^2 \tag{5}
\]

where \( E_o = \frac{\kappa}{E_o} \), \( k = \frac{1}{1 + A(s+1)^4} \).

Substituting (2), (5) in the expression for natural frequency of one-degree-of-freedom system vibrations \( \omega_0^2 = C \), we obtain the formula for the frequency of free vibrations of SVI under load:

\[
\omega^2 = \frac{2 \pi f}{H_o N} \frac{(s+1)^2}{S} \frac{A(s+1)}{1+\alpha} \tag{6}
\]

where \( H_o \) = an initial thickness of the SVI layer
\( N \) = a number of layers.

2.3 Experimental loading both in static and dynamic of rubber specimens made of various grades of rubber proved good correlation between experimental data and theoretical analysis. In tests with a stepped loading with holding time (Fig. 1) experimental results for settlement differed not much then 3% as compared to the theoretical expectations (see Table I).

**Table I. Static loading rubber grade 7-30-I4-102**

<table>
<thead>
<tr>
<th>( N )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{theor} )</td>
<td>5.780</td>
<td>5.281</td>
<td>5.204</td>
<td>4.784</td>
</tr>
<tr>
<td>( \text{exp} )</td>
<td>5.780</td>
<td>5.281</td>
<td>5.204</td>
<td>4.895</td>
</tr>
<tr>
<td>% error</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.05</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>f_t</td>
<td>4.846</td>
<td>4.261</td>
<td>3.975</td>
<td>3.89</td>
</tr>
<tr>
<td>4.704</td>
<td>4.427</td>
<td>4.297</td>
<td>4.083</td>
<td>3.938</td>
</tr>
<tr>
<td>0.2</td>
<td>1.8</td>
<td>0.8</td>
<td>2.65</td>
<td>1.02</td>
</tr>
</tbody>
</table>

\( E_o = 1.144 \text{ MPa}; \ E_e = 6.4 \text{ MPa}; \ E_0 = 1.72 \text{ MPa}; \ E_e = 9.6 \text{ MPa}. \)

Creep in the process of stepped loading was considered by means of relationship (3).

2.4 Under dynamic loading of a specimen theoretically deduced and experimentally obtained frequencies of free vibrations compared. Results of the comparison for rubber grade 7-30-I4-102 are shown in Fig. 2.

**Figure 1.**

**Figure 2.** Dynamic testing of rubber grade 7-30-I4-102

For the case of unfinished creeping using the equation (3) solution under constant load for the expression for natural frequency becomes more precise:

\[
\omega^2 = \frac{2 \pi f}{H_o N} \frac{(s+1)^2}{S} \frac{A(s+1)}{1+\alpha} \tag{7}
\]

where \( f_t = \frac{1}{1 - \frac{1}{\alpha} \frac{E_o}{E_e}} \), \( \alpha = \frac{E_e}{E_o} \).

2.5 Natural frequency of shear vibrations of compressed SVI is defined taking into account possible increase of support area and vibrations in shape coefficient

\[
\omega^2 = \frac{C \cdot \omega_e}{H_o N} \frac{1 + S^2}{S} \frac{S_0}{K^2 E_e} \tag{8}
\]

Reduction of horizontal vibration frequency is governed by the coefficient

\[
\alpha^2 = \left( \frac{\omega_f}{\omega_e} \right)^2 = 3 \left[ 1 + A(s+1)^4 \right] \tag{9}
\]
3.1 The proposed approach permits to estimate the advantages and scope of application for layered SVI. As it follows from (6) free vibration frequency as a function of load has extremum (minimum), which position and magnitude depends on the number of layers and the relation of dynamic modulus of elasticity to static (Fig. 3).

![Graph showing the relationship between load and frequency for layered SVI.]

Figure 3.

Increase in bearing capacity at the expense of rubber stiffening and growing of $E \sim \infty$ is unfavorable as value $\gamma_1$ grow goes up as well. That is why it seems more advantageous to make more numbers of layers and keep $\gamma_1 \approx 1.2 \cdot 1.3$.

3.2 SVI optimization depends on specific engineering task.

If SVI dimensions are known then the load at which natural frequency is minimum is determined from condition

$$0,4 \left( S-1 \right) + A \left( S+1 \right) \left( S-0.4 \right) = 0 \quad (10)$$

In particular, for solid one-layered SVI $S_{opt}$ equals 0.535 at $E \approx 0.3$.

The problem of determination of SVI dimensions at a specified frequency $\omega$ and load $P$, being a vital problem in practice, is solved by either from the condition of minimum volume or non-excess of ultimate deformation $\varepsilon = \Delta / h_0 = 0.3$, determined in the practice of structure service life.

3.3 In the first case the following system of equations is solved by means of iteration method:

$$H_0 = BR_0 \left( S+1 \right)^2 \left( 1 + T \right)$$

$$R_0 = K / 2$$

where $B = E_0 \cdot g \cdot \pi / (g_1^2 \cdot N \cdot P)$

$N$ - a number of layers, $A = \frac{mR}{E_0}$

$$K = \frac{1}{1 + \frac{\beta}{2}} \left( \frac{P}{T \cdot E_{in}} \right) \left( 2 + 3S \cdot T \cdot 2 + 3A \right)^{0.5} \left( 1 + T \right)$$

For the second case design relation has a form:

$$\beta = \frac{1}{1 + \frac{\beta}{2}} \left( \frac{P}{T \cdot E_{in}} \right) \left( 2 + 3S \cdot T \cdot 2 + 3A \right)^{0.5} \left( 1 + T \right)$$

$$\beta = \frac{1}{1 + \frac{\beta}{2}} \left( \frac{P}{T \cdot E_{in}} \right) \left( 2 + 3S \cdot T \cdot 2 + 3A \right)^{0.5} \left( 1 + T \right)$$

$$R_0 = \sqrt{\frac{P}{T \cdot E_{in}}}$$

$$H_0 = \frac{mR}{2}$$

Design nomogram for the first case is shown in Fig. 4.

![Nomogram for the first case of SVI optimization.]

Figure 4. $N=2$

CONCLUSIONS

1. The proposed method has been applied for analysis and realization of the replaceable vibration isolation system for a multi-storied building, being erected quite close to the shallow subway line. The level of vibration turned 25 times less, that confirmed the validity of the proposed method.

2. The fact that non-linear and rheological factors may be consider-
ed within the frame of a simple model open broad prospects in the future for application of the above method in engineering practice.

REFERENCES


