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Direct output feedback control for multiple-degree-of-freedom seismic structures

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ABSTRACT: This paper demonstrates the effectiveness of direct output feedback control algorithm in reducing the dynamic responses of a multiple-degree-of-freedom structure subjected to earthquake excitation. The optimal feedback gain is obtained such that certain prescribed performance index is minimized. The control force is then calculated by multiplying this feedback gain by the output measurements. Since the full order structural model is considered throughout the derivation, spillover in control and observation is no longer a problem. Numerical results show that the performance of direct velocity feedback control with one controller and one sensor, which is not necessarily colocated, is as good as that of state feedback. On-line calculation in this method is very simple that is favorable to real implementation. Finally, the allocation pattern of sensor and controller is suggested.

1 INTRODUCTION

Active structural control is one of the promising alternatives to reduce structural responses and hence enhance structural reliability under stochastic environmental loadings like earthquake. In recent years, considerable amount of work has been carried out in the feasibility study of active vibration control of civil engineering structures (Reinhorn and Manolis 1985; Yang and Soong 1988; Soong 1990). Implementability was also studied through a series of model and full-scale experiments (Chung et al. 1988, 1989). It has become clear that some practical and important problems remain unsolved such as limited number of sensors and controllers, etc.

Since structure is a continuum, there are infinite number of degrees of freedom. Even the structure is simplified by a discrete-parameter model, it still posesses a large number of degrees of freedom. Generalized displacements and velocities of all degrees of freedom are defined as the state of structural system. Economy, data processing and on-line calculation considerations make it impractical and impossible to require full state measurements and feedback. Only output measurements, which are usually the combination of responses at few degrees of freedom, are available for control force calculation. Direct output feedback is thus necessary from a practical point of view.

Three methods have been proposed to compensate for the problem of limited number of sensors:

1. Observers. States are estimated based on the measured outputs by using dynamic observers. Then, the control force is calculated based on the estimated states. In this method, the observers make on-line calculation complicated (O'Reilly 1983).

2. Modal Control. Assumed that output measurements are contributed by the structure's first few modes. The number of controlled modes selected is equal to or less than the number of measurements. Then, control force is calculated based on the modal information. However, the assumptions will introduce control and observation spillovers (Balas 1978).

3. Direct Output Feedback. Control force is calculated based directly on the output measurements (Balas 1979). In our previous paper (Lin et al, 1991), an optimal direct output feedback control algorithm was developed in simple fashion.

The purpose of this paper is to demonstrate the effectiveness of proposed direct output feedback control algorithm in reducing the dynamic responses of a multiple-degree-of-freedom (MDOF) structure under earthquake excitation. Since the full order model of the structure is considered throughout the derivation, no model reduction is assumed. Therefore, spillover in control and observation as mentioned by Balas (1978) is no longer a problem. Moreover, control force is calculated by multiplying the output measurements by a time-invariant feedback gain. Thus, on-line calculation is very simple which is favorable to real implementation. Finally, the allocation pattern of sensor and controller is suggested.

2 CONTROL ALGORITHM

Consider an n-DOF discrete-parameter structure under dynamic loading $\underline{w}(t)$ and active control force $\underline{U}(t)$. The equation of motion takes the form

$$M\ddot{\underline{x}}(t) + C\dot{\underline{x}}(t) + K\underline{x}(t) = B_1\underline{U}(t) + E_1\underline{w}(t) \quad (1)$$

where M is the n×n mass matrix, C is the n×n viscous damping matrix, K is the n×n stiffness matrix, $\underline{x}(t)$ is the n×1 displacement vector relative to the moving base. B_1 is the n×m location matrix of m control forces, and E_1 is the n×k location matrix of k external loadings.

Equation (1) can be rewritten in state-space representation as

$$\dot{\underline{z}}(t) = A\underline{z}(t) + B\underline{U}(t) + E\underline{w}(t) \quad (2)$$

where

$$\underline{z}(t) = \begin{pmatrix} \underline{x}(t) \\ \dot{\underline{x}}(t) \end{pmatrix}$$

is the 2n×1 state vector,

$$A = \begin{pmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{pmatrix}$$

is the 2n×2n system matrix,

$$B = \begin{pmatrix} 0 \\ M^{-1}B_1 \end{pmatrix}$$

is the 2n×m controller location matrix, and

$$E = \begin{pmatrix} 0 \\ M^{-1}E_1 \end{pmatrix}$$

is the 2n×k external loading location matrix.

Because of limited number of sensors, say p and p<2n, output $\underline{y}(t)$ is just some combinations of states $\underline{z}(t)$. The output equation is expressed as

$$\underline{y}(t) = D\underline{z}(t) \quad (3)$$

where D is the p×2n output matrix. Control forces are calculated by multiplying output vector by a constant feedback gain

$$\underline{U}(t) = G\underline{y}(t) \quad (4)$$

where G is the m×p feedback gain matrix.

The problem becomes that determining the constant feedback gain matrix G such that the quadratic performance index

$$J = \int_0^\infty [\underline{z}^T(t)Q\underline{z}(t) + \underline{U}^T(t)R\underline{U}(t)]dt \quad (5)$$

is minimized. In equation (5), Q is the 2n×2n positive semi-definite weighting matrix for the states and R is the m×m positive definite weighting matrix for the control forces. T is the transpose operator.

Suppose that the structure is only subjected to an initial disturbance $\underline{z}(0)=\underline{z}_0$, substituting equations (3) and (4) into (2), the state equation takes the form

$$\dot{\underline{z}}(t) = A'\underline{z}(t), \qquad \underline{z}(0) = \underline{z}_0 \quad (6)$$

where A'=A+BGD is the equivalent system matrix with the application of control forces. Solving equation (6), we get

$$\underline{z}(t) = e^{A't}\underline{z}_0 \quad (7)$$

The substitution of equations (3-4) and (7) into (5) yields

$$J = \underline{z}_0^T[\int_0^\infty e^{A'^T t}(Q + D^T G^T RGD)e^{A't}dt]\underline{z}_0 \equiv \underline{z}_0^T H \underline{z}_0 \quad (8)$$

where

$$H = \int_0^\infty e^{A'^T t}(Q + D^T G^T RGD)e^{A't}dt \quad (9)$$

The integral exists if and only if A' is a stable matrix. Differentiating H with respect to t, we get

$$A'^T H + HA' + Q + D^T G^T RGD = 0 \quad (10)$$

Now the problem is converted to one that minimizes the performance index J subject to the constraint of equation (10). The performance index can be rewritten as

$$J = \underline{z}_0^T H \underline{z}_0 = tr(HZ) \quad (11)$$

where tr(·) is the trace of a matrix and $Z=\underline{z}_0\underline{z}_0^T$. Coupling with the constraint, the Lagrangian can be expressed as

$$J' = tr(HZ) + tr\{L[A'^T H + HA' + Q + D^T G^T RGD]\} \quad (12)$$

where L is the 2n×2n Lagrangian multiplier matrix.

By using the necessary condition for minimization of the performance index, we get

$$\frac{\partial J'}{\partial L} = A'^T H + HA' + Q + D^T G^T RGD = 0 \quad (13)$$

$$\frac{\partial J'}{\partial H} = Z + LA'^T + A'L = 0 \quad (14)$$

$$\frac{\partial J'}{\partial G} = 2B^T HLD^T + 2RGDLD^T = 0 \quad (15)$$

By solving the simultaneous algebric equations (13 – 15), the optimal feedback gain matrix G can be obtained iteratively.

3 NUMERICAL VERIFICATION

An idealized three-DOF structure with tendon control device in place shown in Fig. 1 is studied to demonstrate the effectiveness of proposed control algorithm. The first 10 second acceleration of El Centro earthquake (N-S component, 1940) shown in Fig. 2, which includes the strong motion part, is used as the base excitation. The control force produced by adjusting actuator displacement, $\underline{u}(t)$, is transmitted to the structure through four pretensioned tendons. The state equation of the controlled system is described by equation (2) with control force vector

$$\underline{U}(t) = 4k_c \cos\alpha\,\underline{u}(t) \quad (16)$$

where k_c and α are the stiffness and inclination angle of the tendons with respect to the base, respectively. Here we assume that the actuator is placed at floor with the same inclination angle. The weighting matrices are given as

$$Q = \begin{pmatrix} K & 0 \\ 0 & 0 \end{pmatrix}, \quad R = \frac{\beta}{16k_c\cos^2\alpha}I$$

to make J be the summation of system strain energy and applied control energy, in which K is structural stiffness matrix and I is the m×m identity ma-

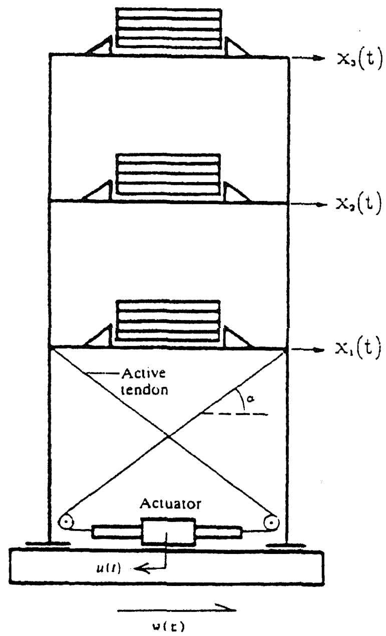

Fig. 1. Schematic Diagram of 3DOF Structure.

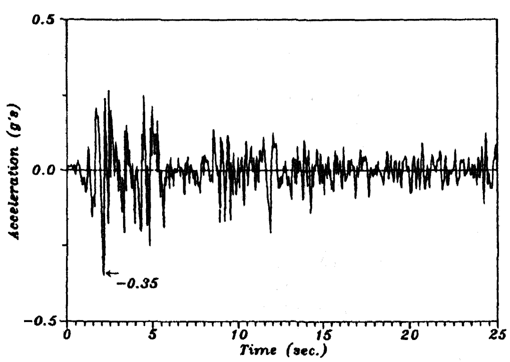

Fig. 2. El Centro Earthquake acceleration (N-S, 1940).

trix. The coefficient β determines the relative importance of control effectiveness (response reduction) and economy (control force requirements). When $\beta < 1$, control effectiveness has more weight and, when $\beta > 1$, economy is more important. When $\beta = 1$, they are equally important. $\beta = \infty$ represents uncontrolled case. The relevant system properties and ϕoutput matrix are listed in Table 1.

Matrix Z is specified to be an identity matrix. Starting from zero initial feedback gain matrix G, the optimal G matrix is obtained by solving the algebric equations (13-15) iteratively for different control cases. As concluded for the control study of SDOF structures (Lin et al, 1991), direct displacement feedback is found to be ineffective. Hence, only direct velocity feedback control is studied in this paper. To examine the influence of reducing numbers

Table 1. System properties and output matrix of 3DOF structure.

M (lb-sec²/in.)	$\begin{bmatrix} 5.6 & 0 & 0 \\ 0 & 5.6 & 0 \\ 0 & 0 & 5.6 \end{bmatrix}$
K (lb/in.)	$\begin{bmatrix} 15649 & -9370 & 2107 \\ -9370 & 17250 & -9274 \\ 2107 & -9274 & 7612 \end{bmatrix}$
C (lb-sec/in.)	$\begin{bmatrix} 2.185 & -0.327 & 0.352 \\ -0.327 & 2.608 & -0.015 \\ 0.352 & -0.015 & 2.497 \end{bmatrix}$
$\underline{\omega}$ (Hz)	$\begin{bmatrix} 2.24 \\ 6.83 \\ 11.53 \end{bmatrix}$
$\underline{\xi}$ (%)	$\begin{bmatrix} 1.62 \\ 0.39 \\ 0.36 \end{bmatrix}$
k_c (lb/in.)	2124
α (°)	36
Φ	$\begin{bmatrix} 0.262 & 0.743 & 0.583 \\ 0.568 & 0.373 & -0.728 \\ 0.780 & -0.555 & 0.360 \end{bmatrix}$
D_1	$\begin{bmatrix} 4.116 & -1.785 & 0.240 \\ -1.642 & 2.254 & -0.686 \\ 0.060 & -1.210 & 1.125 \end{bmatrix}$

of sensors and controllers on control effectiveness, direct velocity feedback with various numbers and locations of sensors and controllers is investigated.

In general, J decreases as β decreases. For a given value of β, J increases as the number of sensor or controller reduces. Modal damping ratios increase significantly and, thus, the reduction in responses is guaranteed. For $\beta = 10$, the results for different control cases, compared with those of uncontrolled case and state feedback, are given in Table 2 and summarized in the following:

1. The control effectiveness with three velocities measurement and feedback is as good as that of state feedback.

2. Reduced the number of controller, the performance of U1v123 is worse than that of U123v123, but still quite effective.

3. Comparing case U1v1 with U1v123, number of sensors is reduced from three to one, its control effect is significant and acceptable.

4. For the extreme case of one controller and one sensor, it is found that U1v1 is effective because modal damping ratios increase. However, U1v3 reduces modal damping ratios even with great control effort. This agrees with the finding obtained by Balas (1979) that the sensor and controller must be colocated.

The simulation results mentioned above are based on the assumption that a sensor only measures the response contributed by the degree of freedom in which it is placed. However, in real situation, each sensor measurement is usually the combination of responses at several degrees of freedom with output matrix in the form of

Table 2. Control effect of different control cases.

Control Cases	J	Modal Frequency (Hz)	Modal Damping Ratio(%)
Uncontrolled	44910	$\begin{bmatrix} 2.24 \\ 6.80 \\ 11.49 \end{bmatrix}$	$\begin{bmatrix} 1.61 \\ 0.39 \\ 0.36 \end{bmatrix}$
U123STATE	2093	$\begin{bmatrix} 2.95 \\ 7.15 \\ 11.70 \end{bmatrix}$	$\begin{bmatrix} 46.04 \\ 21.64 \\ 13.43 \end{bmatrix}$
U123v123	2135	$\begin{bmatrix} 2.24 \\ 6.80 \\ 11.49 \end{bmatrix}$	$\begin{bmatrix} 40.94 \\ 21.13 \\ 13.31 \end{bmatrix}$
U1v123*	3875	$\begin{bmatrix} 2.27 \\ 6.83 \\ 11.27 \end{bmatrix}$	$\begin{bmatrix} 13.74 \\ 13.37 \\ 6.87 \end{bmatrix}$
U1v1	4081	$\begin{bmatrix} 2.26 \\ 6.87 \\ 11.24 \end{bmatrix}$	$\begin{bmatrix} 8.56 \\ 20.15 \\ 5.69 \end{bmatrix}$
U1v3	44419	$\begin{bmatrix} 2.24 \\ 6.80 \\ 11.49 \end{bmatrix}$	$\begin{bmatrix} 1.51 \\ 0.46 \\ 0.34 \end{bmatrix}$
U1v1[D1]	4451	$\begin{bmatrix} 2.24 \\ 6.90 \\ 11.31 \end{bmatrix}$	$\begin{bmatrix} 2.78 \\ 11.83 \\ 7.56 \end{bmatrix}$
U1v3[Da]	3875	$\begin{bmatrix} 2.27 \\ 6.83 \\ 11.27 \end{bmatrix}$	$\begin{bmatrix} 13.74 \\ 13.37 \\ 6.87 \end{bmatrix}$

*U1v123: Controller placed at 1st floor
Velocity measured at 1st, 2nd, 3rd floor

$$D = \begin{pmatrix} D_1 & 0 \\ 0 & D_1 \end{pmatrix} \tag{17}$$

where D_1 is shown in Table 1. Taken this into consideration, the control performance of U1v1[D_1] becomes worse than its corresponding ideal case. But, control effectiveness can still be achieved. However, if the output matrix can be adjusted to become scalar multiplication of the matrix GD which is obtained from the control case of U1v123 with the same value of β, its control performance can be dramatically improved and be the same as that of U1v123. Furthermore, the sensor is not necessarily colocated with the controller as the case of U1v3[D_a] given in Table 2.

The transfer functions of relative displacement and absolute acceleration at top floor with respect to ground acceleration for uncontrolled, U1v1[D_1] and U1v3[D_a] control cases are shown in Figs. 3–4. The relative displacement and absolute acceleration at top floor and drift between first and second floor under El Centro earthquake are shown in Figs. 5–7 for uncontrolled, U1v1[D_1] and U1v3[D_a] control cases. The peak responses and peak control force are also listed in Table 3. It is seen that the responses are reduced significantly due to system dampings increase.

4 CONCLUSIONS

The theoretical development of the proposed control algorithm and simulated results given indicate that direct velocity feedback with one controller and one sensor, which is not necessarily colocated, is effective in reducing the structural responses as that of state feedback for MDOF structures under earthquake excitation. Since the full order model of the structure is considered throughout the derivation, the spillover effect in control and observation is eliminated. Moreover, since on-line calculation in this method is very simple, the contribution of such a control algorithm to the real implementation is quite significant.

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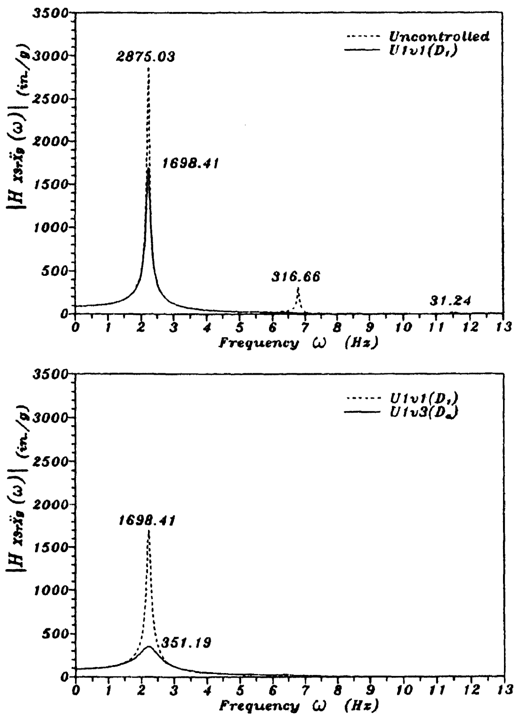

Fig. 3. Top-floor relative displacement transfer functions (with and without control).

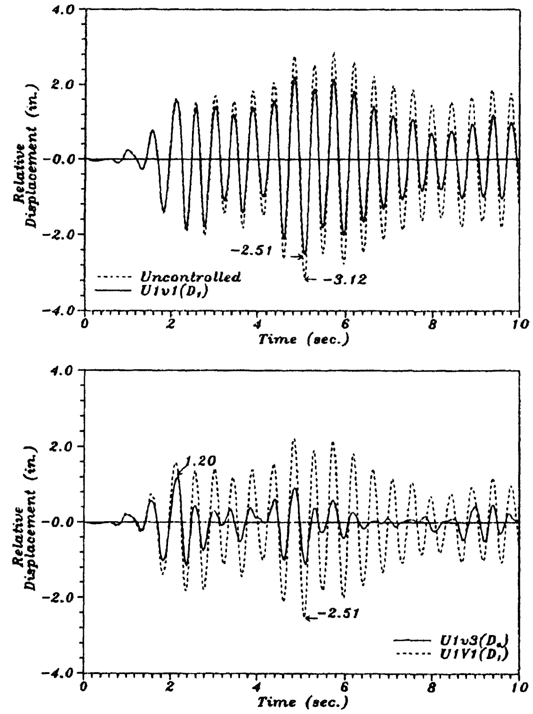

Fig. 5. Top-floor relative displacement under El Centro earthquake (with and without control).

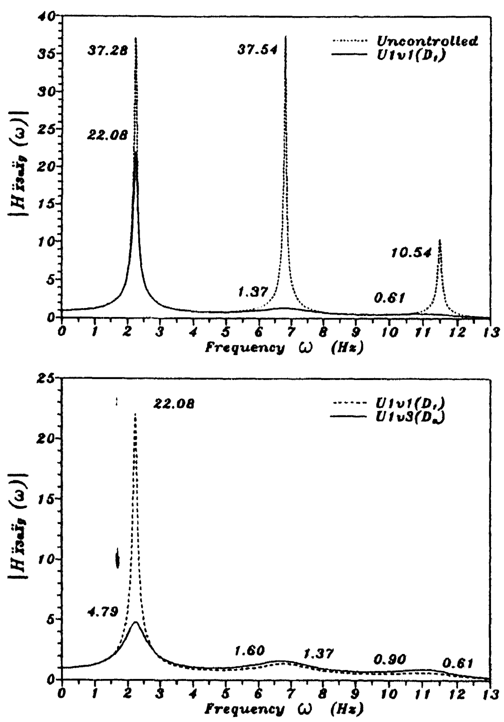

Fig. 4. Top-floor absolute acceleration transfer functions (with and without control).

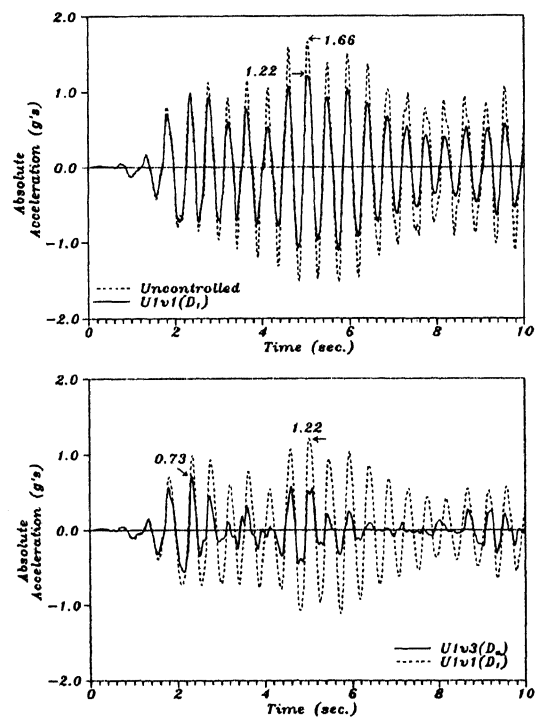

Fig. 6. Top-floor absolute acceleration under El Centro earthquake (with and without control).

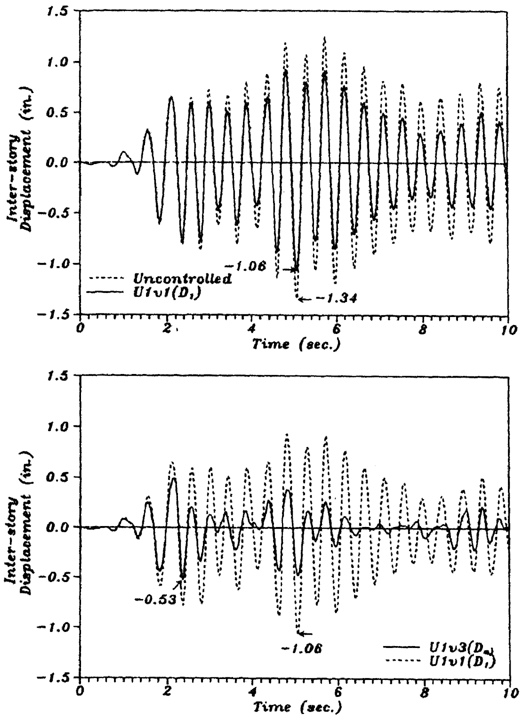

Fig. 7. Drift between 1st and 2nd floor under El Centro earthquake (with and without control).

Table 3. Peak responses and peak control force for different control cases.

Control Cases	$\underline{X}_{r,max}$ (inch)	$\ddot{\underline{X}}_{a,max}$ (g's)	$\underline{X}_{d,max}$ (inch)	$\underline{U}_{max}$ (pounds)
Uncontrolled	[1.08, 2.42, 3.12]	[0.87, 1.31, 1.66]	[1.34, 0.70]	——
U123STATE	[0.20, 0.38, 0.46]	[0.37, 0.39, 0.40]	[0.19, 0.08]	[301.11, 530.79, 692.11]
U123v123	[0.34, 0.70, 0.87]	[0.39, 0.43, 0.45]	[0.36, 0.18]	[295.26, 528.25, 704.53]
U1v123	[0.47, 0.96, 1.20]	[0.44, 0.59, 0.73]	[0.53, 0.30]	[1564.89, ——, ——]
U1v1	[0.54, 1.16, 1.45]	[0.46, 0.70, 0.79]	[0.62, 0.33]	[975.44, ——, ——]
U1v3	[1.11, 2.47, 3.18]	[0.86, 1.34, 1.68]	[1.36, 0.71]	[32.41, ——, ——]
U1v1[D1]	[0.94, 2.00, 2.51]	[0.62, 1.10, 1.22]	[1.06, 0.52]	[359.38, ——, ——]
U1v2[Da]	[0.47, 0.96, 1.20]	[0.44, 0.59, 0.73]	[0.53, 0.30]	[1564.91, ——, ——]

$\underline{X}_{r,max}$: Peak relative displacement
$\ddot{\underline{X}}_{a,max}$: Peak absolute acceleration
$\underline{X}_{d,max}$: Peak inter-story displacement
$\underline{U}_{max}$: Peak control force