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# Reliability problem of active control algorithms caused by time delay

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ABSTRACT: Time delay causes reliability problems for optimal active control algorithms which are widely used in earthquake engineering. It is shown that under certain circumstances there exist critical values of delay time for which the active control algorithms will fail. A distribution map and an explicit formula are given to determine these critical delay times. The ratio of the critical delay time to the natural period of the structure may be small, indicating that the time delay effect may not be neglected for small time delay. The critical delay times may be classified as periodic families where the critical values appear periodically. For a given delayed system, there exist at most two such periodic families. A new algorithm is proposed to take into account effects of time delay.

## 1. INTRODUCTION

Considerable research effort has recently been devoted to the application of structural control to improve structural safety and/or functionality and to mitigate seismic hazards during severe earthquake events. A detailed review may be found in Soong (1988) and Kobori (1988). The state-of-the-art of the progresses and the research needs in this field are summarized in Housner and Masri (1990, Editors).

The emphasis of most previous research has been on the application of classical control theory to highly idealized systems. Many problems of practical importance have yet to be examined. The present paper addresses one such important issue, i.e. the reliability problem of the optimal active control algorithms caused by time delay. In real active control systems, time is consumed by acquisition of response and excitation data, on-line computation to obtain the required theoretical control force, and application of the control force. Therefore, there will always be a delay between the time at which the control force is assumed to be applied and the time when the control force is actually applied. This delay is more significant in control of massive civil structures. The time delay may be minimized by employing more sophisticated hardware and software, or its effects may be partly taken into account by introducing higher order effects into the theoretical analysis such as the interaction of the structure and actuator. However, time delay cannot be eliminated altogether with present-day technology.

It has been pointed out that time delays may not only render an active control system ineffective but may also cause instability of the controlled system (Soong, 1988). However, no details were given as to how the delay time actually influence performance of the control system. The time delay effect has been neglected in most applications of control algorithms to civil engineering structures based on the argument that flexible structures usually have a fairly long natural period compared with the time delay, which makes the delay effect negligible (Abdel-Rohman, 1985). In order to take the delay effect into account, some compensation techniques have been proposed in structural control. Chung, Reinhorn, and Soong (1987) used a phase shift method in their experimental studies of active control of seismic structures to compensate for this effect. In their approach, the control gains are modified such that the real system and the ideal system have the same active stiffness and active damping. It is reported that the phase shift approach works effectively in their experimental studies (Chung et al, 1987, 1989; McGreevy et al, 1988). Abdel-Rohman (1985) employed the truncated Taylor-Series expansion method to design the control forces taking time delay into consideration. It was demonstrated that the active tendon mechanism used is sensitive for delay effect during forced vibration. Better performance of the control law may be achieved if in the design of the control law the delay effect is included. The approach is valid only if the delay time is small as compared with the system's natural period.

In the present study, the reliability problem of active control algorithms caused by the time delay is investigated. A distribution map and an explicit formula are given for the critical delay times for which the optimal control algorithms will fail. It is shown that the time delay effect cannot be neglected even for small delay time. A reliable control algorithm is proposed to take into account effects of time delay.

## 2. FORMULATION

Consider a single-degree-of-freedom oscillator excited by an external excitation f(t) and controlled by a control force F(t). The governing equation of the controlled system can be written as

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) + F(t) = f(t) \quad (1)$$

where $m, c$ and $k$ describe the inertia, damping and stiffness properties of the system and $x(t)$ represents the structural response. The control force is assumed to take the form

$$F(t) = \beta\dot{x}(t-\tau) + \gamma x(t-\tau) \quad (2)$$

which implies that the feedback control force consists of terms proportional to the velocity and displacement with constant gains $\beta, \gamma$, and the control force is applied with a delay time $\tau$. For the sake of simplicity, it is assumed that the delay time is the same for both velocity and displacement, but the analysis may be extended to the case of different delay times for different terms.

The control gains $\beta$ and $\gamma$ are determined by optimizing an objective function defined as

$$J = \int_0^{t_f} \{\frac{R_s}{2}(kx^2 + m\dot{x}^2) + R_f F^2\}dt \quad (3)$$

in which $t_f$ is the time duration of the excitation, and $R_s$ and $R_f$ are the weighting coefficients indicating the relative importance between safety and economy. The optimization procedure leads to a matrix Riccati equation. An explicit expression for $\beta$ and $\gamma$ is given by Meirovitch and Silverberg (1983) as

$$\begin{aligned} \beta &= 2\omega_0(-\omega_0 + \sqrt{\omega_0^2 + R_f^{-1}}) \\ \gamma &= \omega_0(\sqrt{\omega_0^2 + R_f^{-1}} - \omega_0) \end{aligned} \quad (4)$$

for an undamped system with natural frequency $\omega_0$.

## 3. CRITICAL DELAY TIMES

Insightful conclusions can be obtained by studying the steady state response of the delayed system subjected to a harmonic excitation

$$f(t) = f_0 e^{i\omega_f t} \quad (5)$$

where $f_0$ and $\omega_f$ are the amplitude and frequency of the excitation.

### 3.1 Steady-state response

Define the following nondimensional parameters:

$$\begin{aligned} &\delta = \frac{\omega_f}{\omega_0} \qquad r = \frac{\tau}{T_0} \qquad \lambda = \frac{\gamma}{k} \\ &\mu = 2\zeta\frac{\beta}{c}\frac{\omega_f}{\omega_0} \qquad \theta = \omega_f\tau = 2\pi r\delta \end{aligned} \quad (6)$$

where $T_0$ and $\zeta$ are respectively the natural period and the critical damping ratio of the system. Then, the steady-state solution of the delayed system may be rewritten as

$$x(t) = X_0 e^{i(\omega_f t + \phi)} \quad (7)$$

where $X_0$ is the amplitude of the steady-state response and $\phi$ is the phase shift caused by the system damping as well as the time delay. It can be shown that

$$X_0 = \frac{f_0/k}{\sqrt{A^2 + B^2}}, \qquad \phi = -\arctan\frac{B}{A} \quad (8)$$

where

$$\begin{aligned} A &= 1 - \delta^2 + \lambda\cos\theta + \mu\sin\theta \\ B &= 2\zeta\delta + \mu\cos\theta - \lambda\sin\theta \end{aligned} \quad (9)$$

In the special case where there is no the feedback control force, i.e. $\beta = \gamma = 0$, the above results reduce to classical ones.

Fig 1 shows the steady-state amplitude response of a single-degree-of-freedom system with $m = 1.0, k = 1.0$, and $\zeta = 0.05$ for different relative delay times $\tau/T_0 = 0.0, 0.01, 0.25$, and $0.5$. The feedback control force is assumed such that $\beta = 0$ and $\gamma = 0.5$. Note that $\tau/T_0 = 0$ implies a regular system without time delay. It is observed that all the curves for the non-zero delay times show similar characteristics as that for the non-delayed control force, but their magnitude and position of the peak of the response amplitude change with delay time. A small delay time does not necessarily mean that its effect on the peak amplitude response will be correspondingly small. In fact, the amplitude response with smallest time delay $\tau/T_0$ in Figure 1 has the highest peak.

### 3.2 Resonance frequency ratio

Resonance of the steady-state response is achieved when the denominator of $X_0$ in Eq. (8) becomes zero. It can be shown that the resonance occurs if and only if

$$(1-\delta^2)^2 + (2\zeta\delta)^2 = \lambda^2 + \mu^2 \quad (10)$$

and

$$\theta - \psi = (2k+1)\pi \quad (11)$$

where k is an arbitrary integer and

$$\psi = \arctan\frac{(1-\delta^2)\mu - (2\zeta\delta)\lambda}{(1-\delta^2)\lambda + (2\zeta\delta)\mu} \quad (12)$$

The resonance frequency ratio $\delta$ can be determined from the equation

$$\delta^4 + p\delta^2 + q = 0 \tag{13}$$

where

$$p = 4\zeta^4(1 - \beta'^2) - 2 \qquad q = 1 - \gamma'^2 \tag{14}$$

in which

$$\beta' = \frac{\beta}{c} \qquad \gamma' = \frac{\gamma}{k} \tag{15}$$

denote the relative control gains. Note that in the above equation, both $\lambda$ and $\mu$ depend on $\delta$, the ratio of the natural frequency of the system to the frequency of the harmonic external excitation. It can be shown that the possible number of resonance frequency ratios may be zero, one, or two, depending on system parameters and the feedback control gains.

### 3.3 Critical delay times

Let a resonance frequency ratio, if it exists, be denoted by $\delta^*$. The critical values of the delay time can be explicitly expressed by

$$\tau^* = \frac{1}{\delta^*}(k + \frac{1}{2} + \frac{\psi_0}{2\pi}) \tag{16}$$

where $\psi_0$ is the principal value for Eq. (12) and $k$ is any integer which makes the so-obtained $\tau^*$ non-negative. It is clear that if there exists one critical value of the delay time, there will be an infinite number of critical values which appear periodically with a period $1/\delta^*$. Such critical values are catalogued as one periodic family of the critical delay times. The number of such periodic families is equal to the number of the resonance frequency ratios.

Distribution of the possible critical delay times can be graphically illustrated by a distribution map in the $\gamma' - \beta'$ plane. This map can be determined, to large extent, by the critical damping ratio $\zeta$. A distribution map for a lightly-damped system with $\zeta < 1/\sqrt{2}$ is shown in Figure 2.

In Figure 2. the $\beta' - \gamma'$ plane is divided into three regions by curves

$$\begin{aligned} \gamma'^2 &= 4\zeta^2(1 - \beta'^2)(1 - \zeta^2) \\ \gamma'^2 &= 1. \end{aligned} \tag{17}$$

For a control system with the relative control gains in region I, i.e. $\gamma'^2 > 1$, there exists only one periodic family of critical delay times. Region II in Figure 2 corresponds to the case where two periodic families exist. For sufficiently small control gains as in region III, there is no critical delay time. Therefore, for sufficiently small control gains the time delay will not cause additional reliability problems for the active control algorithms. The radius of region III along the $\gamma'$ axis is given by

$$r = 2\zeta\sqrt{1 - \zeta^2} \tag{18}$$

For an undamped system, the closed region shrinks to the origin, which implies that critical delay times always exist for the undamped system.

As an example, the peak-peak response versus delay time for a control system with the relative control gains in region II is given in Figures 3. The system has unit mass, unit stiffness, and the critical damping ratio $\zeta = 0.05$. The feedback control force is assumed such that $\beta' = 0$ and $\gamma' = 0.2$. The external excitation has unit amplitude. The peak of the amplitude response becomes unbounded at the critical delay times. Two families of the critical delay time are observed. For the response frequency ratio $\delta_1^* = 1.2186$, the observed relative critical delay times are 0.03215, 0.8528, 1.1673, 1.6734, 2.4940, 3.3147 etc. with an increment being 0.8206. For $\delta_2^* = 0.7107$, these critical values are 0.6716, 2.0787, 3.4858, etc. with a period equals to 1.4071. It is observed that the relative critical delay time may be very small, which challenges the conventional argument that the time delay effect may be neglected if the ratio of the delay time to the natural period of the system is small.

## 4. RELIABILITY OF ACTIVE CONTROL ALGORITHMS CAUSED BY TIME DELAY

The optimal control strategy has been widely used in structural control. In the optimal active control algorithms, control gains are determined by solving the Riccati matrix equation. In many engineering applications, the Riccati matrix can be assumed to be time-independent (Yang et al., 1987). Consequently, the control gains are constant. When the feedback control force is unsynchronously applied with a time delay, the control algorithm will fail if the delay time happens to be equal to or close to one of these critical values and the dominant frequency of the excitation is close to the resonance frequency.

Figure 4 presents such an example. The system studied has unit mass and unit stiffness and is originally undamped. The weighting coefficients are assumed to be one. The amplitude response of this uncontrolled system is unbounded at the resonance frequency $\omega_f/\omega_0 = 1$, as shown by curve I. The controlled system with undelayed control force determined by solving the Riccati matrix equation or directly given by Eq. (4) has a bounded amplitude response, as shown by curve II, which is greatly reduced due to the application of the control force. However, if the control force is now actually applied with a time delay, say $\tau/T_0 = 0.12$ which is close to $\tau/T_0 = 0.11836$, one of the critical values corresponding the resonance frequency

$\omega_f/\omega_0 = 1.897$, the amplitude response becomes nearly unbounded, as shown by curve III.

The phase shift approach has been proposed to compensate for the time delay effects and satisfactory results are reported in the experimental studies on active control of seismic structures (Chung, Reinhorn, and Soong, 1987). By this approach, the original feedback control gains are modified such that both the real system and the ideal system have the same active stiffness and active damping. It can be shown that the phase shift approach gives an exact solution for the steady-state response of delayed systems. However, it may not eliminate the reliability problem caused by the time delay. For given $\gamma$ and $\beta$, there may exist an infinite number of critical delay times. And, if they exist, they remain in effect for the modified control system. If the delay time is close to one of the critical values and the dominant frequency of the excitation is close to the resonance frequency, the control algorithm will fail, as seen from the previous example.

The problem can be solved by directly optimizing the performance index of the control system with the delayed control force. In the case of steady-state response, the performance index may be written as

$$J = R_s X_0^2 + R_f(\lambda^2 + \mu^2) \qquad (19)$$

For given delay time and harmonic excitation, control gains $\beta$ and $\gamma$ are obtained such that $J$ in Eq. (19) is minimized. The simplex method is employed for the optimization (Himmeblan, 1972). For the previous example, the controlled steady-state amplitude response by this approach is presented by curve IV in Figure 4. As expected, this control algorithm remains reliable for all the frequency ratios and the amplitude response becomes bounded at the frequency ratio $\omega_f/\omega_0 = 1.8$ where the standard control algorithm fails.

## 5. CONCLUSIONS

Time delay causes reliability problems for optimal active control algorithms which are widely used in earthquake engineering. It is shown that under certain circumstances there exist critical values of delay time for which the active control algorithms will fail. A distribution map and an explicit formula are given to determine these critical delay times. The ratio of the critical delay time to the natural period of the structure may be small, indicating that the time delay effect may not be neglected for small time delay. The critical delay times appear periodically if they exist. For a given delay system, there exist at most two periodic families. For given critical damping ratio and sufficiently small control gains, the critical delay time does not exist. The reliability problem caused by time delay can be avoided by directly optimizing the objective function for delayed systems.

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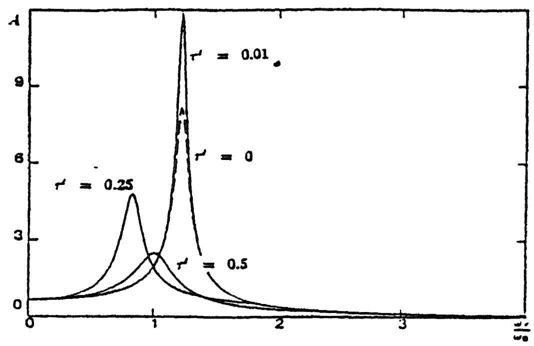


Figure 1. Comparison of the steady-state amplitude responses of a linear SDOF system subjected to a harmonic excitation for different delay times. $\omega_0 = 1.0$ and $\zeta = 0.05$. $\beta = 0$ and $\gamma = 0.5$. The relative delay time $\tau' = \frac{\tau}{T_0} = 0.0, 0.01, 0.25, 0.5$. Dashed curve represents the result for non-delayed control ($\tau'$=0).

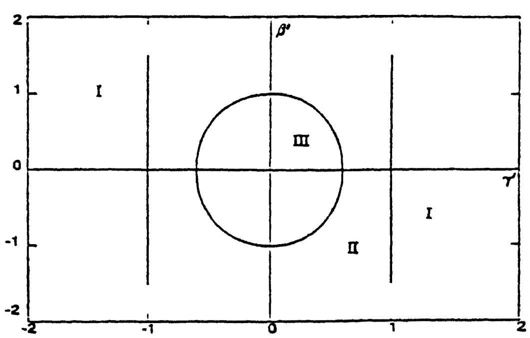


Figure 2. Distribution map of critical values of delay time for lightly delayed systems ($\zeta < \frac{1}{\sqrt{2}}$). Region I: one periodic family of the critical delay times; Region Ii: two periodic families; Region III: no critical delay time.

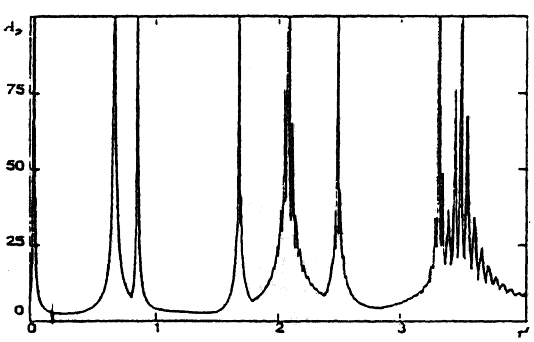


Figure 3. A representative curve for peak-peak response versus delay time for the case where two families of the critical delay times exist. $\omega_0 = 1.0, \zeta = 0.05$; $\beta' = 0.0, \gamma' = 0.2$; The observed critical delay times corresponding the resonance frequency $\frac{\omega}{\omega_0} = 1.219$ are: 0.0322, 0.853, 1.673, 2.494, and 3,315. Those corresponding the second resonance frequency $\frac{\omega}{\omega_0} = 0.711$ are: 0.672, 2.079, and 3.486.

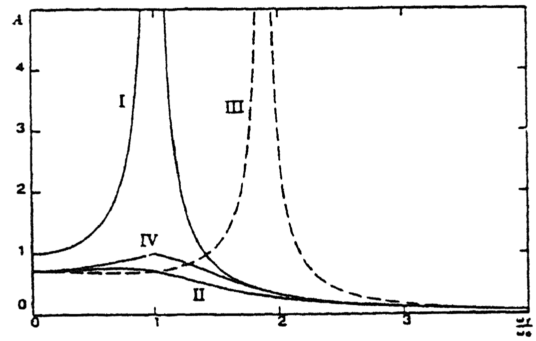


Figure 4. Comparison of the steady-state response curves for controlled and uncontrolled system. $\omega_0 = 1.0, \zeta = 0.0$; $\beta$ and $\gamma$ are calculated to minimize the performance index with $R_s = R_f = 1.0$. I - uncontrolled system; II - controlled system with non-delayed Riccati control force; III - controlled system with Riccati control force delayed by 0.12; IV - controlled system with a optimized delayed control force with $\frac{\tau}{T_0}$=0.12.