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Active control of the seismic response of tall non-uniform buildings

J. Enrique Luco
University of California, San Diego, La Jolla, Calif., USA

Akira Mita
Ohsaki Research Institute, Shimizu Corporation, Tokyo, Japan

ABSTRACT: Two control strategies to reduce the seismic response of tall non-uniform buildings are studied in this paper. The first approach relies on the use of an active control force applied at the top of the building and selected to simulate an absorbing boundary at the top of the building. This control force eliminates resonance in all modes and strongly reduces the response of the structure. In the second approach, in addition to the active absorbing boundary at the top of the building, active control forces are applied at the elevations where structural properties change abruptly. These control forces are determined so that no downward reflections occur at the points of discontinuity. Numerical results in the frequency and time domains for 10- and 50-storey buildings show that this second strategy leads to additional reductions of the structural response.

1. INTRODUCTION

Recently, the authors have shown (Mita and Luco, 1990a; Luco et al, 1992) that it is possible to reduce the seismic response of tall uniform buildings by use of an active control force applied at the top of the building and such that it simulates an absorbing boundary at the top of the structure. The active absorbing boundary eliminates resonance in all modes and strongly reduces the response of the structure. This control strategy is based on the work of Vaughan (1968) and von Flotow (1986), for aerospace structures, in which the energy flow within the structure is modified by controlling the reflection and/or transmission of waves at end points or at joints. The authors (Mita and Luco, 1990b) have also studied the case of structures modelled as shear beams with smoothly varying properties. In this case, control by means of an absorbing boundary at the top also eliminates resonance but cannot eliminate the amplification of motion associated with the reduction of stiffness with height.

The objective of this paper is to extend the previous results to discrete models of tall non-uniform buildings characterized by abrupt changes in stiffness and mass at a few elevations. These abrupt changes in properties result in the reflection of some of the waves propagating through the building. Typically, in the absence of control, the seismic response increases with height as a result of resonance and of the reduction of stiffness with elevation.

Two control strategies to reduce the seismic response of tall non-uniform buildings are considered in this paper. The first approach is to use an active absorbing boundary at the top of the structure. In the second approach, additional control forces are located at points where structural properties experience abrupt changes. All of the control forces in this second case are determined by imposing the condition that only upward propagating waves should be obtained at all elevations.

Expressions for the control rules for both strategies are derived for discretized models of building consisting of several segments each involving several floors with equal masses and stiffnesses. The models ignore the soil-structure interaction effects on the response and control rules. These effects have been discussed elsewhere (Wong and Luco, 1992). Numerical results in the frequency and time domains for non-uniform 10- and 50-storey buildings are used to illustrate the reductions in response resulting from the use of these two types of control.

2. FORMULATION AND SOLUTION

2.1 Basic equations

We consider a discretized model of the structure consisting of M segments with N_r ($r = 1, M$) floors in each segment. Within each segment the masses, stiffnesses and hysteretic damping ratios are uniform and equal to m_r, k_r and ξ_r ($r = 1, M$), respectively. The equations of motion within the r-th segment for harmonic vibrations with time dependence $e^{i\omega t}$, where ω is the frequency, are given by

$$-\omega^2 m_r u_j^{(r)} + \bar{k}_r(2u_j^{(r)} - u_{j-1}^{(r)} - u_{j+1}^{(r)}) = 0 \qquad j = 1, N_r - 1 \tag{1}$$

where $\bar{k}_r = k_r(1 + 2i\xi_r)$ and $u_j^{(r)} e^{i\omega t}$ is the absolute displacement of the j-th floor ($j = 1, N_r$) within the r-th segment ($r = 1, M$). The corresponding equations at the interfaces between two segments and at the top of the structure are

$$\begin{aligned} &-\omega^2 m_r u_{N_r}^{(r)} + \bar{k}_r(u_{N_r}^{(r)} - u_{N_r-1}^{(r)}) \\ &\quad + \bar{k}_{r+1}(u_1^{(r+1)} - u_0^{(r+1)}) = F_r \\ &\qquad (r = 1, M-1) \end{aligned} \tag{2}$$

and

$$-\omega^2 m_M u_{N_M}^{(M)} + \bar{k}_M(u_{N_M}^{(M)} - u_{N_M-1}^{(M)}) = F_M \tag{3}$$

where F_r ($r = 1, M$) are the control forces applied at the interfaces ($r = 1, M-1$) or at the top ($r = M$) of the building.

The prescribed motion of the base given by $U_0 e^{i\omega t}$ and the continuity of displacements at the interfaces between segments lead to to the conditions

$$u_0^{(1)} = U_0 \tag{4}$$

$$u_{N_r}^{(r)} = U_0^{(r+1)} = U_r \qquad (r = 1, M-1) \tag{5}$$

where we designate by $U_r e^{i\omega t}$ the motion at the interface between the r-th and the (r+1)-th segment and by $U_M e^{i\omega t}$ the motion at the top of the building.

The general solution for Eq. (1) can be written in the form

$$u_j^{(r)} = A_r e^{-i\gamma_r j} + B_r e^{i\gamma_r j} \qquad (j = 0, N_r\ ;\ r = 1, M) \tag{6}$$

where

$$\gamma_r = 2\text{Arc sin}\left(\frac{\omega}{2\omega_r}\right) \tag{7}$$

and

$$\omega_r = \left[\bar{k}_r/m_r\right]^{1/2}. \tag{8}$$

The terms associated with A_r and B_r correspond to 'waves' propagating int the upward and downward directions within the r-th segment, respectively. The unknown coefficients A_r, B_r ($r = 1, M$) are determined by Eqs. (2), (3), (4) and (5).

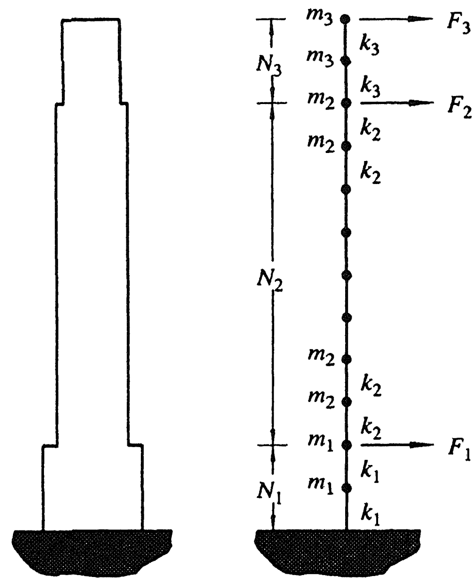

Fig.1. Discretized model of tall non-uniform building.

2.2 Response in the absence of control (Case 1)

In the absence of control forces, i.e., when $F_r = 0$ ($r = 1, M$), it is convenient to write the general solution given by Eq. (6) in terms of the displacements at the bottom (U_{r-1}) and top (U_r) of the segment

$$u_j^{(r)} = U_{r-1}\frac{\sin[\gamma_r(N_r - j)]}{\sin(\gamma_r N_r)} + U_r\frac{\sin(\gamma_r j)}{\sin(\gamma_r N_r)}\ , \quad (j = 0, N_r) \tag{9}$$

In this case, Eqs. (2) and (3) with $F_r = 0$ ($r = 1, M$) lead to the system of equations

$$a_r U_{r-1} + b_r U_r + a_{r+1} U_{r+1} = 0\ , \qquad (r = 1, M-1) \tag{10}$$

$$a_M U_{M-1} + c_M U_M = 0 \tag{11}$$

where

$$a_r = -\frac{\omega\omega_r m_r}{\sin(\gamma_r N_r)}\cos\left(\frac{\gamma_r}{2}\right)\ , \tag{12}$$

$$b_r = \frac{\omega\omega_r m_r \cos[\gamma_r(N_r + \frac{1}{2})]}{\sin(\gamma_r N_r)} + \frac{\omega\omega_{r+1} m_{r+1} \cos[\gamma_{r+1}(N_{r+1} - \frac{1}{2})]}{\sin(\gamma_{r+1} N_{r+1})} \tag{13}$$

$$c_M = \frac{\omega\omega_M m_M \cos[\gamma_M(N_M + \frac{1}{2})]}{\sin(\gamma_M N_M)} \, . \tag{14}$$

The system of equations (10), (11) can be solved for U_r ($r = 1, M$) in terms of the motion of the base U_0.

2.3 Response with an active absorbing boundary at the top (Case 2)

We consider next the case in which control by means of absorbing boundary at the top of the structure is used and no control forces are used at the other interfaces ($F_r = 0$, $r = 1, M-1$). The absorbing boundary at the top is selected so that only upward waves are present in the top segment. In this case, $B_M = 0$ in Eq. (6) and the required control force F_M at the top of the building is obtained from Eq. (3) resulting in

$$F_M = -\omega_M m_M e^{-i\frac{1}{2}\gamma_M}(i\omega U_M) \tag{15}$$

The system of equations for U_r ($r = 1, M$) in terms of U_0 is also given by Eqs. (10), (11) but c_M in this case is given by

$$c_M = \frac{\omega\omega_M m_M}{\sin(\gamma_M N_M)} \cos\left(\frac{\gamma_M}{2}\right) e^{i\gamma_M N_M} \tag{16}$$

2.4 Response with control forces at all interfaces (Case 3)

Finally, we consider the case in which control forces are applied at all interfaces and at the top of the building. The control forces are selected so that only upward propagating 'waves' are present in each segment, i.e., we require that $B_r = 0$ ($r = 1, M$). In this case, Eqs. (4), (5) and (6) lead to

$$u_j^{(r)} = U_{r-1} e^{-i\gamma_r j} \qquad (r = 1, M) \tag{17}$$

$$U_r = U_{r-1} e^{-i\gamma_r N_r} \qquad (r = 1, M) \tag{18}$$

The required control forces obtained by substitution from Eqs. (17), (18) into Eqs. (2), (3) are

$$F_r = -\left[\omega_r m_r e^{-i\frac{1}{2}\gamma_r} - \omega_{r+1} m_{r+1} e^{-i\frac{1}{2}\gamma_{r+1}}\right](i\omega U_r) \qquad (r = 1, M-1) \quad , \tag{19}$$

$$F_M = -\omega_M m_M e^{-i\frac{1}{2}\gamma_M}(i\omega U_M) \tag{20}$$

3. NUMERICAL RESULTS

To illustrate the effects of control on the seismic response of non-uniform structures and to determine the magnitudes of the required control forces we consider models of 10- and 50-storey buildings consisting of three segments ($M = 3$) with the properties listed in Table 1 in which m_0 and k_0 are the average mass and stiffness per floor. For the 10-storey building the three segments end at the second, eighth and tenth floor, respectively. In the 50-storey building the segments end at the tenth, fortieth and fiftieth floors.

Table 1. Properties of 10- and 50-storey building models

r	m_r/m_0	k_r/k_0	ξ_r	N_r (10)	N_r (50)
S 1	1.34	1.70	0.03	2	10
2	0.95	0.94	0.03	6	30
3	0.81	0.48	0.03	2	10

The values of m_r/m_0 and k_r/k_0 were obtained from correlations for tall steel buildings in Japan. These correlations indicate that most of the variations in mass and stiffness take place in the lower and upper one-fifth of the buildings. In the calculations it was assumed that $\sqrt{k_0/m_0} = \omega_0 = 40.0$ rad/sec. The ground excitation was taken to correspond to the NS El Centro 1940 record (peak ground acceleration = 0.348 g, peak ground velocity = 33.4 cm/sec).

The amplitudes of the absolute displacement transfer functions for a 10-storey building are shown in Fig. 2 for the three cases corresponding to absence of control (Case 1), control by an absorbing boundary at the top (Case 2) and control by non-reflecting boundaries at the two discontinuities (2nd and 8th floor) and the top of the building (Case 3). The results for $|U_3/U_0|$ (10th floor), $|U_2/U_0|$ (8th floor), and $|U_1/U_0|$ (2nd floor) are shown in Figs. 2a, 2b and 2c, respectively. These results indicate that the use of an absorbing boundary at the top (Case 2) eliminates resonance but cannot eliminate the (smaller) amplification associated with the reduction of stiffness with height. Control by means of non-reflecting boundaries at the discontinuities and the top of the building (Case 3) eliminates all resonance and the amplification due to increased flexibility and leads to a structural response slightly lower than the motion of the base.

The amplitudes of the transfer functions $|U_3/U_0|$ (50th floor), $|U_2/U_0|$ (40th floor), and $|U_1/U_0|$ (10th floor) for a 50-storey building are shown in Figs. 3a, 3b and 3c, respectively. These results also show the beneficial effects of control.

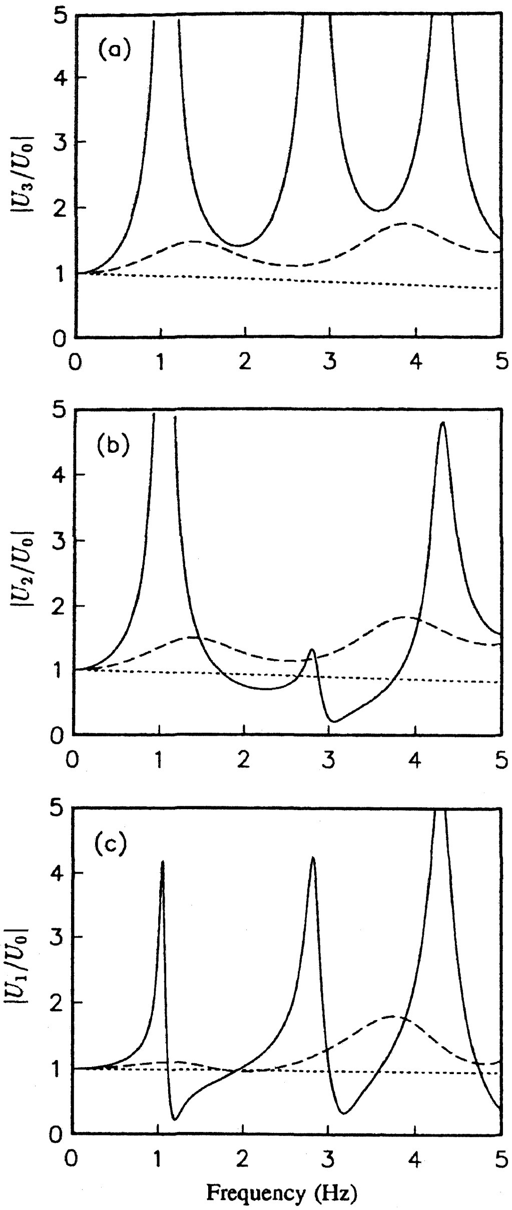

Fig. 2. Effect of control on the absolute displacement transfer functions for a 10-storey building. Solid, segmented and dotted lines represent, respectively, the results for Case 1 (absence of control), Case 2 (control by an absorbing boundary at the top) and Case 3 (control forces at top and at points of discontinuity). The results in Figs. 2a, 2b and 2c correspond to $|U_3/U_0|$ (10th floor), $|U_2/U_0|$ (8th floor) and $|U_1/U_0|$ (2nd floor), respectively.

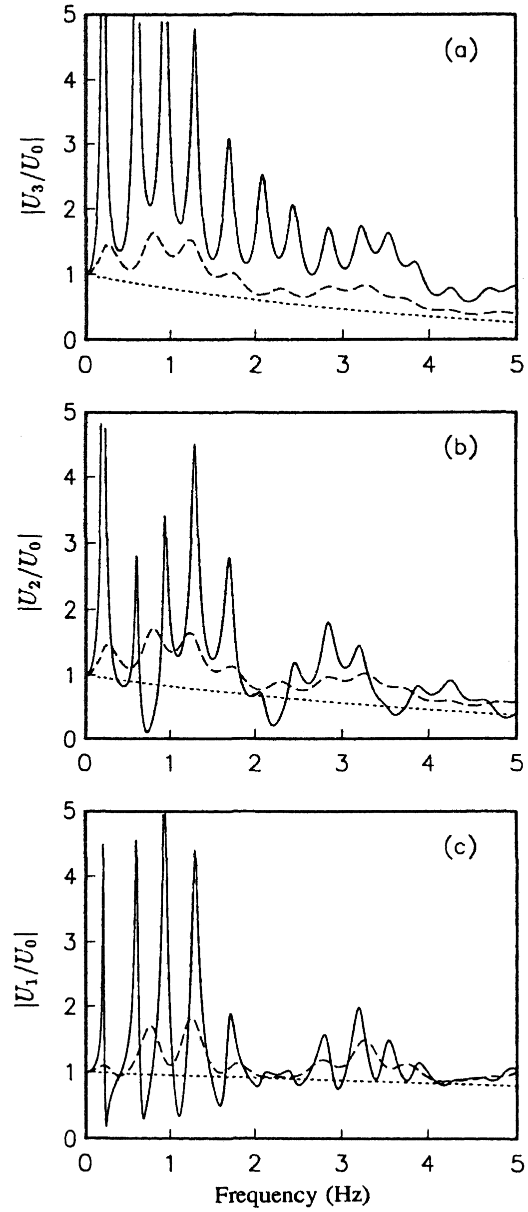

Fig. 3. Effect of control on the absolute displacement transfer functions for a 5-storey building. Solid, segmented and dotted lines represent, respectively, the results for Case 1 (absence of control), Case 2 (control by an absorbing boundary at the top) and Case 3 (control forces at top and at points of discontinuity). The results in Figs. 2a, 2b and 2c correspond to $|U_3/U_0|$ (50th floor), $|U_2/U_0|$ (40th floor) and $|U_1/U_0|$ (10th floor), respectively.

The amplitudes of the normalized values of the control forces $F_r/(i\omega\omega_0 m_0 U_0)$ (r = 3 for Case 2 and r = 1, 2, 3 for Case 3) for a 10-storey building are presented in Fig. 4. It is apparent that the required control forces (F_1, F_2, F_3) in Case 3 are smaller than the control force F_3 required in Case 2.

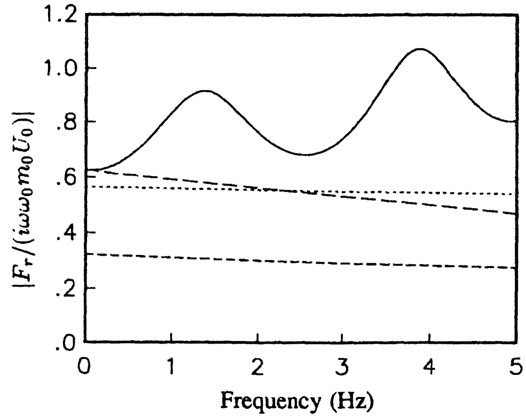

Fig. 4. Amplitudes of the normalized control forces $|F_r/(i\omega\omega_0 m_0 U_0)|$ (r=1, 3) for a 10-storey building. The solid line represents the control force F_3 (10th floor) for Case 2. The long segmented, short segmented and dotted lines represent, respectively, the control forces F_3 (10th floor), F_2 (8th floor) and F_1 (2nd floor) for Case 3.

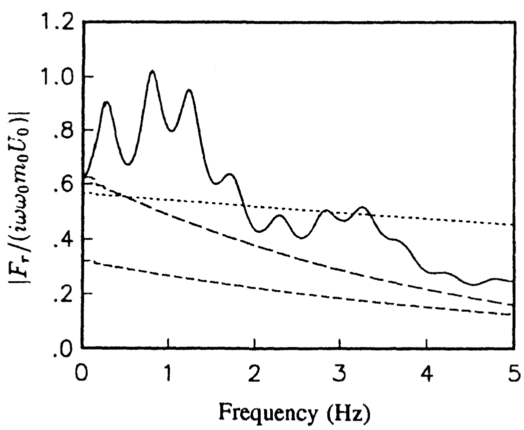

Fig. 5. Amplitudes of the normalized control forces $|F_r/(i\omega\omega_0 m_0 U_0)|$ (r=1, 3) for a 50-storey building. The solid line represents the control force F_3 (50th floor) for Case 2. The long segmented, short segmented and dotted lines represent, respectively, the control forces F_3 (50th floor), F_2 (40th floor) and F_1 (10th floor) for Case 3.

The corresponding control forces for a 50-storey building are shown in Fig. 5. In this case, the control force F_1 on the 10th floor discontinuity (Case 3) can be larger, at frequencies above 2 Hz, than the control force F_3 acting on the 50th floor in Case 2.

The effects of control on the peak velocities at different floors of 10- and 50-storey buildings subjected to the NS El Centro 1940 acceleration record are presented in Table 2. The results in Table 2 are obtained by Fourier synthesis and indicate that control significantly reduces the response of the structure. In Case 3, the peak absolute velocity within the building never exceeds the peak ground velocity (33.4 m/sec).

Table 2. Effects of control on peak velocities (cm/sec) of 10- and 50-storey buildings

No. of Floors	Floor	Case 1	Case 2	Case 3
10	10	160.9	43.8	29.5
	8	132.9	45.2	30.4
	2	44.7	36.4	32.6
50	50	63.5	34.3	22.1
	40	58.9	36.4	24.0
	10	41.6	36.1	30.1

The peak amplitudes of the required control forces (normalized by the weight of the average floor $m_0 g$) for the NS El Centro 1940 excitation are listed in Table 3. In Case 2, the peak of the control force at the top of the structure is of the order of the weight of the average floor. In Case 3, the peak values of the control forces are a smaller fraction of $m_0 g$.

Table 3. Peak amplitudes of control forces (normalized by $m_0 g$).

No. of Floors	Floor	Case 2	Case 3
10	10	1.118	0.757
	8	—	0.407
	2	—	0.747
50	50	0.879	0.569
	40	—	0.320
	10	—	0.697

Velocity time-histories at the 10th and 8th floors of a 10-storey building are shown in Fig. 6 for Cases 1, 2 and 3. These results for the El Centro 1940 excitation clearly illustrate the effects of control.

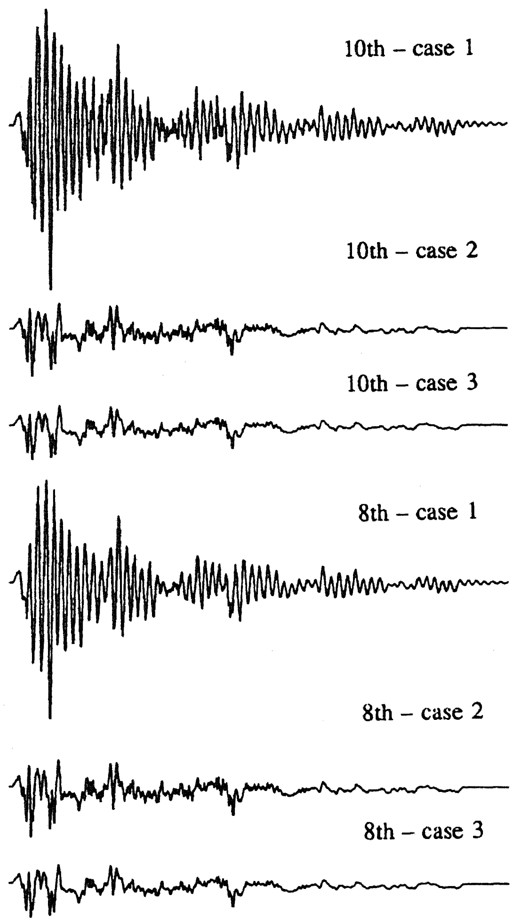

Fig. 6. Velocity time-histories (60 seconds) at the 10th (top three curves) and 8th (lower three curves) floors of a 10-storey building for Case 1 (absence of control), Case 2 (control by an absorbing boundary at the top) and Case 3 (control forces at top and at points of discontinuity).

4. CONCLUSIONS

It has been found that control by means of an absorbing boundary placed at the top of a non-uniform structure drastically reduces the seismic response of the structure. For a typical 10-storey building this control strategy leads to a reduction of the peak velocity at the top of the building by a factor of the order of 3.7. For the El Centro 1940 excitation, the peak of the required control force at the top of the building is of the order of the weight of the average floor.

Further reductions of the seismic response can be obtained by use of additional control forces located at elevations where changes in stiffness and/or mass take place. If the control forces are selected so that no downward propagating waves are obtained at these discontinuities, the absolute response of the building is smaller than the motion of the base. With this type of control the peak response at the top of a 10-storey building is reduced by a factor of the order 5.5 with respect to the response when no control forces are used. This additional reduction is obtained at the expense of introducing additional control forces. These forces are somewhat smaller than the force required when control is achieved by use of only an absorbing boundary at the top of the building.

5. ACKNOWLEDGEMENTS

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