

## Effects of soil-structure interaction on the seismic response of structures subjected to active control

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**ABSTRACT:** The effects of soil-structure interaction on the form of the control rule and on the effectiveness of active control of the seismic response of structures are examined. The structure is modeled as a uniform shear beam supported on a rigid foundation embedded in an elastic soil and subjected to vertically incident waves. Active control in the form of an absorbing boundary located at the top of the structure is considered. The active absorbing boundary cancels the reflection of waves at the top of the structure and eliminates resonance within the superstructure. It is found that the rocking of the foundation changes the form of the control rule. However, the effectiveness of this form of active control is not degraded when soil-structure interaction effects are included.

### 1. INTRODUCTION

A common assumption in most studies on active control of the seismic response of structures is that soil-structure interaction effects are small and, in particular, that the rocking motion of the base is negligible. The objective of this study is to remove these assumptions and to consider the seismic response of tall structures subjected to active control when the flexibility of the soil is included in the analysis. In this paper, the structure is modeled as a uniform shear beam supported on a rigid foundation embedded in the soil represented by a uniform viscoelastic half-space (Fig. 1). The seismic excitation is represented in the form of vertically incident SH-waves. The kinematic interaction effects associated with the embedment of the foundation together with the inertial interaction effects result in a base motion that includes translational and rocking response components. The seismic response of the structure including soil-structure interaction effects is modified by use of a control force acting at the top of the structure. The active control strategy used here is based on the work of Vaughan (1968) and von Flotow (1986) in which the energy flow within the structure is modified by controlling the reflection and/or transmission of waves at end points or at joints. In this study, the active control force is selected to simulate an absorbing boundary such that all upward propagating waves are absorbed at the top of the structure and no downward propagating waves are reflected at that point.

Applications of this approach to the active control of the seismic response of tall structures in the absence of soil-structure interaction effects have been presented by Mita and Luco (1990a,b) and Luco et al (1992).

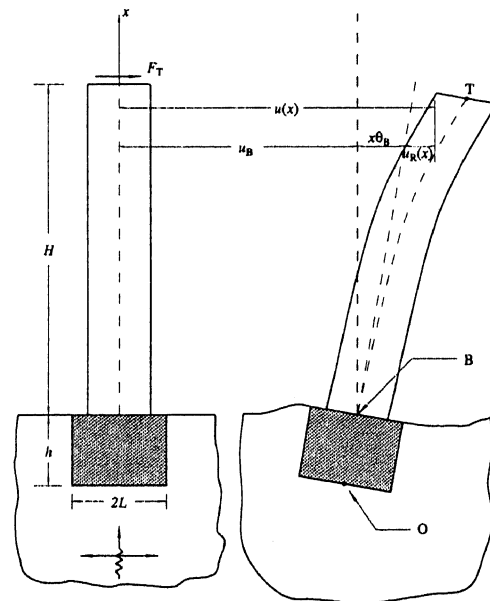


Figure 1. Description of the Model

## 2. FORMULATION OF THE PROBLEM AND SOLUTION

### 2.1 Basic equations

We consider first the motion of the superstructure for a given translation and rotation of the base. The equation of motion of the structure and the boundary conditions for harmonic vibrations with time dependence  $e^{i\omega t}$  are given by

$$u'' + (\omega/\beta_B)^2 u = 0, \quad 0 < x < H \quad (1)$$

$$u(0) = u_B \quad (2)$$

$$\rho_B A_B \beta_B^2 [u'(H) - \theta_B] = F_T \quad (3)$$

where the prime denotes derivative with respect to  $x$ ,  $u(x)e^{i\omega t}$  is the total translation of the structure with respect to an inertial frame of reference,  $u_B e^{i\omega t}$  is the horizontal motion of the base of the structure (top of the rigid foundation),  $\theta_B$  is the rocking angle of the base,  $F_T e^{i\omega t}$  is the active control force applied at the top ( $x = H$ ) of the structure,  $\rho_B$  and  $A_B$  are the density and cross-sectional area of the shear wall, respectively, and  $\beta_B = \beta_B(1 + 2i\xi_B)^{1/2}$ , in which  $\xi_B$  is the hysteretic damping ratio in the structure and  $\beta_B$  is (approximately) the shear wave velocity within the structure. The general solution of Eq. (1) is

$$u(x) = A e^{-i(\omega x/\beta_B)} + B e^{i(\omega x/\beta_B)} \quad (4)$$

in which the first and second terms represent upward and downward propagating waves, respectively.

### 2.2 Structural response without absorbing boundary (Case 1)

To evaluate the effectiveness of active control we consider first the response of the structure when no control is provided. In this case, the boundary condition at the top of the structure is given by

$$F_T = \rho_B A_B \beta_B^2 [u'(H) - \theta_B] = 0 \quad (5)$$

Substitution of the general solution given by Eq. (4) into the boundary conditions given by Eqs. (2) and (5) permits us to determine the unknown coefficients  $A$  and  $B$  with the result

$$u(x) = \left[ \cos(\omega x/\beta_B) + \tan(\omega H/\beta_B) \sin(\omega x/\beta_B) \right] u_B + (\beta_B/\omega L) \frac{\sin(\omega x/\beta_B)}{\cos(\omega H/\beta_B)} L \theta_B \quad (6)$$

### 2.3 Structural response with absorbing boundary (Case 2).

If the control force  $F_T$  is selected so that an absorbing boundary is obtained at the top of the structure, then no reflected waves are obtained and  $B = 0$ . In this case, Eqs. (2) and (4) with  $B = 0$  lead to

$$u(x) = u_B e^{-i(\omega x/\beta_B)} \quad (7)$$

The required control force  $F_T$  is obtained by substitution from Eq. (7) into Eq. (3). The resulting expression is

$$F_T = -\rho_B A_B \beta_B \dot{u}(H) - \rho_B A_B \beta_B^2 \theta_B \quad (8)$$

where the dot denotes differentiation with respect to time [ $\dot{u}(H) = i\omega u(H)$  for harmonic time dependence]. The control rule given by Eq. (8) differs from previous expressions derived under the assumption of negligible rocking (Mita and Luco, 1990a,b) in that it depends not only on  $\dot{u}(H)$  but also on  $\theta_B$ .

### 2.4 Structural response with a simplified absorbing boundary (Case 3).

It is of interest to consider the response of the structure when the control rule given by Eq. (8) is replaced by the simpler expression

$$F_T = -\rho_B A_B \beta_B \dot{u}(H) \quad (9)$$

Eq. (9) corresponds to the form of the control rule when rocking of the base is ignored (Mita and Luco, 1990a,b). In this case, substitution of the general solution, given by Eq. (2), into the boundary conditions given by Eqs. (2) and (3) with  $F_T$  given by Eq. (9) results in the solution

$$u(x) = u_B e^{-i(\omega x/\beta_B)} + \theta_B [\sin(\omega x/\beta_B)/(\omega/\beta_B)] e^{-i(\omega H/\beta_B)} \quad (10)$$

The solution in this case includes both upward and downward propagating waves in the structure.

### 2.5 Soil-structure interaction equations

It can be shown (Luco and Wong, 1982) that the generalized displacement of the bottom of the foundation  $\{\tilde{U}_0\} = (u_0, L\theta_0)^T$  normalized by the half-width  $L$  of the foundation is given by

$$\{\tilde{U}_0\} = ([I] + [C(\omega)])^{-1} \left( [\tilde{K}_{B0}(\omega)] - (a_0^2/\rho_s L^3) [\tilde{M}_0] \right) \{\tilde{U}_0^*\} \quad (11)$$

where  $[I]$  is the  $2 \times 2$  identity matrix,  $[C(\omega)]$  is the  $2 \times 2$  normalized foundation compliance matrix,  $[M_0]$  is the normalized mass matrix for the rigid foundation,  $a_0 = \omega L / \beta_s$  is a dimensionless frequency normalized by  $L$  and by the shear wave velocity of the soil  $\beta_s$ ,  $\rho_s$  is the density of the soil and  $\{U_0^*\} = (u_0^*, L\theta_0^*)^T$  is the effective input motion to the foundation. In this study, we assume that the seismic excitation corresponds to vertically incident shear waves characterized by the free-field ground motion on the soil surface  $u_g(\omega)e^{i\omega t}$ . In this case,

$$u_0^* = S(a_0)u_g \quad , \quad L\theta_0^* = R(a_0)u_g \quad (12)$$

where  $S(a_0)$  and  $R(a_0)$  are scattering coefficients which depend on the dimensionless frequency  $a_0$  and on the characteristics of the foundation and the soil. The scattering coefficients and the elements of the compliance matrix (or, of its inverse the impedance matrix) can be obtained from published results (e.g., Mita and Luco, 1989). The  $2 \times 2$  matrix  $[\tilde{K}_{B0}(\omega)]$  appearing in Eq. (11) relates the generalized force  $\{\tilde{F}_{B0}\} = (F_{B0}, M_{B0}/L)^T$  that the superstructure exerts at the bottom of the foundation with the motion  $\{\tilde{U}_0\}$  of the bottom of the foundation. The elements of the matrix  $[\tilde{K}_{b0}(\omega)]$  are obtained by considering the linear and angular momenta of the superstructure and are listed in a previous paper by the authors [Wong and Luco (1991)]. In Eqs. (11) and (12),  $[C(\omega)]$ ,  $[\tilde{K}_{B0}(\omega)]$ ,  $\{\tilde{U}_0^*\}$ ,  $S(a_0)$  and  $R(a_0)$  are referred to the center of the bottom of the foundation.

Once the motion  $\{\tilde{U}_0\}$  of the bottom of the foundation has been obtained by use of Eqs. (11), the motion  $\{\tilde{U}_B\} = (u_B, L\theta_B)^T$  at the base of the superstructure, the response  $u(x)$  at any point in the structure, the base shear force  $F_B$ , the base overturning moment  $M_B$  and the required control force  $F_T$  can be easily calculated.

### 3. NUMERICAL RESULTS

To study the effects of soil-structure interaction on the effectiveness of active control of the seismic response of structures we have considered simplified models of a 10- and 50-storey buildings founded on soils with different rigidities. The characteristics of the models for the structures and foundations are listed in Table 1. The foundation was modeled as rigid rectangular block of base dimension  $2L \times 2L$  embedded to a depth  $h$  in the soil. The soil was modeled as a uniform viscoelastic half-space characterized by complex wave velocities  $\alpha_s = \alpha_s(1 + 2i\xi_\alpha)^{1/2}$  and  $\beta_s = \beta_s(1 + 2i\xi_\beta)^{1/2}$  for P- and S-waves, respectively. To take advantage of the numerical results presented by Mita and Luco (1989)

for the impedance functions and scattering coefficients of square embedded foundations it was assumed that  $\alpha_s = 2\beta_s$  ( $\nu \approx 1/3$ ),  $\xi_\alpha = 0.0005$  and  $\xi_\beta = 0.001$ . Three values of the soil shear wave velocity  $\beta_s$  corresponding to  $\beta_s = 1500$  m/sec, 300 m/sec and 150 m/sec were used to represent stiff, intermediate and soft soil conditions.

Table 1. Characteristics of Structures and Foundations Considered.

	10-storey building	50-storey building
$T_1$ (sec)	1.00	5.00
$\xi_B$	0.02	0.02
$\beta_B$ (m/sec)	150.00	150.00
$H/L$	3.75	9.38
$(\rho_B A_B H / \rho_s L^2 H)$	1.00	1.00
$(\rho_B I_B H / \rho_s L^5)$	0.	0.
$L$ (m)	10.00	20.00
$h/L$	0.50	0.50
$(M_F / \rho_s L^3)$	0.70	0.70
$(I_F / \rho_s L^5)$	0.35	0.35

Numerical results for the 10- and 50-storey building models were obtained for three soil conditions and for three cases corresponding to the absence of control (Case 1), control by the exact absorbing boundary defined by Eq. (8) (Case 2), and control by the simplified absorbing boundary defined by Eq. (9) (Case 3). Results in the frequency domain for a 10-storey building and intermediate soil conditions  $\beta_s = 300$  m/sec are shown in Fig. 2. The amplitudes of the transfer functions  $|u_T/u_g|$  where  $u_T = u(H)$  is the total motion at the top of the structure and  $u_g$  is the free-field motion of the ground surface are shown in Fig. 2(a). A first observation is that both the exact (Case 2) and the simplified (Case 3) absorbing boundaries drastically reduce the response and eliminate all resonant behavior. Both absorbing boundaries lead to almost the same response at the top. The results for  $|u_T/u_g|$  also indicate that the beneficial effects introduced by active absorbing boundaries are not reduced in any way by soil-structure interaction effects. The normalized values for the amplitude of the required control force  $|F_T/(-i\omega\rho_B A_B \beta_B u_g)|$  shown in Fig. 2(b) indicate that both types of absorbing boundaries lead to almost the same control force. The amplitudes of the transfer functions  $|u_B/u_g|$  and  $|L\theta_B/u_g|$  for the translational and rocking response at the base of the structure shown in Figs. 2(c) and 2(d), respectively, indicate that the use of absorbing boundaries reduces the inertial interaction effects on the base translation and rotation.

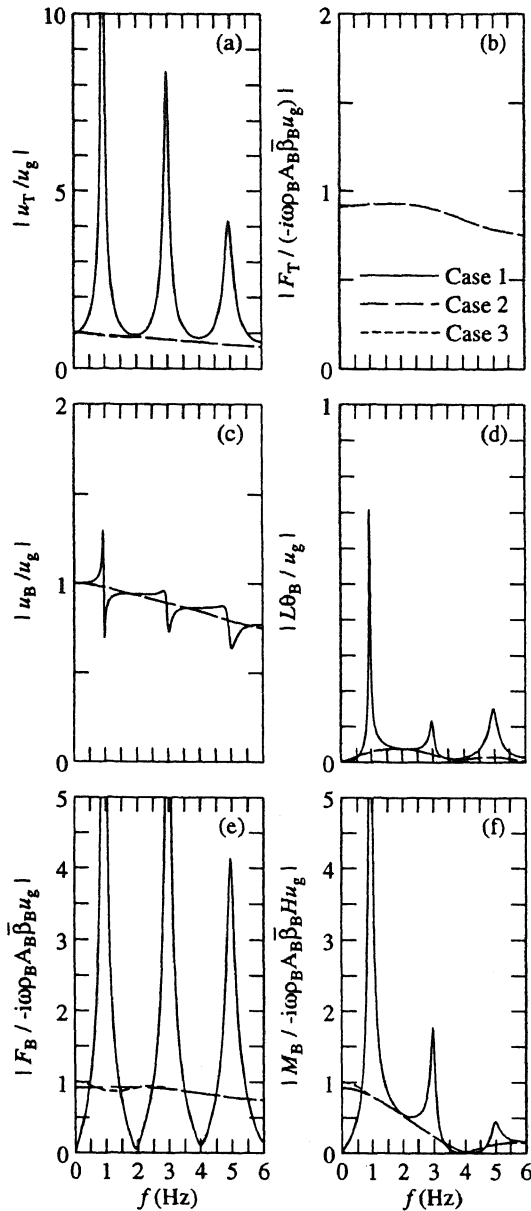


Fig. 2. Normalized Amplitudes of: (a) Top Translation  $|u_T/u_g|$ , (b) Control Force  $|F_T/(-i\omega\rho_B A_B \beta_B u_g)|$ , (c) Base Translation  $|u_B/u_g|$ , (d) Base Rotation  $|L\theta_B/u_g|$ , (e) Base Shear Force  $|F_B/(-i\omega\rho_B A_B \beta_B u_g)|$  and (f) Base Overturning Moment  $|M_B/(-i\omega\rho_B A_B \beta_B H u_g)|$  for a 10-Storey Building on an Intermediate Soil ( $\beta_s = 300$  m/sec). Cases 1, 2 and 3 Correspond, Respectively, to Absence of Control, Control by an Exact Absorbing Boundary and Control by a Simplified Absorbing Boundary.

In particular, the rocking response is drastically reduced when absorbing boundaries are used. As in the case of the response at the top, both types of absorbing boundaries (Cases 2 and 3) lead to almost the same response at the base. The normalized amplitudes  $|F_B/(-i\omega\rho_B A_B \beta_B u_g)|$  and  $|M_B/(-i\omega\rho_B A_B \beta_B H u_g)|$  of the base shear force and base overturning moment are shown in Figs. 2(e) and 2(f), respectively. These results indicate that the use of absorbing boundaries strongly reduces the base shear force and the base overturning moment in the vicinity of the characteristic frequencies of the system without control (Case 1). At other frequencies these quantities may be increased by the use of control through absorbing boundaries.

The effects of soil-structure interaction on the amplitude of the normalized displacement  $|u_T/u_g|$  at the top of the building and of the normalized control force  $|F_T/(-i\omega\rho_B A_B \beta_B u_g)|$  for a 50-storey building are summarized in Fig. 3. These results indicate that the normalized amplitudes of the top displacement and of the required control force decrease as the soil becomes softer. It appears that this reduction is mainly associated with kinematic interaction effects and, consequently, will be stronger for larger structures founded on deeper foundations.

Some typical results in the time domain are presented in Fig. 4 for a 10-storey building and an intermediate soil ( $\beta_s = 300$  m/sec). These results were obtained by Fourier synthesis for a free-field ground motion on the ground surface corresponding to the NS component of the El Centro 1940 record. The results in Fig. 4 illustrate the effects of control on the translational ( $\dot{u}_B$ ) and rocking ( $L\dot{\theta}_B$ ) velocities at the base of the superstructure and on the velocity at the top of the structure ( $\dot{u}_T$ ). It is apparent from Fig. 4 that control by means of an absorbing boundary drastically reduces the rocking response at the base and the translational response at the top. Also shown in Fig. 4 is the time history of the required control force (expressed as a fraction of the total weight of the superstructure.)

The effects of control and soil-structure interaction are illustrated in Table 2 listing the peak values of the velocity response at the top ( $\dot{u}_T$ ) and base ( $\dot{u}_B$ ) of the structure, the peak rocking velocity ( $L\dot{\theta}_B$ ) at the base and the peak value of the required control force  $F_T$ . These results show that:

- (i) control drastically reduces the rocking response at the base and the translational response at the top,
- (ii) the exact and approximate absorbing boundaries lead to almost the same results, and
- (iii) the peak amplitude of the required control force decreases as the soil becomes softer.

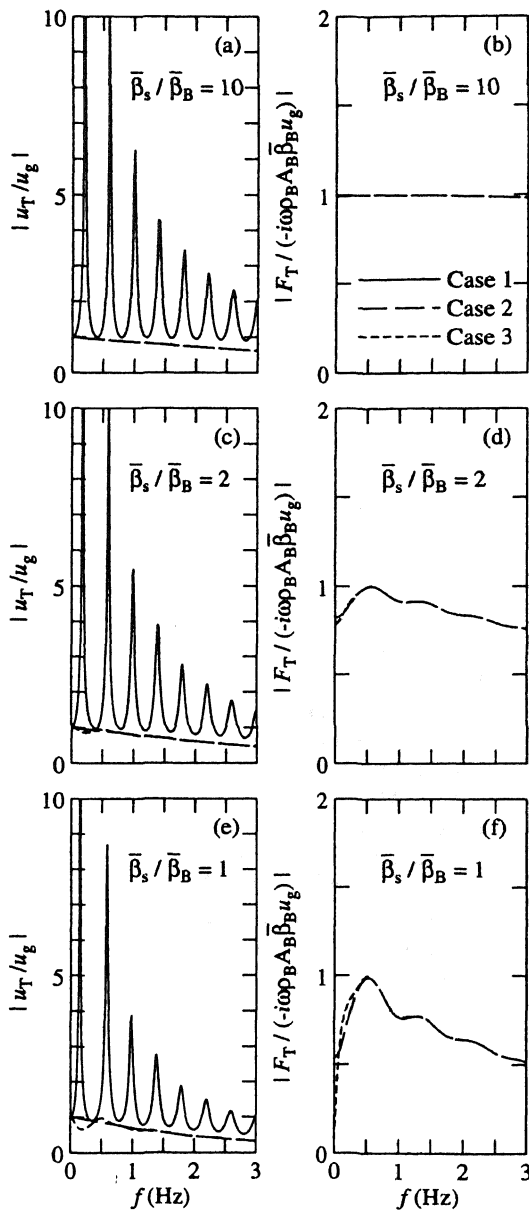


Fig. 3. Normalized Amplitudes of the Top Translation  $|u_T/u_g|$  and of the Control Force  $|F_T/(-i\omega\rho_B A_B \beta_B u_g)|$  for a 50-Storey Building and Three Soil Conditions ( $\beta_s/\beta_B = 10, 2$  and  $1$ ). The Cases Shown Correspond to Absence of Control, Control by an Exact Absorbing Boundary and Control by an Approximate Absorbing Boundary.

Table 2. Effect of Active Control on the Peak Values of the Response of Ten- and Fifty-Storey Buildings on Soft, Intermediate and Stiff Soils [El Centro 1940 NS Excitation].

		$N = 10$			$N = 50$		
		$\beta_s$ (m/sec)			$\beta_s$ (m/sec)		
Comp	Case	150	300	1500	150	300	1500
$\dot{u}_T$ ( $\frac{cm}{sec}$ )	1	102.9	142.5	160.4	61.04	59.69	59.32
	2	25.39	28.82	31.04	19.58	23.00	24.60
	3	22.83	27.90	31.02	19.41	22.63	24.59
$\dot{u}_B$ ( $\frac{cm}{sec}$ )	1	26.46	29.81	33.00	21.48	27.02	32.69
	2	26.21	30.44	33.04	23.00	27.09	32.69
	3	26.13	30.47	33.04	23.09	27.09	32.69
$L\dot{\theta}_B$ ( $\frac{cm}{sec}$ )	1	7.01	3.17	0.145	3.52	1.357	0.089
	2	2.51	0.881	0.041	1.67	0.775	0.088
	3	2.38	0.863	0.041	1.62	0.783	0.088
$F_T/Mg$	1	—	—	—	—	—	—
	2	.103	.122	.135	.0199	.0223	.0267
	3	.102	.122	.135	.0201	.0223	.0267

#### 4. CONCLUSIONS

The effects that the interaction between the structure and the soil may have on the possibility of using active control techniques to modify the seismic response of structures have been studied. It has been found that the rocking of the foundation resulting from the kinematic and inertial interaction effects changes the form of the control rule required to obtain an active absorbing boundary at the top of the structure. Active control by means of an absorbing boundary which includes the rocking effects and by a simplified absorbing boundary which ignores the rocking effects result in large reductions of the structural response even when soil-structure interaction effects are included. In fact, the amplitudes of the required control force and of the structural response decrease as the soil becomes softer. The use of control by means of absorbing boundaries also reduces the inertial interaction effects and, in particular, drastically reduces the rocking response of the structure.

#### 5. ACKNOWLEDGEMENT

The work described here was supported by grants from the Ohsaki Research Institute, Shimizu Corporation to the University of Southern California and the University of California, San Diego.

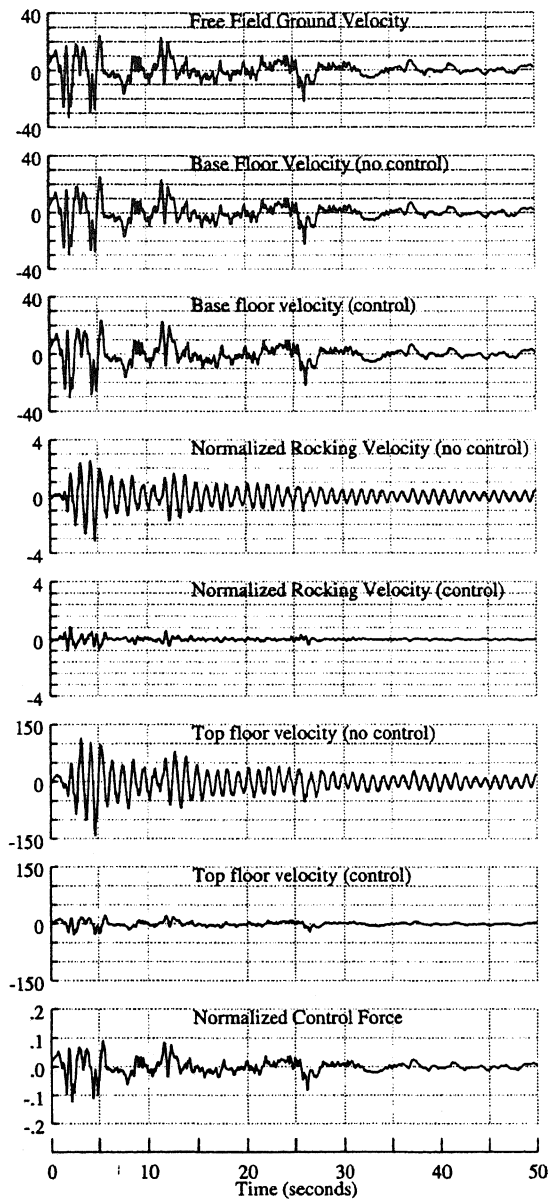


Fig. 4. Velocity Response (cm/sec) and Control Force (Fraction of Building Weight) for a 10-storey Building on an Intermediate Soil ( $\beta_s = 300$  m/sec) Subjected to the El Centro 1940 Free-Field ground Motion.

## 6. REFERENCES

- Luco, J. E. and H. L. Wong 1982. Response of Structures to Nonvertically Incident Seismic Waves. *Bull. Seism. Soc. Am.*, 72, 275-302.
- Luco, J. E., H. L. Wong and A. Mita 1992. Active Control of the Seismic Response of Structures by Combined Use of Base Isolation and Absorbing Boundaries. *Earthquake Engineering and Structural Dynamics*, (in press).
- Mita, A. and J. E. Luco 1989. Impedance Functions and Input Motions for Embedded Square Foundations. *Journal of Geotechnical Engineering*, GED, ASCE, 115, 491-503.
- Mita, A. and J. E. Luco 1990a. Active Vibration Control of a Shear Beam with Variable Cross Section. *Proceedings of the 1990 Dynamics and Design Conference, Japan Society of Mechanical Engineers* (Kawazaki, Japan, July 9-12), 276-279.
- Mita, A. and J. E. Luco 1990b. New Active Control Strategy for Tall Buildings. *Proceedings of the Eighth Japan Earthquake Engineering Symposium*, Tokyo, Japan, 1869-1874.
- Vaughan, D. R. 1968. Application of Distributed Parameter Concept to Dynamic Analysis and Control of Bending Vibrations. *Journal of Basic Engineering*, June, 157-166.
- von Flotow, A. H. 1986. Travelling Wave Control of Large Spacecraft Structures. *Journal of Guidance, Control and Dynamics*, 9, 462-468.
- Wong, H. L. and J. E. Luco 1991. Structural Control Including Soil-Structure Interaction Effects. *Journal of Engineering Mechanics*, 1, 2237-2250.