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A study of applicability of vibration control to nonlinear structure for seismic excitation

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ABSTRUCT: The optimal control force could be decided instantaneously from the observed ground motion and structural responses by minimizing the sum of the time-dependent performance index according to the instantaneous optimal control algorithms that was introdused by J.N.Yang (A.S.C.E. 1987). By applying this algorithm, the state space equation including the nonlinearity of structure could be introduced. In this study, nonlinear characteristic was expressed as bilinear-type, and it can be modeled by a sum of stiffness of an elastic spring and another elastic spring with coulomb slider. By using the proposed method to an exsiting five story structure, several analysis were carried out, and about their result were discussed.

1 INTRODUCTION

Recently, several kind of mechanisms have been proposed for vibration control of high-rise structure excited by wind. However, in the case that was affected by strong earthquake motion, control force should be tremendously large to keep the structure within the limit of elasticity. Therefore, to introduce the method of vibration control to take into account nonlinearity of the structure is more reasonable.

In this study, the instantaneous optimal control algorithm to take into account nonlinearity of structure was established by using time-dependent performance index J, which is introduced by J.N.Yang. And the results by applying the method to an existing five story structure were analytically carried out and discussed.

2 CONTROL ALGORITHMS ON NONLINEAR SYSTEM

In this study, a structure with active control is considered as shown in Fig.1. In this figure, $\ddot{z}_G$ is a one-dimensional earthquake ground acceleration, m_i is mass of i-th story, and x_i is relative story displacement between i-th and (i-1)th story. Nonlinear characteristics that applied here is bilinear-type, and it can be modeled by a sum of stiffness of an elastic spring k_{ei} and another elastic spring with a coulomb slider k_{si}. An initial stiffness of each stories can be expressed as $k_{ei}+ k_{si}$. And under the condition that the deformation is over the yield point on each stories, the stiffness can be expressed only by using elastic spring k_{ei}, because coulomb slider begins to slip at yield point and the stiffness of spring with coulomb slider becomes zero.

Relative story displacement x_i can be expressed as a sum of the expansion of

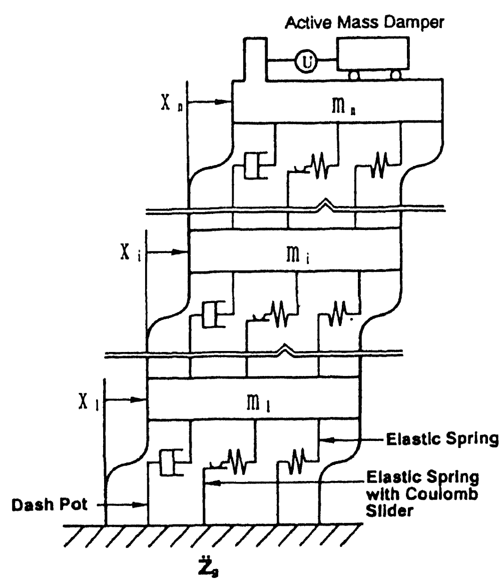


Fig.1 Vibration Model of Structurare According to Nonlinearity

spring with coulomb slider $w_i(t)$ and slip length of coulomb slider $v_i(t)$ as follows.

$$x_i(t) = w_i(t) + v_i(t) . \quad (1)$$

A shear force of each story can be obtained as follows.

$$\Phi_i(x_i(t)) = k_{ei} x_i(t) + k_{si} w_i(t) . \quad (2)$$

This system has elasto-plastic property, and the hysteresis should be satisfied following differential equation.

$$\dot{w}_i(t) = x_i(t) \{ 1 - H(x_i(t))H(\dot{w}_i(t) - Y_i) - H(-x_i(t))H(-\dot{w}_i(t) - Y_i)\}$$

$$\equiv g_{wi} (x_i(t), \dot{w}_i(t)) , \quad (3)$$

in which, H (·) means Heaviside step function, and Y_i means the expansion of spring when the coulomb slider begins to slide.

From Eq.(3), the motion of this system can be explained as follows. In the period while $w_i(t)$ is smaller than Y_i, coulomb slider does not slide, and the differentiation of slip length $v_i(t)$ becomes zero and the stiffness of the story can be given by $k_{ei} + k_{si}$. When the coulomb slider begins to slide, that is, $w_i(t) = Y_i$, the differentiation of expansion of spring $w_i(t)$ becomes zero, and the stiffness of the story is k_{ei}. As the slip motion of coulomb slider and expansion motion of spring have same direction, following relation as $w_i(t) \cdot v_i(t) > 0$ can be obtained, and it can be considered they are independent each other in this condition.

To take into account nonlinearity of structure, equation of motion that was combined with Eq.(3) with active control can be obtained as state space representation as follows.

$$\dot{Z}(t) = A Z(t) + B U(t) + F_0(t) + W_1 z_g(t) , \quad (4)$$

in which

$$\left.\begin{aligned} Z(t) &= \begin{Bmatrix} X(t) \\ \dot{X}(t) \\ \Omega(t) \end{Bmatrix} , \\ X(t) &= \{ \cdots x_i(t) \cdots \}^T , \\ \dot{X}(t) &= \{ \cdots \dot{x}_i(t) \cdots \}^T , \\ \Omega(t) &= \{ \cdots w_i(t) \cdots \}^T , \end{aligned}\right\} \quad (5)$$

$$A = \begin{bmatrix} 0 & I & 0 \\ -M^{-1}Ke & -M^{-1}C & -M^{-1}Ks \\ 0 & 0 & 0 \end{bmatrix} , \quad (6)$$

$$B = -\begin{bmatrix} 0 \\ M^{-1}H \\ 0 \end{bmatrix} , \quad (7)$$

$$H = \begin{bmatrix} \cdot & & & & \\ & \cdot & & & \\ & & \delta_i & & \\ & & & \cdot & \\ & & & & \cdot \end{bmatrix} , \quad (8)$$

$$U(t) = \{ \cdots u_i(t) \cdots \}^T , \quad (9)$$

$$\left.\begin{aligned} F_0(t) &= \begin{Bmatrix} 0 \\ 0 \\ G_w(t) \end{Bmatrix} , \\ G_w(t) &= \{ \cdots \dot{w}_i(t) \cdots \}^T , \end{aligned}\right\} \quad (10)$$

$$\left.\begin{aligned} W_1 &= -\begin{Bmatrix} 0 \\ M^{-1}m \\ 0 \end{Bmatrix} , \\ m &= \{ \cdots m_i \cdots \}^T , \end{aligned}\right\} \quad (11)$$

in which, X(t) is relative story displacement vector, $\dot{X}(t)$ is relative story velocity vector and $\Omega(t)$ is vector that expresses expansion of spring with coulomb slider. A is expressed as shown in Eq.(6) by using mass matrix M (in lumped mass system), viscous damping matrix C and stiffness matrix K_e and K_s. B is expressed as Eq.(7) by using a matrix H which specifies the location of active controllers, and U(t) is control vector. In Eq.(8), if active controllers put on i-th mass, δ_i is set as 1, other terms is set as 0. $F_0(t)$ is the vector which indicates hysteresis character. W_1 is expressed as Eq.(11) by using the vector m which convert inertia force to external force of the structure.

In this study, we used following time-dependent performance index J that was introduced by J.N.Yang. By applying this algorithm to the system described at Eq.(4), optimal control algorithm including nonlinearity can be established.

$$J(t) = Z^T(t)Q(t)Z(t) + U^T(t)RU(t) \rightarrow \text{Min}, \qquad (12)$$

in which Q and R are weighting matrix.

We adopt a principle that the control force can be decided at every time step by minimizing the total energy of structure, which is obtained from the sum of vibration energy and the control energy. Therefore, Q is utilized as follows considering the time-dependent property of stiffness because of its nonlinearity.

$$Q = 1/2 \begin{bmatrix} Ke & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & Ks \end{bmatrix}, \qquad (13)$$

$$R = \begin{bmatrix} \cdot & & & & \\ & \cdot & & & \\ & & R_i & & \\ & & & \cdot & \\ & & & & \cdot \end{bmatrix}. \qquad (14)$$

in which R_i is the weighting coefficient of control force on i-th mass.

3 MODELING THE EXISTING STRUCTURE

Some case of analysis were carried out. The model of structure that was used in this analysis had five stories, 45m × 90m plan and 35m height. Static inelastic analysis was carried out about this model, five stories mass-spring vibration model shown in Table.1 was obtained. Load-deformation relations in this model were shown in Fig.2.

Table 1 Structural Properties of a nonlinear model

No.	Mass (t)	stiffness (tf/cm)		Yield point
i	m_i	$k_{e i}$	$k_{s i}$	(cm)
1st	1.22	111	15	11.4
2nd	0.35	605	50	3.6
3rd	2.66	1049	130	3.6
4th	3.16	2490	290	1.8
5th	6.22	4814	400	1.35

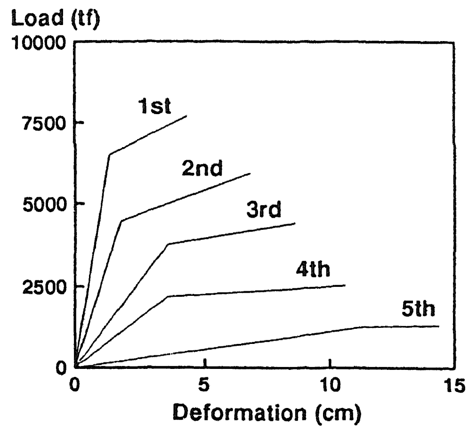


Fig.2 Load-Deformation Relation of Each Stories

In this model, stiffness of fifth story is relatively weakened, so it can be predicted that the story vibrates as like a rod swings.

From the result of eigenvalue analysis the characteristics of vibration mode of linear system was obtained. 1st to 3rd modes were shown in Fig.3. 1st natural period is 0.813sec, and this mode shows that the top of mass vibrate largely.

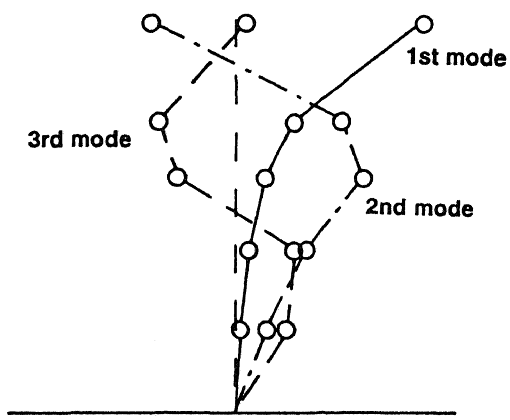


Fig.3 Result of Eigenvalue Analysis

Time histories of the response of the relative story displacement without vibration control are shown in Fig.4. In these figures, result with and without considering nonlinearity of structure are shown, respectively. As the input excitation, record of Imperial Valley (El Centro(NS)) occurred at 1940 was utilized amplifying its maximum acceleration to 511gal (50kine). Only the response of fifth story becomes very large and it indicates

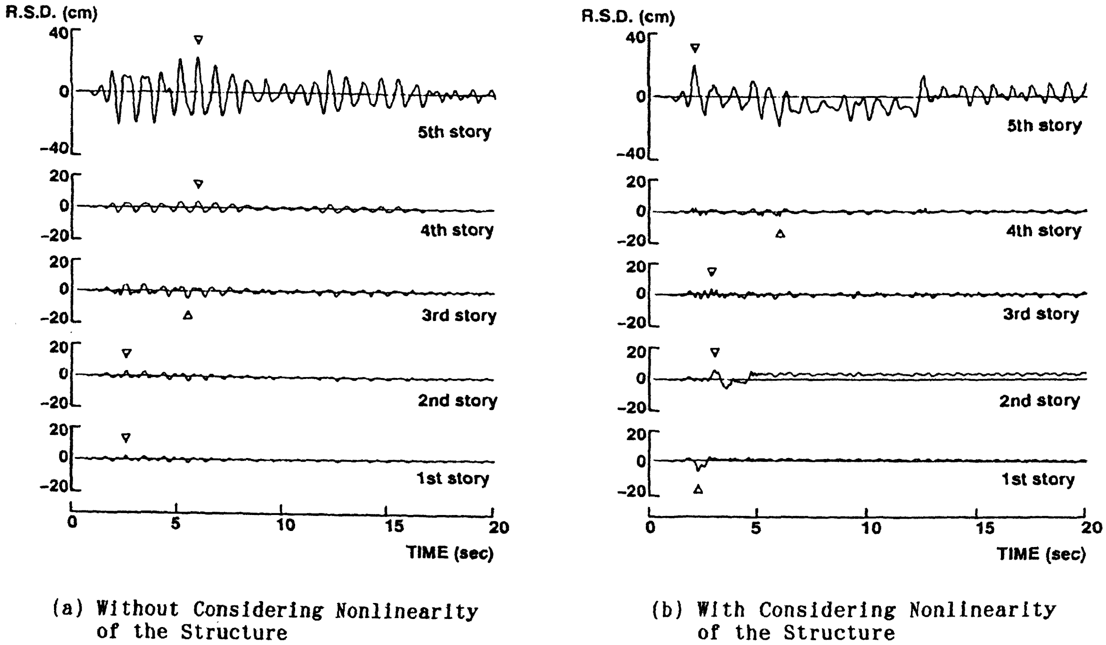


Fig.4 Time Histories of the Response of Relative Story Displacement on each stories without vibration control

that it behaves as like a rod swings. In the case of considering nonlinearity, first, second, and fifth stories behave over the elastic limit under the such severe earthquake motions. It can be seen that the response to take into account the nonlinearity is different from that of elastic analysis.

4 RESULT OF ANALYSIS

4.1 COMPARISON OF SEVERAL KIND OF VIBRATION CONTROL SYSTEM

Abilities of the vibration control for the structure by using passive system, semi-active system and active system were compared under the condition with and without considering the nonlinearity of the structure. In semi-active system, An control force is supplemented between modal mass and tuned mass damper. These 3 type systems are shown in Fig.5. The results of each system for maximum value of relative story displacement and control force were shown in Table 2.

As a passive system, tuned mass damper was considered (mass ratio to whole mass of structure is 0.0068, and its natural period is tuned to 1st natural period of the model 0.813sec). In the passive system, without considering nonlinearity of structure, the maximum value of relative story displacement was smaller than that of without control system. But with considering nonlinearity of structure, the maximum value of relative story displacement was lager than that of without control system. It is shown that the lengthening of natural period of the structure by the effect of nonlinearity was occurred, and the tuning condition in optimum was out of order.

In the semi-active system, even in the case with considering nonlinearity of structure, its ability of vibration control was no less than that of linear system. It is shown that the control force acted effectively as self-tuning mechanism to the structure that was lengthened its natural period by the effect of nonlinearity.

In the active system, the maximum value of response reduced to about one half in proportion to that of noncontrolled case in both case without considering nonlinearity and with.

From upper study, it can be said that the active system is the most suitable system in this case that nonlinear behavior of the structure is remarkable.

With considering nonlinerity of structure, the maximum values of both relative story displacement and control force were smaller than that of without considering. It is explained that the stiffness of structure was changing every time step according to the nonlinear property, the control force could be small.

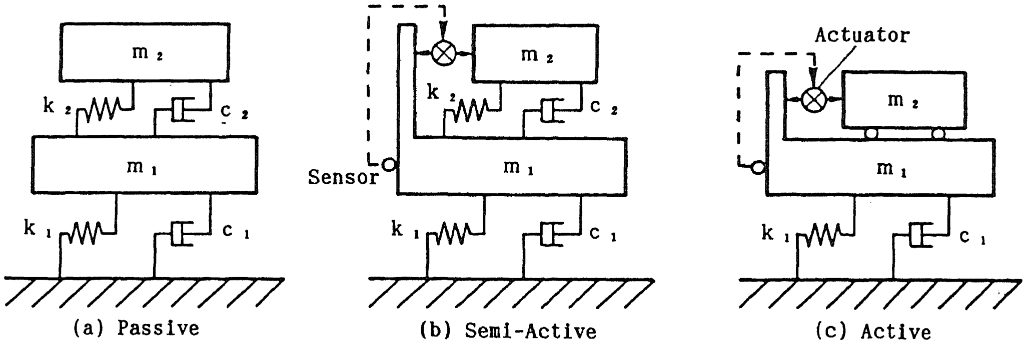


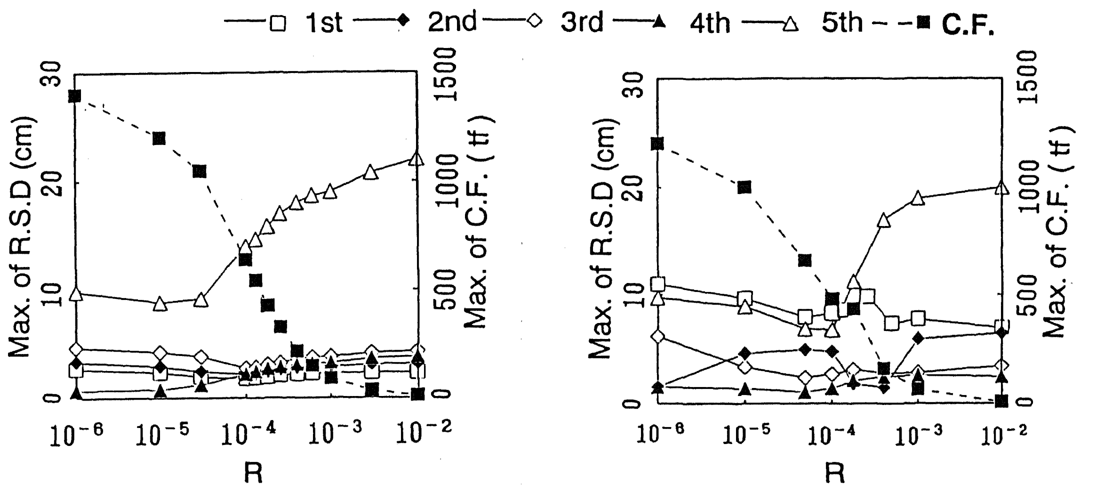
Fig.5 Each System of Control Method

Table 2 Compared with Each System for Relative Story Displacement and Control Force

		LINEAR	NONLINEAR
NON CONTROL	MAX R.S.D.	22.6cm	20.0cm
PASSIVE	MAX R.S.D.	19.4cm	24.5cm
SEMI-ACTIVE	MAX R.S.D.	17.0cm	16.4cm
	MAX C.F.	60tf	59tf
ACTIVE	MAX R.S.D.	13.6cm	11.3cm
	MAX C.F.	642tf	437tf

4.2 THE CHARACTER OF ACTIVE CONTROL SYSTEM

In generally, as the weighting factor R which relate to control force becomes smaller, the control force becomes larger and the response of the structure becomes smaller. Fig. 6 shows that effect of weighting factor R influenced to maximum value of the response (relative story displacement) and the control force, in both case with considering nonlinearity or not. With equal weighting factor R, the control force with considering nonlinear of the structure was more small than that of without considering. As mentioned before, due to stiffness degrading of structure according to the nonlinearity. Other tendencies are much the same to that of linear analysis result. Weighting factor R becomes smaller, control force becomes larger and the relative story displacement of fifth story becomes smaller. However, on stories from first to forth, relative story displacement increased gradually. It would be able to consider that the boundary condition of the top of the structure change



(a) Without Considering Nonlinearity of the Structure

(b) With Considering Nonlinearity of the Structure

Fig.6 Effect of Weighting Factor R for Relative Story Displacement and Control Force from free to nearly fixed condition by the effect of vibration control, and vibration energy is accumulated in the system.

Time histories of the response of relative story displacement with active control to take into account nonlinearity of the structure were shown in Fig.7. Time histories of control force is shown in Fig.8. Maximum value of the relative story displacement of fifth story became one half of noncontrolled case as shown in Fig.3. As mentioned above, the response of first story showed nonlinear characteristics.

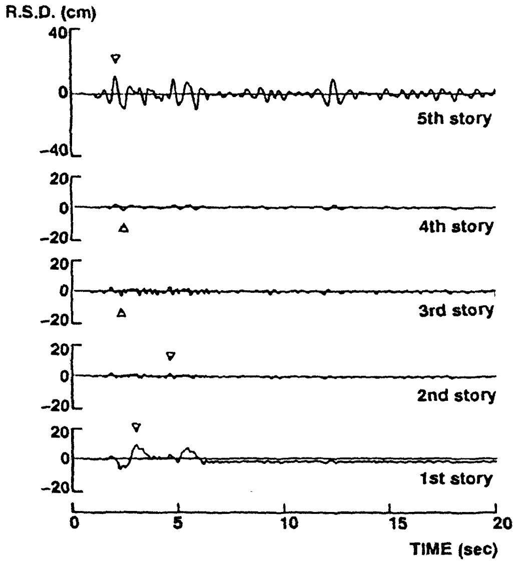


Fig.7 Time Histories of the Response Relative Story Displacement on Each Stories with Vibration Control

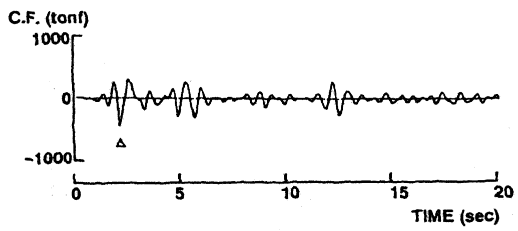


Fig.8 Time Histories of the Response on Control Force of actuator

5 CONCLUSION

To obtain more reasonable vibration control method against earthquake motions, active control algorithm to take into account nonlinearity of structure was introduced and some analysis were carried out. From this study, following conclusions were obtained.

1) Bilinear hysteresis characteristics can be idealized as a sum of elastic spring with a coulomb slider and an elastic spring. By using this model, equation of motion considering nonlinear property can be expressed as state space representation.

2) By using the time-dependent performance index J introduced by J.N.Yang, the instantaneous optimal control algorithm to take into account nonlinearity of structure can be introduced.

3) In the passive system, the ability of the vibration control becomes less than that of without control system because of the lengthening of natural period of the structure by the effect of nonlinearity.

4) In the semi-active system, its control force acts effectively as self-tuning mechanism to the structure which lengthened its natural period by the effect of nonlinearity, and the ability of vibration control can be sustained in the condition of nonlinearity.

5) In the active system, as the control force becomes larger, the boundary condition of the free end becomes nearly to one of the fixed end. Therefore, the vibration energy has accumulated in the system, and the lower stories of the structure show in more nonlinear properties.

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