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New methods in the intelligent control of building structures subjected to strong earthquake ground motion

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ABSTRACT: This paper deals with a new method for the active control of building structures subjected to strong earthquake ground shaking. The structure is modelled as a continuous shear beam. By using the properties of the mode shapes of vibration of the system, it is shown that the modal response at a given location in the system can be exactly reconstructed from the **time delayed** modal responses at (at most) three different locations in the system. This result is then used to motivate a closed-loop control design which is capable of stabilizing the system and dampening the vibrations in **all** its modes, while using **dislocated** sensor and actuator locations. Simple finite-dimensional controllers, commonly used in control design, are found to suffice. Explicit conditions are provided to obtain the bounds on the controller gains to ensure stability of the closed loop control design with no spill-over effects. Simulation results, which validate the control methodology and the theoretical bounds on the controller gain, are also presented.

INTRODUCTION

The development of new methods for the control of building structures subjected to strong ground motions has become a prominent topic of research and development(e.g., Meirovitch, 1980; Udwadia, 1981; Yang, 1987). Different methods for achieving such control have been developed using, for example, pulses of short duration applied to the struture at appropriate times, the use of proof masses set into motion to reduce the overall amplitude of structural response, hybrid control etc. One of the main concerns in the intelligent control of large-scale continuous structural systems (modelled by linear equations) such as, building structures (or its elements), is the fact that there is always a time delay between the signals (responses) measured at the sensor locations and the actuator locations. This time delay is known to be detrimental in actively controlling the system for it leads to the 'spill-over effect.' Thus attempts to control certain modes of the system guarantee that some other modes will become unstable (Balas, 1979). The only solution to this situation that has so far been available is to collocate the sensors and the actuators. However practical considerations often make such collocation impossible in the actual intelligent control of large-scale systems. In this paper we present a new method of control which does not require the constraint that the sensors and actuators be collocated.

This paper deals with the active control of building structures, or elements therefo, modelled as shear beams. By using the physical properties of the mode shapes of vibration of the system it is shown that the modal response at a given location in the system can be exactly reconstructed from the **time delayed** modal responses at (at most) three different locations in the system. We show that noncollocated point control of such a building structure, using finite dimensional controllers, can lead to complete controllability of the system without any spillover effects. Furthermore, it is shown that a large variety of simple and commonly used controllers, among them velocity feedback controllers and lead-lag compensators, are more than sufficient to perform such control. This is achieved by the judicious placement of sensors in the system and the acquisition of data from those locations with specific time delays. From a practical standpoint, the use of time delays reduces the bandwidth needed for the controller, while ensuring that **all** modes of the structural system are controlled.

SYSTEM MODEL

Consider a general structural system subjected to a time varying force f(x,t) described by

$$z_{tt} = c^2 z_{xx} + f(x,t) \tag{1}$$

where the space parameter x extends from 0 to L. The wave speed in the medium is denoted by c, and f(x,t) is the force normalized with respect to the inertial mass per unit length of the medium. The subscripts t and x refer to differentiation with respect to time, t, and space, x. This equation, though simplistic, governs the motion, z(x,t), of diverse systems like the torsional vibrations of tall tubular structures, the axial vibrations of rods, the horizontal motions of buildings, etc. We shall assume that the boundary conditions are given by

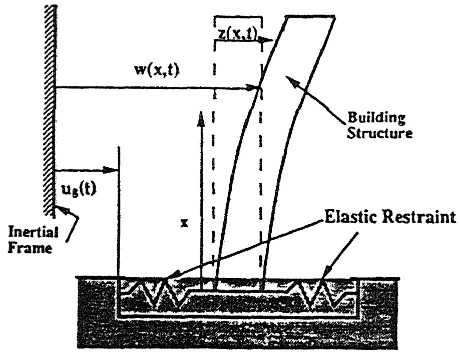

Figure 1: Building structure modelled as a shear beam.

$$z_x(0,t) - h_1 z(0,t) = 0 \qquad (2)$$

and

$$z_x(L,t) + h_2 z(L,t) = 0 \qquad (3)$$

and that the initial conditions are $z_t(x, 0) = z(x, 0) = 0$. The parameters h_1 and h_2 will be taken to be non-negative.

A building structure undergoing horizontal vibrations induced by strong earthquake ground shaking is often modelled by equations (1)-(3) (see Figure 1) where we interpret: **(a)** the displacement z(x,t) to be the relative displacement between the structure and the ground, **(b)** the force f(x,t) to be independent of x and equal to the ground acceleration, generated by the earthquake, and **(c)** h_2 to be generally equal to zero, signifying a free-end at x=L. Thus if w(x,t) is the absolute motion of the structure at time t, then,

$$z(x,t) = w(x,t) - u_g(t) \text{ and } f(x,t) = -\ddot{u}_g(t),$$

where $\ddot{u}_g(t)$ is the ground acceleration. The parameter h_1 refers to the extent of fixity at the end x=0. It is meant to model soil-structure interaction effects. The mass per unit length of the structure is assumed to be constant, and is normalized to unity.

The eigenvalue problem associated with equation (1) may then be obtained as

$$u_{xx} + \beta^2 u = 0; \quad \beta^2 = \frac{\omega^2}{c^2} \qquad (4)$$

with,

$$u_x(0) - h_1 u(0) = 0 \qquad (5)$$

and

$$u_x(L) + h_2 u(L) = 0. \qquad (6)$$

The eigenfunctions $u_n(x)$ and the corresponding eigenvalues β_n can now be found in the standard way. Several different boundary conditions are included in equations (2) and (3) depending on the values of h_1 and h_2. For example, when $h_1 = 0$, the end x = 0 is a free end; when $h_1 \rightarrow \infty$, the end x = 0 *is fixed; intermediate values of* h_1 *correspond to* partially restrained systems. The same can be said about the boundary condition at the end x = L. Thus various building structural elements, as well as the entire structure may be modelled, where appropriate, by these eqations.

Using the eigenfunction expansion and taking Laplace Transforms (the transform variable is s) we obtain the response as,

$$z(x,s) = \sum_{n=1}^{\infty} a_n(s) u_n(x) \qquad (7)$$

which yields

$$z(x,s) = \int_0^L g_0(x,\xi,s) f(\xi,s) d\xi . \qquad (8)$$

where $g_0(x,x,s)$ is given by ($N_n := \|u_n\|_2$)

$$g_0(x,\xi,s) = \sum_{n=1}^{n=\infty} \frac{u_n(x) u_n(\xi)}{N_n(s^2 + \omega_n^2)} \qquad (9a)$$

$$= \sum_{n=1}^{\infty} \frac{\mathrm{Sin}(\beta_n x + \phi_n)\,\mathrm{Sin}(\beta_n \xi + \phi_n)}{N_n(s^2 + \omega_n^2)}, \qquad (9b)$$

The open loop transfer function, $g_0(x,x,s)$, has an infinite number of poles at $s = \pm i\omega_n$. We see that these poles always lie along the imaginary axis.

A KEY PROPERTY OF MODAL RESPONSES

We begin by noting that the eigenfunction response (using separation of variables) of the system governed by equation (1) looks like

$$u_n(x,t) = \mathrm{Sin}(\beta_n x + \phi_n) e^{i\omega_n(t+\psi_n)}, \qquad (10)$$

which can be expressed as

$$u_n(x,t) = \frac{e^{i[\beta_n x + \phi_n]} - e^{-i[\beta_n x + \phi_n]}}{2i} \times e^{ic\beta_n(t+\psi_n)}. \qquad (11)$$

Furthermore for any location $x_1 > x_2$ we have

$$u_n\left(x_1, t - \frac{x_2}{c}\right) - u_n\left(x_2, t - \frac{x_1}{c}\right) =$$
$$u_n(x_1 - x_2, t) - e^{-i\beta_n(x_1 - x_2 - ct)} \mathrm{Sin}\phi_n e^{i\omega_n\psi_n}, \qquad (12)$$

and, similarly, for any location $x_a > x_3$ we get

$$u_n\left(x_a, t - \frac{x_3}{c}\right) - u_n\left(x_3, t - \frac{x_a}{c}\right) =$$
$$u_n(x_a - x_3, t) - e^{-i\beta_n(x_a - x_3 - ct)} \mathrm{Sin}\phi_n e^{i\omega_n\psi_n}. \qquad (13)$$

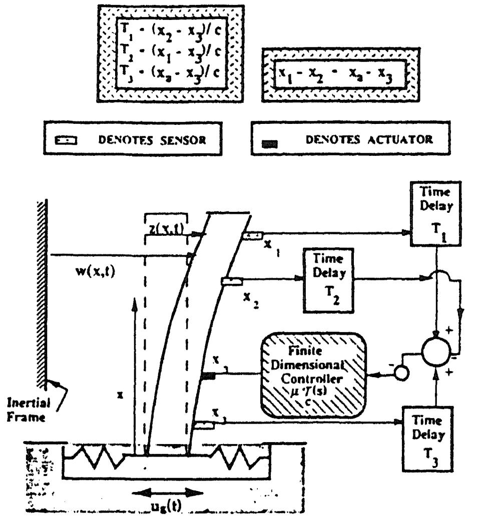

Figure 2: Time-delayed noncollocated control design using finite dimensional controller.

The subscript 'a' on x denotes, as we shall see in the next section, the actuator location. If we further choose the locations $x_a > x_3$ such that $x_1 - x_2 = x_a - x_3$ and subtract equation (13 from (12) we obtain

$$u_n\left(x_a, t-\frac{x_3}{c}\right) = u_n\left(x_1, t-\frac{x_2}{c}\right) + u_n\left(x_3, t-\frac{x_a}{c}\right) - u_n\left(x_2, t-\frac{x_1}{c}\right), \ \forall n, \forall t. \qquad (14)$$

Without loss in generality we can choose $x_a < x_1$ so that $x_3 < x_2$. Then time shifting equation (23) by x_3/c we get,

$$u_n(x_a, t) = u_n(x_1, t-T_1) + u_n(x_3, t-T_3) \qquad (15)$$

where,

$$T_1 = (x_2 - x_3)/c$$
$$T_2 = (x_1 - x_3)/c \qquad (16)$$
$$T_3 = (x_a - x_3)/c\ .$$

The time delays T_1, T_2, T_3 are all positive and thus we have been able to obtain a perfect "predictor" for the nth mode response at location x_a at time t by looking at the nth mode response at locations x_1, x_2, and x_3 at times $(t - T_1)$, $(t - T_2)$, and $(t - T_3)$ respectively. To predict the response of the nth mode (n = 1, 2, . . .) at location x_a we therefore need, in general ***three sensors***. Note that $x_1 - x_2 = x_a - x_3$. The location of x_a relative to x_2 is left open for now. Two possible configurations could arise: (1) C1: $x_3 < x_a < x_2 < x_1$, and (2) C2: $x_3 < x_2 < x_a < x_1$. For both these conditions, relations (15) and (16) are valid. For brevity, we shall use configuration C1 in this brief paper (see Udwadia 1990, for other results).

The time delays T_1, T_2, T_3 in equations (15) and (16) do not involve the mode number and therefore ***the same three locations and the same three delays will provide predictions for all modes.***

CLOSED LOOP CONTROL

Having realized that we can obtain a prediction of $u_n(x_a,t)$ by using at most three sensors at locations x_1, x_2 and x_3 as described in the previous section, we can now design a feedback controller for the distributed system of equation (1), where location, x_a, of the actuator is chosen so that: (a) it does not lie on any node of any mode of the system, and (b) $x_1 - x_2 = x_a - x_3$. Figure 2 shows the control design. The sensors, which are located at x_1, x_2, x_3 are polled, as shown in the figure, at time delays of T_1, T_2, and T_3 respectively. The outputs from the sensors located at x_1 and x_3 are added together, those from x_2 are subtracted, and the combined signal is multiplied by minus one (for negative feedback) and then fed to a finite dimensional controller. The controller has a transfer function of $\mu\, \mathcal{T}_c(s) := \mu K(s)/P(s)$, and the control gain is denoted by μ.

Using equation (8) and letting $f(x,s) = f_d(x,s) - f_c(u,s)$ where f_c represents the feedback control force and f_d the disturbance we obtain

$$z(x,s) = \int_0^L g_0(x,\xi,s)\left[f_d(\xi,s) - f_c(\xi,s)\right] d\xi\ . \qquad (17)$$

Using an actuator at location x_a we obtain (see Figure 2)

$$f_c(\xi,s) = \mu\, \mathcal{T}_c(s)\begin{bmatrix} z(x_1,s)e^{-sT_1} - z(x_2,s)e^{-sT_2} \\ + z(x_3,s)e^{-sT_3} \end{bmatrix}\delta(\xi - x_a). \qquad (18)$$

Equation (17) then becomes,

$$z(x,s) = \int_0^L g_0(x,\xi,s) f_d(\xi,s)\, ds\ - g_0(x,x_a,s)\mu\, \mathcal{T}_c(s). \left[z(x_1,s)e^{-sT_1} - z(x_2,s)e^{-sT_2} + z(x_3,s)e^{-sT_3}\right] \qquad (19)$$

which gives, after considerable algebra, the closed loop transfer function

$$G_{cl}(x,\xi,s) = \left[\frac{\det[A] g_0(x,\xi,s) - \alpha(x,x_a,s)\, \underline{d}^T \underline{G}_0(\xi,s)}{\det[A]}\right] \qquad (20)$$

where,

$$\alpha(x,x_a,s) = \mu\, \mathcal{T}_c(s)\, g_0(x,x_a,s), \tag{21}$$

$$\underline{G}_0(\xi,s) = \begin{bmatrix} g_0(x_1,\xi,s) & g_0(x_2,\xi,s) & g_0(x_3,\xi,s) \end{bmatrix}^T, \tag{22}$$

$$\underline{d} = \begin{bmatrix} e^{-sT_1} & -e^{-sT_2} & e^{-sT_3} \end{bmatrix}^T, \text{ and,} \tag{23}$$

$$\det[A] := \text{determinant}\,[A]$$
$$= 1 + \alpha(x_1,x_a,s)e^{-sT_1} - \alpha(x_2,x_a,s)e^{-sT_2} + \alpha(x_3,x_a,s)e^{-sT_3} \tag{24}$$

STABILITY AND CONTROLLER DESIGN

1. Stability for Small Gains, μ:

To study the stability of the control design, we shall use the following stability criterion. Noting that the open loop poles occur at the frequencies $s_k = \pm i\omega_k$ where ω_k is real, the root locus $s_k(\mu)$ of the kth closed loop pole, which starts for $\mu = 0$ at ω_k on the imaginary axis, will move to the left half s-plane if,

$$\mathrm{Re}\left\{\frac{ds_k}{d\mu}\right\}_{\mu=0+} < 0 \quad , \quad \forall k. \tag{25}$$

If this condition is satisfied, then in the close vicinity of $\mu = 0$, all the poles will have negative real parts and will therefore be stable. Using expansion (9) for $g_0(x_i,x_a,s)$ in (24) we get the following requirement for the closed loop poles in order to ensure stability:

$$\mathrm{Re}\left\{\left.\frac{ds_k}{d\mu}\right|_{\mu=0+}\right\} = -\frac{u_k^2(x_a)}{2N_k}\,.\,\mathrm{Re}\left\{\underset{s\to\pm i\omega_k}{\mathrm{Lt}}\left[\frac{\mathcal{T}_c(s)}{s}\right]\right\} < 0. \tag{26}$$

We can now design controllers which satisfy equation (49) and are therefore stable. Taking the controller's transfer function as

$$\mathcal{T}_c(i\omega) := \frac{K(i\omega)}{P(i\omega)} = a(\omega) + ib(\omega), \tag{27}$$

condition (26) requires that at <u>each</u> open loop pole,

$$\underset{\omega\to\pm\omega_k}{\mathrm{Lt}}\left[\frac{b(\omega)}{\omega}\right] > 0; k = 1,2,..,\infty. \tag{28}$$

Thus if we choose $b(\omega)$ to be a continuous function we then need it to be an <u>odd</u> function of ω), with $b(0) = 0$, $b(\omega) > 0$, $\omega \in (0, \infty)$. Throughout this paper we will assume that $b(\omega)$ of the controller's transfer function has this property, which we shall refer to as property P1.

Examples of a few such simple finite-dimensional controllers that satisfy P1 are:

1. a velocity feedback controller with

$$\mathcal{T}_c(i\omega) = i\omega, \tag{29}$$

2. a lag-lead compensator (with more 'lead' than 'lag')

$$\mathcal{T}_c(i\omega) = \frac{1+i\omega\tau_1}{1+i\omega\tau_2}, \quad \tau_1 > \tau_2 > 0. \tag{30}$$

We have so far proved stability when the controller gain, μ is positive, though only vanishingly small.

2. The Development of an Upper Bound, μ^{max}, for the Controller Gain

We denote by μ^{max} the upper bound on μ up to which the closed loop control described in this paper is stable. In this subsection we show that one can actually provide explicit expressions for a parameter M, M > 0, such that $\mu^{max} \geq M$ for the closed loop time-delayed, noncollocated system to be stable when the controller's transfer function $\mathcal{T}_c$ satisfies property P1. We will consider configuration C1.

It can be shown (after considerable algebra) that (see Udwadia, 1990):

if

$$\underset{s\to 0}{\mathrm{Lt}}\{g_0(x_a,x_a,s\} > \frac{(x_a - x_3)}{c^2}, \tag{31}$$

then stability can be ensured for all values of μ such that

$$\mathrm{Sin}^2\left\{\frac{\pm\omega(x_a - x_3)}{c}\right\} \neq \frac{c}{\mu}\left\{\frac{\omega\, b(\omega)}{a^2(\omega)+b^2(\omega)}\right\}, \text{ for all } \omega. \tag{32}$$

Noting that the maximum of the left hand side of (32) is unity, this relation will be satisfied if

$$M = \mu < \frac{c\,\omega\, b(\omega)}{\{a^2(\omega)+b^2(\omega)\}} \text{ for all } \omega. \tag{33}$$

In many practical situations, it can be shown that stability is ensured when

$$\mu < \left\{\frac{\omega_1^* b(\omega_1^*)}{\{a^2(\omega_1^*)+b^2(\omega_1^*)\}}\right\}.c \tag{34}$$

Table 1: First ten normalized frequencie of the building structure.

Mode Number	β_i
1	1.577
2	4.713
3	7.854
4	10.994
5	14.135
6	17.276
7	20.417
8	23.558
9	26.699
10	29.839

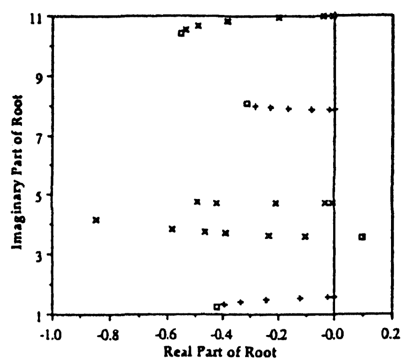

Figure 3a: Root locus of lowest four closed loop frequencies.

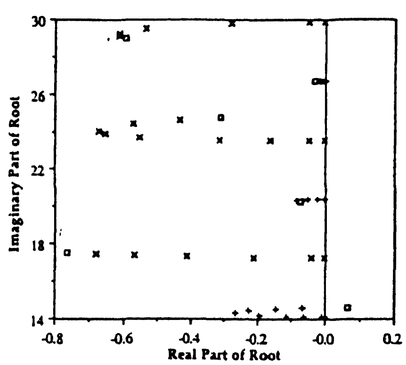

Figure 3b: Root locus of next six closed loop frequencies.

where,

$$\omega_1^*:=\pm\frac{\pi c}{2(x_a-x_3)}. \tag{35}$$

The first condition (i.e., relation (31)) is relatively easy to satisfy by a proper choice of the actuator-sensor distance (x_a-x_3). In practice, condition (33) is easy to verify.

When we use velocity feedback control, then, $a(i\omega)=0$, $b(i\omega)=\omega$ for all ω, and the closed loop control is stable for all $0 < \mu < c$, i.e., $M = c$. This is because the right hand side of relation (33) is now a constant whose value is c.

SIMULATION RESULTS

We now show some simulation examples of the control design that we have discussed in this paper using velocity feedback control. Consider the structure shown in Figure 1. The effect of soil-structure interaction is modelled by a linear elastic spring, as shown. We consider the system described by equations (1)-(3) with the following parameters (assumed to be chosen in consistent units):

$c = 2, L=1,$
$h_1 = 5000, h_2 = 0,$
$x_a-x_3 = 0.34$, and,
$x_a = 0.47$

The first ten frequencies $\beta_i=\omega_i/c$ are shown in Table1. The fundamental period is about 2 seconds.

The transfer function of the controller is given by $T_c(i\omega) = i\omega$. Thus we use a simple velocity feedback controller. Since M=c=2, we are assured stability as long as $0 < \mu < 2$. The upper half s-plane root loci are shown in Figures 3(a) and 3(b) for the first ten frequencies, using equation (24). The roots, $\overline{\beta} = s/c$, for different values of μ (typically μ = 0.01, 0.1, 0.5, 1, 1.5, 2, and 2.5) are shown. The location of the roots for all values of $0 < \mu < M$ are indicated by crosses or pluses. The open squares show the roots for $\mu > M$. We observe that the closed loop poles begin for $\mu = 0.01$ near the open loop poles which lie on the imaginary axis. The root loci of the poles corresponding to the second and fifth modes are seen to curve around and move into the right-half s-plane only for values of $\mu > M$. For $0 < \mu < M$, all the closed loop poles lie in the left half s-plane, as expected. Extensive numerical experimentation using *Mathematica* showed that the smallest value of μ for the controller to be stable is about 2.00449 for $\overline{\beta} = 59.21i$. Thus the value M = 2 found theoretically is a good lower bound and therefore a good approximation to μ^{max}.

CONCLUSIONS

In this paper we have shown ta new method of controlling building structures when the sensors and actuators are noncollocated. Our control force can take into account soil-structure interaction effects, albeit in a rudimentary manner.

1. By properly locating sensors and choosing **appropriate** time delays, the modal response of the system at time t at location x_a can be **exactly predicted** by measurements taken at three sensor locations at appropriate prior times.

2. The control design offered in this paper is different from those proposed in the past in that we use time-delayed inputs to the controller. It is this new feature, which is motivated by our understanding of the physics of the system, which appears to be quite important in bringing about stability of the closed loop system, using finite dimensional controllers.

3. The paper provides explicit bounds on the controller gains for which dampening of all modes is ensured. These bounds are provided in a form that can be easily calculated. More importantly, they are expressed in terms of the actual locations of the sensors and the actuators. Treating collocated controller designs as special cases noncollocated designs, the resultsof previous investigators are put in a more general framework.

4. Simulation results are presented for a building structure, modelled as a shear beam, undergoing horizontal vibrations induced by strong earthquake ground shaking. The control design methodology is validated along with the theoretically obtained bounds on the controller gains.

5. The technique uses noncollocated sensors and actuators, a variety of simple finite dimensional controllers, and feeds back a control force which is related to the time delayed response of the structure. The results show that all modes of vibration are stabilized with no "spill-over" effects as long as the controller gain is less than M.

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