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Predictive control of structures with reduced number of sensors and actuators

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ABSTRACT: In this paper a control system using a reduced number of sensors and actuators is proposed for active aseismic protection of structures. The control algorithm has been obtained generating individual control laws for each mode to be controlled according to a predictive control strategy. The time delay in the control loop can be considered into this formulation. The efficiency of the control action is assessed by means of a numerical example: a full-scale 6-story experimental building undergoing seismic excitation.

1 INTRODUCTION

The interest in using active control systems for seismic protection of structures has increased in the last decade. The basic components in the implementation of an active control system are sensors, actuators and the control methodology driving the system. One of the control methodologies proposed in this context has been the so called predictive control (Rodellar et al. 1987). It has been considered both in numerical (López-Almansa & Rodellar 1989 and Inaudi et al. 1992) and experimental (Rodellar et al. 1990) applications. In López-Almansa & Rodellar (1990), López-Almansa et al. (1991) and Andrade (1992) a methodical assessment about its efficiency has been carried out.

The implementation of active control systems on large structures may not be practical if the number of sensors and actuators have to be equal to the number of degrees of freedom (DOF) of the model. The objective of this paper is to formulate a predictive control law which can be implemented for structures represented by multidegree of freedom systems (MDOF) by using a reduced number of sensor and actuator (s/a) devices.

2 MODAL EQUATIONS

The seismic motion of a structure actively controlled and spatially-discretized by a n-DOF model can be described by the linear differential matrix equation

$$\boldsymbol{M}\,\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}\,\dot{\boldsymbol{y}}(t) + \boldsymbol{K}\,\boldsymbol{y}(t) = \boldsymbol{f}_c(t) - \boldsymbol{M}\,\boldsymbol{r}\,\ddot{y}_g(t) \quad (1)$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the symmetric, constant and positive definite mass, viscous damping and stiffness matrices and $\boldsymbol{y}$ is the relative displacements vector. $\boldsymbol{f}_c$ is the control vector, $\boldsymbol{r}$ is a column vector whose elements are 1 or 0 according whether the degrees of freedom correspond to the direction of the seismic motion or not and $\ddot{y}_g$ is the ground acceleration.

If the number of actuators is m, the $n \times 1$ vector $\boldsymbol{f}_c$ in (1) is related to the $m \times 1$ vector $\boldsymbol{u}$ (which contains the m control signals in the actuators) in the form

$$\boldsymbol{f}_c(t) = \boldsymbol{M}\,\boldsymbol{L}\,\boldsymbol{u}(t - \tau_d) \quad (2)$$

where $\boldsymbol{L}$ is a $n \times m$ matrix whose elements are 1 or 0 depending on the presence or absence of actuators in the degrees of freedom. τ_d is the lag time in the control loop, so $\boldsymbol{u}(t - \tau_d)$ is the control signal generated at instant $t - \tau_d$ which results in an action on the structure at instant t.

The equation of motion (1) can be analyzed by a classical modal analysis approach via the eigenvalue problem:

$$(\boldsymbol{K} - \omega^2 \boldsymbol{M})\,\boldsymbol{\phi} = \boldsymbol{0} \quad (3)$$

Equation (3) is verified for n independent modal vectors $\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_n$ and for n natural frequencies $\omega_1, \ldots, \omega_n$. Mode shapes $\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_n$ are arranged as the columns of $\boldsymbol{\Phi}$, referred as modal matrix.

Equation (1) can be formulated in modal coordinates by premultiplying by $\boldsymbol{\Phi}^T$. It works out to:

$$\boldsymbol{M}^*\,\ddot{\boldsymbol{\eta}}(t) + \boldsymbol{C}^*\,\dot{\boldsymbol{\eta}}(t) + \boldsymbol{K}^*\,\boldsymbol{\eta}(t) = \boldsymbol{f}_c^*(t) + \boldsymbol{f}^*(t) \quad (4)$$

where $\boldsymbol{M}^*$, $\boldsymbol{C}^*$ and $\boldsymbol{K}^*$ are the mass, damping and stiffness matrices in modal coordinates given by

$$M^* = \Phi^T M \Phi \qquad C^* = \Phi^T C \Phi \qquad K^* = \Phi^T K \Phi \tag{5a}$$

In (4), modal coordinates η verify

$$y(t) = \Phi \eta(t) \tag{5b}$$

The modal excitation forces f^* are equal to $-\Phi^T M r \ddot{y}_g$ and, from (2), the modal control forces f_c^* are

$$f_c^*(t) = \Phi^T f_c(t) = \Phi^T M L u(t - \tau_d) \tag{6}$$

M^* and K^* are diagonal and, if the system is classically damped, C^* is diagonal as well and therefore, (4) can be separated into a set of n scalar equations:

$$\ddot{\eta}_i(t) + 2\xi_i \omega_i \dot{\eta}_i(t) + \omega_i^2 \eta_i(t) = \frac{f_{c_i}^*(t)}{m_i^*} + \frac{f_i^*(t)}{m_i^*} \tag{7}$$

where m_i^* is the i-th diagonal element of matrix M^* and ω_i and ξ_i are, respectively, the natural frequency and damping ratio of mode i. The n equations in (7) can be coupled through the control force f_c^*.

3 INDEPENDENT MODAL CONTROL

A procedure formulating the predictive control strategy in the framework of independent modal space control (IMSC) is described in this section. IMSC consists of calculating control force $f_{c_i}^*$ in (7) only in terms of modal components η_i and $\dot{\eta}_i$. In this case, equations in (7) are uncoupled and each mode can be controlled independently. Obviously, mode shapes are not modified and, since (5b), relative displacements y are reduced if modal coordinates η_i are.

By applying the predictive control strategy to equation (7), the value of the modal control force $f_{c_i}^*$ at each discrete time instant k is given by

$$f_{c_i}^*(k) = -\begin{pmatrix} D_{i_1}^* & D_{i_2}^* \end{pmatrix} \begin{pmatrix} \eta_i(k) \\ \dot{\eta}_i(k) \end{pmatrix} - \sum_{j=1}^{\hat{d}} K_{i_j}^* f_{c_i}^*(k-j) \tag{8}$$

where $D_{i_j}^*$ $(j = 1,2)$ and $K_{i_j}^*$ $(j = 1,\dots,\hat{d})$ are, respectively, modal gain and memory factors for mode i. d is the number of delay periods in the discrete time control loop, i.e.

$$d = \frac{\tau_d}{T} \tag{9}$$

T being the sampling period. In (8) $\hat{d}$ is the value of d assumed to generate the control law (both values can differ due to identification errors).

If only p modes have to be controlled, by rearranging the order of the modes, the last $n - p$ control forces $f_{c_i}^*$ are zero. The first p scalar equations in (8) can be written together in matrix form according to

$$f_{c_p}^*(k) = -\begin{pmatrix} D_1^* & D_2^* \end{pmatrix} \begin{pmatrix} \eta_p(k) \\ \dot{\eta}_p(k) \end{pmatrix} - \sum_{j=1}^{\hat{d}} K_j^* f_{c_p}^*(k-j) \tag{10}$$

where vectors $f_{c_p}^*$ and η_p contain, respectively, p first components of f_c^* and η. D_1^*, D_2^* and K_j^* $p \times p$ are diagonal matrices whose elements are, respectively, $D_{i_1}^*$, $D_{i_2}^*$ and $K_{i_j}^*$.

Since modal coordinates η_p are not measurable and modal control forces $f_{c_p}^*$ are not implementable, equation (10) is not useful to be implemented directly. In the next section it is shown how to express the modal control forces in terms of the control signals defined in (2) and in section 5 a procedure to estimate the modal coordinates is described.

4 CALCULATION OF THE CONTROL SIGNALS

If there are as many actuators as degrees of freedom ($n = m$), provided no different control forces apply on the same DOF, then $L = I$ and (6) can be inverted to supply u:

$$u(k-d) = \left(\Phi^T M\right)^{-1} f_c^*(k) \tag{11}$$

Equation (11) is formulated in discrete time taking into account (9).

When $n = m = p$, (11) shows that it is possible to generate control signals u in order to have desired modal control forces $f_{c_i}^*$ $(i = 1,\dots,n)$.

If the number of actuators is smaller than the number of degrees of freedom ($m < n$), equation (6) can not be inverted and it is not possible to find m scalar values in u that supply the n desired values of the components of f_c^*. However, it is possible to obtain the approximate solution of (6) which provides the minimum error according to a least square criterion.

As only the first p modes are controlled the last $n-p$ components of f_c^* correspond to residual modes and are zero. In this case equation (6) may be written in the form

$$f_c^*(k) = \begin{pmatrix} f_{c_1}^*(k) & \dots & f_{c_p}^*(k) & 0 & \dots & 0 \end{pmatrix}^T = \Lambda u(k-d) \tag{12}$$

where $\Lambda = \Phi^T M L$. Λ is a full rank matrix regardless of matrix L, that is to say, independently of the actuators position.

Equation (12) can be separated in two parts (provided $p < n$):

$$f_{c_p}^*(k) = \begin{pmatrix} \phi_1 \\ \dots \\ \phi_p \end{pmatrix} M L \begin{pmatrix} u_1(k-d) \\ \dots \\ u_m(k-d) \end{pmatrix} = \Lambda_p u(k-d) \tag{13a}$$

$$\mathbf{0} = \begin{pmatrix} \boldsymbol{\phi}_{p+1} \\ \cdots \\ \boldsymbol{\phi}_n \end{pmatrix} \boldsymbol{M}\,\boldsymbol{L} \begin{pmatrix} u_1(k-d) \\ \cdots \\ u_m(k-d) \end{pmatrix} = \boldsymbol{\Lambda}_{n-p}\,\boldsymbol{u}(k-d) \tag{13b}$$

where $\boldsymbol{\Lambda}_p$ and $\boldsymbol{\Lambda}_{n-p}$ contain, respectively, the first p and the last $n-p$ rows of matrix $\boldsymbol{\Lambda}$.

Equation (12) has no solution, i.e., it is not possible to find a value of $\boldsymbol{u}$ to generate any desired value of $\boldsymbol{f}_c^*$. But it is possible to obtain the value of $\boldsymbol{u}$ which minimizes the quadratic cost function $H = (\boldsymbol{f}_c^* - \boldsymbol{\Lambda}\,\boldsymbol{u})^T\,\boldsymbol{\Theta}\,(\boldsymbol{f}_c^* - \boldsymbol{\Lambda}\,\boldsymbol{u})$ where $\boldsymbol{\Theta}$ is a symmetric and positive definite weighting matrix. By imposing that $\partial H/\partial \boldsymbol{u} = 0$, the following value of $\boldsymbol{u}$ is obtained:

$$\boldsymbol{u}(k-d) = \boldsymbol{\Gamma}\,\boldsymbol{f}_c^*(k) \tag{14a}$$

where $m \times n$ matrix $\boldsymbol{\Gamma}$ is equal to $(\boldsymbol{\Lambda}^T\,\boldsymbol{\Theta}\,\boldsymbol{\Lambda})^{-1}\,\boldsymbol{\Lambda}^T\,\boldsymbol{\Theta}$. Since last $n-p$ components of $\boldsymbol{f}_c^*$ are zero (14a) is equivalent to

$$\boldsymbol{u}(k-d) = \boldsymbol{\Gamma}_p\,\boldsymbol{f}_{c_p}^*(k) \tag{14b}$$

where $\boldsymbol{\Gamma}_p$ contains the first p columns of matrix $\boldsymbol{\Gamma}$.

If $\boldsymbol{\Theta}$ is diagonal the i-th element θ_i of the diagonal corresponds to the mode i and its value can be chosen according to its relative importance. If $\boldsymbol{\Theta} = \boldsymbol{I}$ ($\boldsymbol{I}$ being the identity matrix) $\boldsymbol{\Gamma}$ is the pseudo-inverse of Λ. If $\theta_{p+1} = \ldots = \theta_n = 0$ uncontrolled modes in (13b) are neglected and only (13a) is considered. If the number of actuators is equal to the number of modes to be controlled ($m = p$) (13a) has an exact solution given by

$$\boldsymbol{u}(k-d) = \boldsymbol{\Lambda}_p^{-1}\boldsymbol{f}_{c_p}^*(k) \tag{15}$$

Since equation (12) is not verified, the last $n-p$ components of $\boldsymbol{f}_c^*$ are not exactly 0 and residual modes are excited by control forces, with the consequent control spillover effects.

5 CALCULATION OF THE MODAL COORDINATES

Modal coordinates η_i and $\dot{\eta}_i$ ($i = 1, \ldots, p$) involved in (10) need al least p displacement and velocity sensors to be measured. In this paper it is assumed that the number of sensors is equal to the number of modes to be controlled p. Vectors $\boldsymbol{y}$ and $\dot{\boldsymbol{y}}$ can be separated in measured and residual (unmeasured) components according to

$$\boldsymbol{y}_p(k) = \boldsymbol{H}_p\,\boldsymbol{y}(k) \qquad \dot{\boldsymbol{y}}_p(k) = \boldsymbol{H}_p\,\dot{\boldsymbol{y}}(k) \tag{16a}$$

$$\boldsymbol{y}_r(k) = \boldsymbol{H}_r\,\boldsymbol{y}(k) \qquad \dot{\boldsymbol{y}}_r(k) = \boldsymbol{H}_r\,\dot{\boldsymbol{y}}(k) \tag{16b}$$

where $\boldsymbol{y}_p$ and $\dot{\boldsymbol{y}}_p$ contain, respectively, the measured displacements and velocities and $\boldsymbol{y}_r$ and $\dot{\boldsymbol{y}}_r$ contain the residual ones. $\boldsymbol{H}_p$ and $\boldsymbol{H}_r$ are matrices which define, respectively, the degrees of freedom with and without sensors. In (16) it has been assumed that the displacement and velocity sensors are placed in the same degrees of freedom.

By writing together (16a) and (16b) and taking into account (5b) it results

$$\begin{pmatrix} \boldsymbol{y}_p(k) \\ \boldsymbol{y}_r(k) \end{pmatrix} = \begin{pmatrix} \boldsymbol{H}_p \\ \boldsymbol{H}_r \end{pmatrix} \boldsymbol{\Phi}\,\boldsymbol{\eta}(k) = \boldsymbol{\Psi}\,\boldsymbol{\eta}(k) \tag{17a}$$

$$\begin{pmatrix} \dot{\boldsymbol{y}}_p(k) \\ \dot{\boldsymbol{y}}_r(k) \end{pmatrix} = \begin{pmatrix} \boldsymbol{H}_p \\ \boldsymbol{H}_r \end{pmatrix} \boldsymbol{\Phi}\,\dot{\boldsymbol{\eta}}(k) = \boldsymbol{\Psi}\,\dot{\boldsymbol{\eta}}(k) \tag{17b}$$

$n \times n$ matrix $\boldsymbol{\Psi}$ can be split in four blocks: $\boldsymbol{\Psi}_{pp}$ ($p \times p$), $\boldsymbol{\Psi}_{pr}$ ($p \times n-p$), $\boldsymbol{\Psi}_{rp}$ ($n-p \times p$) and $\boldsymbol{\Psi}_{rr}$ ($n-p \times n-p$). According to this, the first p scalar equations in (17a) and (17b) can be written as

$$\boldsymbol{y}_p(k) = \begin{pmatrix} \boldsymbol{\Psi}_{pp} & \boldsymbol{\Psi}_{pr} \end{pmatrix} \begin{pmatrix} \boldsymbol{\eta}_p(k) \\ \boldsymbol{\eta}_r(k) \end{pmatrix} \approx \boldsymbol{\Psi}_{pp}\boldsymbol{\eta}_p(k) \tag{18a}$$

$$\dot{\boldsymbol{y}}_p(k) = \begin{pmatrix} \boldsymbol{\Psi}_{pp} & \boldsymbol{\Psi}_{pr} \end{pmatrix} \begin{pmatrix} \dot{\boldsymbol{\eta}}_p(k) \\ \dot{\boldsymbol{\eta}}_r(k) \end{pmatrix} \approx \boldsymbol{\Psi}_{pp}\dot{\boldsymbol{\eta}}_p(k) \tag{18b}$$

neglecting the terms $\boldsymbol{\Psi}_{pr}\,\boldsymbol{\eta}_r$ and $\boldsymbol{\Psi}_{pr}\,\dot{\boldsymbol{\eta}}_r$. This is a logical assumption since upper modes do not usually contribute significantly to the response. Since matrices $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$ contain the same rows, block $\boldsymbol{\Psi}_{pp}$ is non-singular and expressions (18) can be inverted providing

$$\boldsymbol{\eta}_p(k) \approx \boldsymbol{\Psi}_{pp}^{-1}\,\boldsymbol{y}_p(k) \qquad \dot{\boldsymbol{\eta}}_p(k) \approx \boldsymbol{\Psi}_{pp}^{-1}\,\dot{\boldsymbol{y}}_p(k) \tag{19}$$

(19) defines the values of modal coordinates $\boldsymbol{\eta}_p$ and $\dot{\boldsymbol{\eta}}_p$ in terms of measured quantities.

6 CONTROL LAW

The substitution of (10) in (14b) and of (13a) and (19) in the resulting expression yields

$$\boldsymbol{u}(k-d) =$$

$$-\begin{pmatrix} \boldsymbol{G}_{r1} & \boldsymbol{G}_{r2} \end{pmatrix} \begin{pmatrix} \boldsymbol{y}_p(k) \\ \dot{\boldsymbol{y}}_p(k) \end{pmatrix} - \sum_{j=1}^{\hat{d}} \boldsymbol{E}_{rj}\,\boldsymbol{u}(k-d-j) \tag{20}$$

where

$$\boldsymbol{G}_{r1} = \boldsymbol{\Gamma}_p\,\boldsymbol{D}_1^*\boldsymbol{\Psi}_{pp}^{-1} \tag{21a}$$

$$\boldsymbol{G}_{r2} = \boldsymbol{\Gamma}_p\,\boldsymbol{D}_2^*\boldsymbol{\Psi}_{pp}^{-1} \tag{21b}$$

$$\boldsymbol{E}_{rj} = \boldsymbol{\Gamma}_p\,\boldsymbol{K}_j^*\boldsymbol{\Lambda}_p \tag{21c}$$

It is important to note that the way from eq. (10) to eq. (20) involves the number and location of the sensors (p) and the actuators (m) as well as the number n of degrees of freedom. Since all these numbers can be different in the general case, the aforementioned relations represent approximate transformations between variables. Consequently, the actual effect of the control action on each mode will not be the one expected when designing the independent modal control laws defined in (8). In order to analyze the control effect, a procedure based on relating the eigenvalues of the discrete time controlled system with modal frequencies and modal dampings is used in the next section. So the uncontrolled and controlled modal characteristics can be compared in order to extract the effect of the real control on each mode.

If the motion of the controlled structure is not described by a discrete model like (1) but by a continuous model (partial differential equations), similar results are obtained (Meirovitch 1990).

7 EFFICIENCY CRITERIA

The equations of motion (1) can be solved in discrete time (López-Almansa et al. 1988) by taking the sampling period T as a discretization time increment. This provides the following step-by-step state space model

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{B}\,\boldsymbol{u}(k-d) + \boldsymbol{w}(k) \qquad (22)$$

where the state vector $\boldsymbol{x}$ is given by

$$\boldsymbol{x}(k) = \begin{pmatrix} \boldsymbol{y}(k) \\ \dot{\boldsymbol{y}}(k) \end{pmatrix} \qquad (23)$$

$\boldsymbol{A}$ and $\boldsymbol{B}$ are constant matrices and $\boldsymbol{w}$ is a vector related to the seismic excitation.

The motion of the controlled system is governed in discrete time by (22), $\boldsymbol{u}$ given by (20). Defining an extended state $\bar{\boldsymbol{x}}$, both equations can be written together according to

$$\bar{\boldsymbol{x}}(k+1) = \bar{\boldsymbol{A}}\,\bar{\boldsymbol{x}}(k) + \bar{\boldsymbol{w}}(k) \qquad (24)$$

If there are no differences between $\hat{d}$ and d, the $2n+md$ vectors $\bar{\boldsymbol{x}}$ and $\bar{\boldsymbol{w}}$ and the $(2n+md)\times(2n+md)$ matrix $\bar{\boldsymbol{A}}$ are respectively given by

$$\bar{\boldsymbol{x}}(k) = \begin{pmatrix} \boldsymbol{x}(k) \\ \boldsymbol{u}(k-1) \\ \boldsymbol{u}(k-2) \\ \cdots \\ \boldsymbol{u}(k-d) \end{pmatrix} \qquad \bar{\boldsymbol{w}}(k) = \begin{pmatrix} \boldsymbol{w}(k) \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \cdots \\ \boldsymbol{0} \end{pmatrix} \qquad (25a)$$

$$\bar{\boldsymbol{A}} = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{B} \\ -\boldsymbol{G}_r\boldsymbol{F}_p & -\boldsymbol{E}_1 & \cdots & -\boldsymbol{E}_{d-1} & -\boldsymbol{E}_d \\ \boldsymbol{0} & \boldsymbol{I} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ & & \cdots & & \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{I} & \boldsymbol{0} \end{pmatrix} \qquad (25b)$$

$$\boldsymbol{G}_r = (\boldsymbol{G}_{r_1} \quad \boldsymbol{G}_{r_2}) \qquad \boldsymbol{F}_p = \begin{pmatrix} \boldsymbol{H}_p & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{H}_p \end{pmatrix} \qquad (25c)$$

If $\hat{d} \neq d$, quantities in (25) have similar values (López-Almansa 1988).

Equation (24) demonstrates that the controlled system has a linear behavior. Therefore the efficiency of the control action can be assessed through the eigenvalues of matrix $\bar{\boldsymbol{A}}$: if they are inside the unit circle the system is asymptotically stable. Besides, if there are no time delays ($d = \hat{d} = 0$), such values provide information about the modal frequencies and damping ratios of the controlled system (Mickeleborough & Pi 1989).

8 NUMERICAL EXAMPLES

Several numerical analysis have been carried out (Andrade 1992) in order to assess the efficiency of the methodology proposed in this paper. In this section some numerical simulations on a 6-story full-size experimental building placed in Tokyo (Japan) are described. In Table 1 natural frequencies ω_i, damping ratios ξ_i and participation factors Γ_i of the first three modes are shown. More details can be found in Soong et al. (1991) and Andrade (1992).

Three control cases are considered, called I, II and III. In I there is 1 sensor and 1 actuator ($m = p = 1$) placed in the 6th floor, in II there are 2 sensors and 2 actuators ($m = p = 2$) placed in the 1st and 6th floors and in III there are 3 sensors and 3 actuators ($m = p = 3$) placed in the 2nd, 4th and 6th floors. In all these cases no delays have been considered ($d = \hat{d} = 0$). In the case I only the first mode is controlled and in the cases II and III the two and three first modes are controlled, respectively. For such controlled modes the values of the parameters of the predictive control strategy generating control laws in (10) have been chosen according to recommendations given in Andrade (1992). The motion of the building has been described by a 2-D lumped mass model having six degrees of freedom (each one corresponding to the horizontal translation of each floor).

Table 1
EXPERIMENTAL BUILDING

Modal quantities	Mode No. (i) 1	2	3
ω_i [rad/s]	4.11	10.99	18.35
ξ_i [%]	1	1	1
Γ_i [%]	77.20	11.71	4.52

In Table 2 the maximum seismic displacements and control forces are shown for the three control cases (I, II and III) and for the uncontrolled case (0). The excitation has been El Centro

Table 2 SEISMIC RESULTS y_i [cm], u_i [cm/s^2]				
	0	I	II	III
y_1	1.03	0.95	0.48	0.48
y_2	2.61	2.47	1.56	1.13
y_3	3.98	3.79	2.64	1.67
y_4	4.82	4.37	3.27	1.86
y_5	5.45	3.67	3.28	2.34
y_6	7.36	1.65	1.85	2.68
u_1	-	398.11	239.90	62.63
u_2	-	-	278.87	183.09
u_3	-	-	-	186.15

Table 3 EQUIVALENT FREQUENCIES ω_i' [rad/s]			
Control case	ω_1'	ω_2'	ω_3'
I	7.44	16.00	18.87
II	7.74	15.21	19.84
III	7.46	12.47	18.87

Table 4 EQUIVALENT DAMPING RATIOS ξ_i' [%]			
Control case	ξ_1'	ξ_2'	ξ_3'
I	5.74	11.49	27.61
II	11.88	8.05	26.25
III	16.96	7.68	3.25

earthquake (California 1940). Floors are numbered in such a way that y_1 and y_6 are the horizontal relative displacements of the lower and upper floor, respectively. A similar criterion has been considered for numbering control signals u_i.

Results from Table 2 show that the three control cases are efficient since the response is smaller than in the case without control. The efficiency of the three cases is similar but cases with more sensors and actuators need smaller control forces.

Equivalent frequencies and damping ratios of the three first modes for the control cases I, II and III are shown in Tables 3 and 4, respectively. Such values have been computed according to the formulation described in section 7.

Comparison between Table 1 and Tables 3 and 4 confirms that control cases I. II and III are efficient since an important increasing of frequencies and dampings is reached.

9 CONCLUSIONS

The main conclusion is that following the procedure described in this paper it is possible to design control laws with a reduced number of sensors and actuators providing a satisfactory global effect.

In the numerical example considered in this paper it has been shown that an important reduction of the response and a significant increase of modal frequencies and damping ratios has been obtained. If bigger numbers of sensors and actuators are considered smaller control forces are required.

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