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Control devices optimal installment for the new vibration control system of multi-structure connection

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ABSTRACT: The new vibration control system of multi–structure connection is proposed in this peper, which is a high efficiency and great future structural control system for the earthquake resistance. The system optimum design is according to the two stage design way of optimum design for control action distribution and optimum design for system control law. The theory for optimal control law is ripe and vast used, however, the problem of control action optimal distribution for the multi–control action system has not yet resolved. The analysis method for sensitivity of performance index to the control action distribntion is established and the design method for control action optimal distribution for multi–control action system is developed in this paper, with which the theory of system optimum design for the control system of multi–structure connection is perfected.

1 INTRODUCTION

It is for the development of structural earthquake resistant theory that the method of only depending on improving structural strength can not have been meeting the needs of today's earthquake resistant. The structural vibration control method come into being. The structural states can be operated in some degree with the structural control. The control structure systems are formed with the building structures and the control systems combination, which plays a dynamic role in earthquake resistance. Control structure systems can effectively isolate or dissipate the structural energy input of earthquakes, which are called as "intelligent structures".

More than one structures connected with control devices are together support the action of earthquakes, which is defind as the new vibration control system of multi–structure connection (MSC control system). Two or more structures connected with control devices on some floors can effectively distribute to structures and dissipate by control devices of the energy input of earthquakes, which is effective dynamic energy dissipation system. The active or passive control system design can be done by use of such new system.

The action of control devices of MSC system is connectting each single structure, transmitting and assigning the vibration energy between structures and dissipatting the vibration energy by control damping. The control devices provide horizontal control actions between structures and can control the multi–dimentional and rotational vibrations of structures by control devices connected in different directions of structures. The MSC system is a high effeciency control system, by use of which the deficiency of only depending on single structure to bear dynamic load is surmounted.

The first problem to be solved for the vibration control of the new system is the installment of the control devices. It's realy difficult and need not, and also large additional load occur for the system installed control devices on each floor. The installment of control devices has to be selected. The control devices has to be installed on the places of effective action, which is of problem of optimum.

2. THE TWO STAGE DESIGN WAY FOR MSC SYSTEM

For control system optimum design, the optimal control law is reached according to the minimum of the performance index, which is the result of the optimum for system control. It is different for the result of system optimal control because of the difference of system control action distribution, which makes it different for the minimum of performance index for optimal control. There is a problem of optimal distribution for control action installment. The two stge design way is proposed for the system of multi–control actions, which is the optimum design for the system control action distribution and the optimum design for the system control law. For the linear system of Gaus disturbance input, the performance index is always quadrics, which is defind as linear, quadric and Gaus problem (LQG problem). The disturbances as earthquakes are usnally assumed as Gaus distribution. The LQG problem is perfected resolved relatively and vast used random optimal control problem. The algorithm of optimal control is ripe for use and the optimal control action can be obtained by use of appropriate algorithm. However, the problem of optimal distribution for control actions has not been resolved. The resolution for control action system is given in this paper to perfect the optimum design method for MSC system, which is worthful in general for multi–control action system.

3. THE MODEL FOR MSC OPTIMAL CONTROL SYSTEM

As an example, the control system of two structure connection is discussed. The control devices are installed on some floors between n stroey strusture *a*and *m*storey structure *b*. The vibration equation of each structure under the disturbance of earthquake are respectively

$$M_a\ddot{Y}_a + C_a\dot{Y}_a + K_aY_a = -M_aI_a\ddot{X}_g - D_{nm}U \qquad (1)$$

$$M_b\ddot{Y}_b + C_b\dot{Y}_b + K_bY_b = -M_bI_m\ddot{X}_g + D_{mm}U \qquad (2)$$

in which, Y_a, Y_b are the displacement vectors of structure *a*and *b*respectively. M_a, C_a, K_a are the mass matrix, damper matrix and stiffness matrix of structure *a*respectively. M_b, C_b, K_b are the mass matrix, damper matrix and stiffness matrix of structure *b*. $\ddot{X}_g$ is the acceleration of earthquake ground mortion. $U = [u_1, u_2 \cdots u_3]^T$ is the vector of control actions. D_{nm}, D_{mm} are the distribution matrix of control actions. The matrices of control action distribution D_{nm} and D_{mm} express the control action distribution between structure *a*and *b*. The matrix D_{mm} is an diagonal matrix, in which the diagonal elements are 1 or 0 expressing the relative storey between strustures installed or not control action respectively. The matrix D_{mm} is identity when the control actions are installed on each floor. D_{nm} is

$$D_{nm} = \begin{bmatrix} D_{mm} \\ O_{(n-m)\times m} \end{bmatrix}$$

eq. (1) and eq. (2) is combined to state equation

$$\dot{X} = AX + BU + F\ddot{X}_g \qquad (3)$$

in which

$$X = \begin{bmatrix} Y_a^T & Y_b^T & \dot{Y}_a^T & \dot{Y}_b^T \end{bmatrix}^T$$

$$A = \begin{bmatrix} O_n & O_{nm} & E_n & O_{nm} \\ O_{mn} & O_m & O_{mn} & E_m \\ -M_a^{-1}K_a & O_{nm} & -M_a^{-1}C_a & O_{nm} \\ O_{mn} & -M_b^{-1}K_b & O_{mn} & -M_b^{-1}C_b \end{bmatrix}$$

$$B = \begin{bmatrix} O_{nm} \\ O_{mm} \\ -M_a^{-1}D_{nm} \\ M_b^{-1}D_{mm} \end{bmatrix} \qquad F = \begin{bmatrix} O_{n1} \\ O_{m1} \\ -I_{(n+m)} \end{bmatrix}$$

Here the matrix B is the distribution matrix of the control actions in state equation. eq. (3) can be given in time discrelized form

$$X(k+1) = A_dX(k) + B_dU(k) + W(k) \qquad (4)$$

in which

$$A_d = E + TA \quad B_d = TB \quad W(k) = TF\ddot{X}_g(k)$$

For passive control system optimum design, the time continuous state equation eq. (3) is used, when the performance index is

$$J = E\left\{\int_0^{\infty}\left[X^T(t)Q_1X(t) + U^T(t)Q_2U(t)\right]dt\right\} \qquad (5)$$

For active control system optimum design, the time discrelized state equation eq. (4) is used, when the performance index is

$$J = E\left\{\sum_{k=0}^{\infty}\left[X^T(k)Q_1X(k) + U^T(k)Q_2U(k)\right]\right\} \qquad (6)$$

4 THE SENSITIVITY ANALYSIS FOR PERFORMANCE INDIES OF RANDOM OPTIMAL CONTROL SYSTEM TO CONTROL ACTION DISTRIBUTION

The effeciency of system optimal control is influerenced with control action distribution. The control action distribution is determined with the installment matrix B in system state equation. The sensitivity for performance index of random optimal control system to control action distribution is defind as the first derivative of performance index to the installment matrix of control actions, which expresses the influerence of control action distribution to the performance index. It is the basis for analysis the system control action distribution and system design of control action optimal distribution. The resoltions are given by abridging the process of derivation.

4.1 The senstivity for performance index of time discrelized system to the matrix of control action distribution

For the state equation of time discrelized system eq. (4), the tactics of feadback control is applied and the optimal control law is developed by using the performance index eq.(6)

$$U(k) = -LX(k) \tag{7}$$

Hence, the eq. (6) and eq. (4) changed as

$$J = E\left\{\sum_{k=0}^{\infty} X^T(k)QX(k)\right\} \tag{8}$$

$$X(k) = \psi(k)X(0) + \sum_{i=0}^{t} \psi(k-i-1)W(i) \tag{9}$$

in which

$\psi(m) = (A - BL)^m$

is the matrix of state transformation;

$$Q = Q_1 + L^T Q_2 L$$

The sensitivity of performance index eq. (6) as

$$\frac{\partial J}{\partial B} = -2[L(R+P)\psi^T(1)S]^T \tag{10}$$

in which R, P, S are obtained by solving Lyapunov equations respectively

$$R = \psi^T(1)R\psi(1) + R_0 \tag{11}$$

$$P = \psi^T(1)P\psi(1) + P_0 \tag{12}$$

$$S = \psi^T(1)S\psi(1) + Q \tag{13}$$

Here

$$R_o = X(0)X^T(0) \quad P_o = E\left\{\sum_{i=0}^{t} W(i)W^T(i)\right\}$$

4.2 The sensitivity for the performance index of time continuous system to the matrix of control action distribution

For the system state equation eq. (3), the optimal feedback control law for performance index eq. (5) is obtained

$$U = -\tilde{L}X \tag{14}$$

eq.(5) and eq. (3) are transmitted as

$$J = E\left\{\int_0^{\infty} (X^T QX)dt\right\} \tag{15}$$

$$X(t) = \varphi(t)X(0) + \int_0^t \varphi(t-\tau)W(\tau)d\tau \tag{16}$$

in which

$\varphi(t) = e^{(A-B\tilde{L})t}$

is the metrix of state transformation

$Q = Q_1 + \tilde{L}^T Q_2 \tilde{L}$

The sensitivity of performance index eq. (5) as

$$\frac{\partial J}{\partial B} = -2[\tilde{L}(R_0 + \tilde{P})\tilde{S}]^T \tag{17}$$

in which

$$R_0 = X(0)X^T(0)$$

$$\tilde{P} = \int_0^{\infty} EW(t)W^T(t)dt$$

$$\tilde{S} = \int_0^{\infty} t\varphi^T(t)Q\varphi(t)dt$$

The sensitivity of performance index expresses the sensitive of performance index of optimal control to the control action distribution. The sensitivity of system performance index is minimum when the distribution of control actions is optimal. For the control system, the low sensitive of performance index is need, so it is an important index for system quality analysis. The control actions for the performance index of low sensitive can be removed and the economical distribution is obtained for system control.

5. THE INCREMENT ALGORITHM OF THE PERFORMANCE INDEX FOR SYSTEM CONTROL AND THE OPTIMAL DISTRIBUTION OF CONTROL ACTIONS

The qualititive analysis for the influerence of the distribution of control actions to the system optimal control is done with the sensitive analysis for system control, however, the increment of the performance index has to be given for the quantititive analysis of the influerence of the control action distribution to the system optimal control.

The increment of performance index will occur when there is the increment of the matrix of control action distribution. According to the Taylor equation, the increment of performance index is

$$\Delta J = J(B + \Delta B) - J(B) = t_r\left[\left(\frac{\partial J}{\partial B}\right)^T \Delta B\right] \tag{18}$$

When control action i is removed, the increment ΔB_i occur and the increment of performance index obtained

$$\Delta J_i = t_r[(\frac{\partial J}{\partial B})^T \Delta B_i] \tag{19}$$

The relative element $\{b_i\}$ in matrix B put into zero when the control action i is removed, here

$$\Delta B_i = [\{-b_i\}] \tag{20}$$

The increment of performance index in eq (19) is regard as the loss for optimal control when the control action i is removed, hence it is the contribution to the system optimal control of control action i. For each control action, the increment of performance index is obtained when the control action is removed respectively and the contribution of each control action to system optimal control is given. The optimal distribution of control action for the system optimal control is developed when the control actions of lowest increments of performance index is removed. The increment of performance index is regard as the loss of system optimal control when the $s$control actions removed

$$\Delta J^{s} = t_{r}\left[\left(\frac{\partial J}{\partial B}\right)^{T} \Delta B^{s} \right] = \sum_{i}^{s} \triangle J_{i} \qquad (21)$$

If the increment of performance index is minimum when the s control actions removed each, the increment of performance index for the $s$control actions removed given in eq. (21) is minimum. The lowest loss of performance index obtained and the optimal distribution of control actions is developed.

6. CONCLUSION

The sensitivity of performance index for the distribution of control actions is an important quality for system optimal control, which expresses the influerence of control action distribution to the system optimal control. The basis for quantititive analysis of the control action distribution is furnished by the establishment of the increment algorithm of performance index or the contrbutions of each control action to the system optimal control, with which the design method of control action optimal distribution is achieved. According to the two stage design way, the optimum design for the MSC system is divided into two stages of system optimum design for control action distribution and the optimum design for system control law. The method of the formal is contributed in this paper, which complete the theory of system optimum design for MSC system under the random disturbance of earthquakes.

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