Modal seismic response control of high-rise building by active tuned mass damper

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ABSTRACT: For the purpose of reducing seismic response of a high-rise building including high-order vibrations, a plural number of ATMD's were tested to examine the practicability of a vibration control system in each vibration mode. The results obtained are as follows: (1) The concept of modal response control system have been clarified. (2) The system functions properly according to the control law in a function verification test of the ATMD model, and in a vibration control test up to the third vibration mode using 3 ATMD's on a building model, the system is capable of obtaining the same effect of vibration control as analysis. (3) The modal response control method has been proved to be a highly practicable in a simulation analysis of an actual building which was performed to grasp the device scale, performance and power consumption of the system.

1 INTRODUCTION

The seismic response of a high-rise building generally include the response in the higher-order natural vibration modes. In some cases, the response in the second or third mode may exceed that in the basic first mode. To reduce seismic response of building, such high order vibration modes must be controlled. For this purpose, the mass damper system can be applied to such a building without modifying its structure by two methods: one is to provided more than one passive tuned mass damper (TMD) as tuned to each natural mode of vibration, and the other, to provide an active mass damper (AMD) that is capable of directly controlling random responses including the response of higher-order vibration modes.

Both methods involve such problems preventing their actual applications as: the former will require a large-scale (heavier) damper mass to have a sufficient effect on such random forces as earthquake, etc.; and the latter, a large-output driving device for a high-rise building causing a large energy (power) consumption. In this respect, we have studied the practicability of a system capable of controlling each vibration mode by applying active tuned mass damper (ATMD)* for higher performance at a lower energy consumption.

*ATMD: To drive and control the tuned mass damper (TMD). It is provided with a performance much higher than that of TMD. To a tuned vibration mode of a structure, it will develop a similar performance to that of the active mass damper (AMD) and yet require a much smaller driving output than AMD.

2 MODAL RESPONSE CONTROL METHOD

2.1 Separation into modal vibration systems

The concept of separation into the modal vibration systems is considered in an example of a structure or two-mass system on which 2 ATMD's are used for the first and second-order vibrations. The vibration model in this case is shown in Fig. 1. The structure consist of two masses $m_1$ and $m_2$, which are provided with a first-order vibration ATMD having a damper mass of $m_1$ and a secondary vibration ATMD having $m_2$. The most effective location for each ATMD is the point where the vibration mode of the order in the structure become the maximum, as described below in such case:

![Figure 1. Two-mass vibration control model](image-url)
The equation of motion for the vibration control model consisting of a structure (2 masses) and 2 ATMD's, as in Fig. 1, is given as:

\[ [M]\ddot{x} + [C]\dot{x} + [K]x = -[M]\ddot{u} + [U] \]  

(1)

where

\[ [M] = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} = \text{diag}(m_1, m_2) \]

\[ [C] = \begin{bmatrix} [C_{11}] & [C_{12}] \\ [C_{21}] & [C_{22}] \end{bmatrix} = \begin{bmatrix} c_{11} & 0 \\ 0 & c_{22} \end{bmatrix} \]

\[ [K] = \begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{21}] & [K_{22}] \end{bmatrix} = \begin{bmatrix} k_{11} + k_{21} & 0 \\ 0 & k_{22} \end{bmatrix} \]

\[ [U] = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ -u_2 \end{bmatrix} \]

\[ [W] = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ -y_2 \end{bmatrix} \]

\[ [M*] = \text{diag}(m_1, m_2) \]

\[ [C*] = \begin{bmatrix} C_{11} & y_{12} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ \end{bmatrix} \]

\[ [K*] = \begin{bmatrix} K_{11} & y_{12} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ \end{bmatrix} \]

\[ [W*] = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} w_1 \\ -w_2 \end{bmatrix} \]

In the above equation, \( x \) is the displacement vector, \( u \) is the ground acceleration and \( W \), the control force vector, when the displacement of the structure \( x \) is expressed by the linear sum of the mode vector \( y \) obtained by solving Eq. (2) of the structure only, \( x \) can be rewritten as in Eq. (3).

\[ \ddot{x} = [V]\ddot{\xi} \]

(2)

where

\[ [V] = \begin{bmatrix} [Y] & 0 \\ 0 & [I] \end{bmatrix} \]

\[ [Y] = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_{11} \\ y_{12} \\ y_{21} \\ y_{22} \end{bmatrix} \]

\[ [V] = \begin{bmatrix} q_1 & q_2 & x_1 & x_2 \end{bmatrix} \]

In Eq. (3), \( q_1, q_2 \) are the first- and second-order modal displacements of the structure. Substituting Eq. (3) in Eq. (1) and multiplying each term from the left by \([V]\) and using the orthogonality of the mode vector to assume that structure is in proportional damping, the structure’s mass, damping and stiffness matrix will be all diagonalized as:

\[ \begin{bmatrix} [M]\ddot{\xi} + [C]\dot{\xi} + [K]\xi = -[M]\ddot{u} + [U] \]

(4)

where

\[ [B] = \begin{bmatrix} \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} & \begin{bmatrix} m_1 & m_2 \\ m_2 & m_2 \end{bmatrix} \end{bmatrix} \]

In Eq. (4), \( M_1, C_1, \) and \( K_1 \) \((i=1,2)\) are the modal mass, the modal damping coefficient and the modal stiffness of the first- and second-order vibrations of the structure, respectively.

The underlined terms of the damping, stiffness matrix \([C*], [K*]\) and the control force vector \([U*]\) are the coupled terms of the 1st- and 2nd-order vibrations. These coupled terms are assumed to have little effect on one another because the natural frequency of these two vibrations differ from each other and because \( M_1, M_2 > m_1, m_2 \), \( C_1, C_2 > c_{11}, c_{22} \) and \( K_1, K_2 > k_{11}, k_{22} \). Ignoring such an effect and also \( y_{12}^2, y_{21}^2, y_{12}^2, y_{21}^2 \) of \([C*]\) and \([K*]\) for the same reason, the equation of motion in a separated form for each order vibration can be obtained as:

\[ \begin{bmatrix} [M*] \ddot{\xi}_1 + [C*] \dot{\xi}_1 + [K*] \xi_1 = -[B]\ddot{u}_1 + [U*] \]

(5)

where

\[ [M*] = \text{diag}(M_1, M_2) \]

\[ [C*] = \begin{bmatrix} C_{11} & c_{12} \\ c_{21} & C_{22} \end{bmatrix} \]

\[ [K*] = \begin{bmatrix} K_{11} & k_{12} \\ k_{21} & K_{22} \end{bmatrix} \]

\[ [W*] = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} w_1 \\ -w_2 \end{bmatrix} \]

\[ [B*] = \begin{bmatrix} [y_1, m_1, y_{12}, m_2, m_2] \\ [y_2, m_2, y_{21}, m_1, m_2] \end{bmatrix} \]

\[ [U*] = \begin{bmatrix} y_{11} \\ -y_{22} \end{bmatrix} \]
2.2 Vibration control system

With the separated each vibration system as in Eq. (5), the vibration control system forms an optimal regulator consisting of state variables of modal displacement $q_j$, modal velocity $\dot{q}_j$, and displacement $x_n$, velocity $\dot{x}_n$, of the damper mass, and performs feedback control.

In this case, the modal component of each order of the structural response to be required is detected by the following equations.

$$\ddot{q}_j = \frac{1}{M_j} \left[ \ddot{x}_n \right] - \left( \begin{array}{c} \ddot{x}_n \end{array} \right), \quad (j=1, n)$$

$$\dot{q}_j = \int \ddot{q}_j \, dt, \quad \ddot{q}_j = \int q_j \, dt$$

$$\left( \begin{array}{c} \ddot{x}_n \\ \ddot{u}_n \end{array} \right) = \left( \begin{array}{c} \ddot{x}_n \\ \ddot{u}_n \end{array} \right), \quad (j=1, n)$$

where $\ddot{x}_n$ is the relative acceleration of each mass of the structure as against the ground, $\ddot{u}_n$ is the absolute acceleration of each mass, and $n$, the number of the mass points that connects the driving device with the damper mass, and which forms a universal joint.

4 FUNCTION VERIFICATION TEST

A large ATMD model (damper mass weight: 3 tons) was set on a shaking table that excites the model at a magnitude converted from that to be applied on a super-high-rise building in storm. Fig. 4 shows a comparison between the experimental and theoretical values. The movement of the damper mass was in good agreement between them. Thus, it is confirmed that the operation conforms to the control law.

5 PERFORMANCE VERIFICATION TEST

5.1 Building and ATMD model

For the purpose of controlling vibration up to the third-order, the building model (8 m in height, 22 tons in total weight) constructed in a 5-layer pendulum of alternately combined steel blocks and PC steel bars. The model is suspended from the top of the frame installed on a shaking table (Fig. 5). The shaking table and the whole frame are excited by driving a hydraulic actuator. Table 1 shows the natural vibration characteristics of the building model.

![Figure 3. Multi-stage pendulum type ATMD](image)

**Figure 4. Result of function verification test**

![Figure 2. Concept of multi-stage pendulum](image)
Table 1. Vibration characteristics of the building model

<table>
<thead>
<tr>
<th>Order of vibration</th>
<th>Natural frequency (Hz)</th>
<th>Modal mass (kg)</th>
<th>The floor of maximum mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.522</td>
<td>6.65*10^4</td>
<td>5FL</td>
</tr>
<tr>
<td>2</td>
<td>1.24</td>
<td>1.26*10^4</td>
<td>3FL</td>
</tr>
<tr>
<td>3</td>
<td>1.73</td>
<td>7.89*10^4</td>
<td>4FL</td>
</tr>
</tbody>
</table>

Three ATMD models are used which have been tuned to the frequencies of the first- to the third-order vibrations of the building model, respectively. The weight of damper mass and the location of each ATMD are, as follows:

- 1st-order vibration: 50 Kg on 5FL
- 2nd-order vibration: 94 Kg on 3FL
- 3rd-order vibration: 56 Kg on 4FL

The effective weight ratio to the modal mass of the building model are all about 0.8%, respectively.

5.2 Results of experiment

An example of the results of the experiment of an uncontrolled and a controlled ATMD performed, exciting the shaking table by the scaled El Centro NS wave, is shown in Fig. 6, including the modal acceleration in each vibration order, and the displacement and the acceleration of the 3rd and 5th floors of the building model.

This figure also indicate the results of simulation analyses performed for each vibration, and also the response of 3rd and 5th floors superposed on the result of each vibration mode. Table 2 shows the maximum values of each response and the ratio of the controlled case to that of the uncontrolled case. Both values of the experiment and the analysis are almost in agreement with each other, confirming that the actual response control performance is exactly assessed theoretically.

Table 2. Results of seismic response control test

<table>
<thead>
<tr>
<th>Maximum displacement(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st mode</td>
</tr>
<tr>
<td>Uncontrolled</td>
</tr>
<tr>
<td>Controlled</td>
</tr>
<tr>
<td>Reduction ratio</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Maximum acceleration(cm/s^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st mode</td>
</tr>
<tr>
<td>Uncontrolled</td>
</tr>
<tr>
<td>Controlled</td>
</tr>
<tr>
<td>Reduction ratio</td>
</tr>
</tbody>
</table>

* absolute acceleration

6 SIMULATION ANALYSIS OF AN ACTUAL BUILDING

6.1 Specifications of building and ATMD

The building used is a high-rise building of steel construction, about 160 m in height and 70,000 tons in weight, with the vibration characteristics, as shown in Table 3. The damping ratio of the building is assumed to be 2% for all vibration order. Five ATMD's for the first- to the fifth-order vibrations are used, and installed on the pent house, as all the natural vibration modes become maximum at the top. The damper mass weight of each ATMD's are as follows:

- 1st order vibration: 150 tons
- 2nd order vibration: 100 tons
- 3rd order vibration: 120 tons
- 4th order vibration: 120 tons
- 5th order vibration: 120 tons

The total weight ratio of the ATMD's to the building is 0.9%.

Table 3. Vibration characteristics of the building of study

<table>
<thead>
<tr>
<th>Order of vibration</th>
<th>Natural frequency (Hz)</th>
<th>Modal mass (ton)</th>
<th>The floor of maximum mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.27</td>
<td>1.95*10^4</td>
<td>top</td>
</tr>
<tr>
<td>2</td>
<td>0.63</td>
<td>1.33*10^4</td>
<td>top</td>
</tr>
<tr>
<td>3</td>
<td>1.02</td>
<td>1.56*10^4</td>
<td>top</td>
</tr>
<tr>
<td>4</td>
<td>1.37</td>
<td>1.02*10^4</td>
<td>top</td>
</tr>
<tr>
<td>5</td>
<td>1.76</td>
<td>1.34*10^4</td>
<td>top</td>
</tr>
</tbody>
</table>

6.2 Results of simulation analysis

Table 4 and Fig. 7 show the responses to the input of the maximum velocity taken as 10 cm/s of the El Centro NS wave and the TAFT EW wave. The reducing ratios of the individual modal components in controlled are scattered according to the displacement, the relative acceleration and the seismic waves, and the vibration control effects are not clear.
Figure 6. Results of modal response control test

In the response of the building superposed the 1st- to 5th-order vibrations, the reduction in displacement is not so large (0.59-0.84) but the absolute acceleration is reduced to as low as...
Figure 7. Response by simulation analysis (TAFT EW 10 cm/s)

Table 4. Result of the simulation analysis for control effectiveness

<table>
<thead>
<tr>
<th>Order of vibration</th>
<th>El Centro</th>
<th>TAFT EW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncontrolled</td>
<td>Controlled</td>
</tr>
<tr>
<td>1</td>
<td>15.73</td>
<td>9.03</td>
</tr>
<tr>
<td>2</td>
<td>3.65</td>
<td>1.98</td>
</tr>
<tr>
<td>3</td>
<td>1.62</td>
<td>1.19</td>
</tr>
<tr>
<td>4</td>
<td>0.89</td>
<td>0.64</td>
</tr>
<tr>
<td>5</td>
<td>0.56</td>
<td>0.44</td>
</tr>
<tr>
<td>Σ (1-5)</td>
<td>6.31</td>
<td>3.92</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Order of vibration</th>
<th>El Centro</th>
<th>TAFT EW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncontrolled</td>
<td>Controlled</td>
</tr>
<tr>
<td>1</td>
<td>107.2</td>
<td>107.8</td>
</tr>
<tr>
<td>2</td>
<td>116.5</td>
<td>109.3</td>
</tr>
<tr>
<td>3</td>
<td>131.8</td>
<td>109.9</td>
</tr>
<tr>
<td>4</td>
<td>106.9</td>
<td>96.8</td>
</tr>
<tr>
<td>5</td>
<td>80.7</td>
<td>70.5</td>
</tr>
<tr>
<td>Σ (1-5)</td>
<td>527.7</td>
<td>511.4</td>
</tr>
</tbody>
</table>

* indicates a large vibration control effect.

7 CONCLUSIONS

For the purpose of controlling seismic response of a high-rise building, a modal vibration control method was examined for practical application. The following results were obtained:

1. The modal vibration control system has been clarified.
2. ATMD has function according to the control law and proved that it is capable of obtaining the same vibration control effect as estimation by theory.
3. From a simulation analysis of a high-rise building, it has been made clear that this vibration control method is effective for controlling seismic response and also highly practicable from the phase of its driving device and power consumption.

Even if an earthquake at 25 cm/s level, therefore, the existing motor capacity and emergency power facilities will be sufficiently within a range of practical applicability.

0.45, indicate a large vibration control effect.
At this time, the power consumption for 1-minute duration of earthquake is only 1.5-2 kwh.
This means that a little power will be required